Ruhr Economic Papers

Yiquan Gu

Imperfect Certification

#78

Ruhr Graduate School in Economics
ECON
technische universität dortmund

Ruhr Economic Papers

Published by

Ruhr-Universität Bochum (RUB), Department of Economics
Universitätsstraße 150, 44801 Bochum, Germany

Technische Universität Dortmund, Department of Economic and Social Sciences
Vogelpothsweg 87, 44227 Dortmund, Germany

Universität Duisburg-Essen, Department of Economics
Universitätsstraße 12, 45117 Essen, Germany

Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI Essen)
Hohenzollernstrasse 1/3, 45128 Essen, Germany

Ruhr Economic Papers #78

Responsible Editor: Wolfgang Leininger

ISSN 1864-4872 (online) – ISBN 978-3-86788-084-8

Ruhr Economic Papers

#78

Yiquan Gu

Imperfect Certification

Ruhr Graduate School in Economics
ECON

technische universität dortmund

Bibliografische Information der Deutschen Nationalbibliothek
Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über http://dnb.d-nb.de abrufbar.

ISSN 1864-4872 (online)
ISBN 978-3-86788-084-8

Yiquan Gu*

Imperfect Certification

Abstract
This paper proposes a model for a certification market with an imperfect testing technology. Such a technology only assures that whenever two products are tested the higher quality product is more likely to pass than the lower quality one. When only one certifier with such testing technology is present in the market, it is found that this monopoly certifier can be completely ignored in equilibrium, in contrast to the prediction of a model with perfect testing technology. A separating equilibrium is also supported in which only relatively high quality types (products) choose to pay for the certification service. It is true that in such an equilibrium having a certificate is better than not. The exact value of a certificate, however, depends both on the prior distribution of product quality and the nature of the testing technology. Welfare accounting shows that the monopolistic certifier's profit maximizing conduct can lead to under or over supply of certification service depending on model specification. Optimal certification fee is always positive and such that it makes all positive types choose to test. In the case of two competing certifiers with identical testing technologies, the intuition of Bertrand competition does not necessarily hold. Segmentation equilibrium in which higher seller types choose the more expensive certification service and not so high types choose the less expensive service can be supported. As an application, we argue that the fee differentiation between major and non-major auditing firms need not be a result of any differences in their auditing technologies.

JEL Classification: C72, D82, L15

Keywords: Asymmetric information, imperfect certification

November 2008

---

* Ruhr Graduate School in Economics and Technische Universität Dortmund. – I am deeply indebted to my supervisor Wolfgang Leininger for his encouragement and comments. I would also like to thank Carlos Alós-Ferrer, Anette Boom, Jan Heufer, Georg Kirchsteiger, Gisele Umbhauer, Tobias Wenzel as well as participants of EARIE 2008 conference in Toulouse, Spring Meeting of Young Economists in Lille, workshops in Dortmund and Konstanz for helpful comments. Financial support from the RGS is gratefully acknowledged. – All correspondence to Yiquan Gu, University of Dortmund (TU), Department of Economics and Social Science, Chair of Microeconomic Theory, Vogelpothsweg 87, 44227 Dortmund, Germany, e-mail: yiquan.gu@uni-dortmund.de.

1 Introduction

Consider a market in which sellers know more about product quality than buyers do as in Akerlof (1970). It is well understood that serious consequences including market breakdown may result from information asymmetry in this fashion. Other than building up reputation (Klein and Leffler, 1981) and providing warranty (Grossman, 1981), sellers sometimes resort to third-party intermediaries. This paper studies such markets featuring one type of pure information intermediaries known as certifiers.[1] By using a testing technology certifiers normally are able to assess the quality of tested products. After the assessment, a certifier decides whether to grant the tested product a certificate. With the additional information of a product's certification status, buyers should then know more about its quality. Examples of such certification services are numerous. Laboratories test and certify consumer products; credit rating agencies assign credit ratings to issuers of debt obligations; universities issue diploma to students who meet their graduation criteria; educational testing services carry out tests evaluating testees' scholastic aptitudes;[2] many software solution companies also run certification programs of technical expertise through which job applicants can obtain relevant credentials.[3]

In studies of certification markets, more significantly so in those with strategic certifiers, it is often assumed that a perfect testing technology is available to the certifiers. That is, they are able to know the exact quality of each tested product without a single mistake. Though this simplification is helpful to many other research topics, it is of both practical and theoretical interest to see how certifiers set prices and how markets perform when testing technologies are imperfect. Justifications for imperfectness in testing technologies are as many as the applications. Laboratories make honest mistakes in certifying consumer products; credit rating agencies only have imperfect knowledge about debt issuers' credit worthiness; there are cases that students fail to graduate because of non-productivity related factors; and luck plays a role in any expertise certification process. Yet, real life experiences indicate that those certification services are helpful in reducing information asymmetry. For example, an university degree usually is a good signal of a worker's ability although some students may have obtained their degrees just out of luck and some high ability students failed to graduate.

[1]Intermediaries who buy and sell products may also improve buyers' information on product quality. This point is studied in Biglaiser (1993) and Biglaiser and Friedman (1994).

[2]The Educational Testing Service (ETS) is, of course, one of such institutions.

[3]Currently Microsoft runs four such certification programs: Microsoft Certified Technology Specialist (MCTS), Professional Developer (MCPD), IT Professional (MCITP) and Architect (MAC). Many other software companies such as Sun, Cisco, Oracle, etc., provide their own certification service.

Many certification services are imperfect but effective in differentiating products of different qualities. This paper attempts to model such certification technologies in a general way. Our main assumption is the following: tested by such a technology, a product may or may not pass but for any two products the higher quality one has a higher probability than the lower quality one to pass. In the context of education, it amounts to say that a student may or may not graduate from a university but for any two students the one of the higher ability is more likely to succeed in earning a diploma than the other. As shown in the following, when utilized, such a testing technology is sufficient to render a certification service informative although only to a limited extent.

1.1 Main results

The deviation from perfect certification generates new results. For example, a monopoly certifier with an imperfect technology can now be completely ignored, in contrast to the prediction of a model with perfect testing technology. A certificate is informative in a separating equilibrium in which only relatively high quality types (products) choose to pay for the certification service. Though having a certificate is preferable, the exact value of a certificate depends both on the product quality distribution and the nature of the testing technology. Welfare accounting shows that the monopolistic certifier's profit maximizing conduct can lead to under or over supply of certification service depending on model specification. Optimal certification fee is always positive and such that it makes all positive types choose to test.

In the duopoly case, the intuition of Bertrand competition between two identical suppliers (of certificates) need not hold. Facing two certifiers with identical but imperfect testing technologies, higher seller types may choose the certifier who charges the higher fee and not so high types choose the other. In such a segmentation equilibrium, neither the lower fee certifier nor the higher fee one monopolizes the entire market of testing. Moreover, lowering one's certification fee does not necessarily generate a higher demand nor a higher profit. This observation suggests the possibility of positive profits for both certifiers even when their testing technologies are essentially identical. Consequently, competition need not drive the certification fee to zero which would be the case if both certifiers had perfect testing technologies (see Lizzeri 1999). Applied to the case of financial auditing services, we cannot rule out the possibility that auditors charging vastly different fees may have similar auditing abilities.

The rest of the paper is organized as follows. Section 2 reviews the related literature and section 3 sets up the model. Section 4, 5 and 6 investigate the monopoly case and section 7 the duopoly case. Section 8 concludes. All proofs are relegated to the Appendix.

2 Related literature

There are a few studies of strategic certifiers, but mostly with perfect testing technologies. Lizzeri (1999) builds up a canonical model of certifiers upon which our model is constructed. In that paper the model is used to study certifiers' strategic behavior in information revelation assuming that they are able to know the exact value of every tested product's quality. Based on a similar model, Albano and Lizzeri (2001) investigate sellers' incentive in quality provision when the possibility of certification is available and the certifier may reveal the quality information in a strategic way. Strausz (2005) studies another important aspect of certification service, namely the credibility of certifiers. Our paper on the other hand, focuses on certifier's testing technology. We propose a general representation of imperfect testing technology that only requires a few basic assumptions. By constructing our model on Lizzeri (1999)'s perfect testing model, we'll be able to do a direct comparison of respective results and highlight the implication of imperfectness in testing technologies.[4]

Imperfect testing technology is studied in some other papers of certification markets. In this strand of literature, however, certifiers do not strategically set their prices and there are normally only two possible levels of product quality, either high or low. These papers include, for example, Heinkel (1981), De and Nabar (1991), and Mason and Sterbenz (1994). Heinkel (1981) investigates sellers' incentive in improving product quality in a setup with exogenously provided imperfect tests. Mason and Sterbenz (1994) analyze how imperfect test affects market size. Compared to De and Nabar's (1991) paper, which like ours also studies the equilibria of certification markets with imperfect testing technologies, we introduce strategic certifiers and allow product quality to be drawn from a continuous interval. A shortcoming of limiting quality space to a binary set in modeling imperfect certification is that in an information-revealing separating equilibrium the testing technology becomes "perfect".

Hvide (2005) models strategic certifiers and introduces a zero-mean, normally distributed error term into testing technology. When a product is tested by this technology, a certifier observes the sum of its true quality and the realization of a white

[4]It has to be noted that in this paper we are mainly interested in testing technologies. We do not model certifier's strategic behavior in information revelation.

noise. If this reading exceeds the certifier's passing score, the tested product will be awarded a certificate. Modeled in this way, as it is in Hvide (2005), for any given passing score such a technology exhibits the property of our approach, namely, the higher the tested product quality is, the more likely it passes. This "measurement error" approach hence amounts to a special case of our modeling of imperfect testing technology.[5]

In a setting of rating agencies, Boom (2001) assumes an investment project's probability of getting a favorable rating is the same as its success probability.[6] With this rating technology, she shows that in a market with a monopolistic rating agency there can be over or under supply of rating services compared to the socially optimal level. Though differing in details, our paper shows that both market provision and socially optimal level of certification service depend on product quality distribution and the testing technology; we also establish a necessary condition for market equilibrium to be socially optimal and show that when this condition is not satisfied market either undersupplies or oversupplies certification service depending on model specification.

To explain the significant fee differentiation between major and non-major auditing firms in financial service market, Hvide (2005) argues major auditing firms adopt stricter test standards (higher passing scores in the "measurement error" approach) than non-major auditing firms. With the help of the stricter standards, major auditing firms are then able to charge higher auditing fees and make higher profits. In this paper we provide an alternative explanation. In our model, we need not assume differences in their auditing processes. Even with identical standards, i.e., identical tests, Bertrand Competition need not happen and segmentation equilibrium may be supported in which firms charge different prices.

3 The model

Following the setup of Lizzeri (1999), we analyze the market situation as a non-cooperative game with incomplete information.

[5]Note that the reading gives the expected quality of the tested product. The certifiers have incentive to reveal more information than just the certificate. For instance, revealing the reading itself can attract testees. In our current model, however, this information is not available to the certifiers.

[6]It will become clear in the following that this is also a special case of our modeling of imperfect testing technology, namely $G(t) = t$. See Equation (1) in Section 3.

3.1 Players

We have four players in the model, one seller, one certifier and two buyers.
The seller wants to sell a product to the buyers. The product has a value equal to its quality t (type) to both of the buyers but is worth nothing to the seller and the certifier. The type t is originally only known to the seller; the buyers and the certifier, however, know the prior distribution of t represented by cumulative distribution function, $F(t)$. $F(t)$ is assumed to be continuous, differentiable and strictly increasing on interval $[a, b]$, where $a < 0 < b$.[7] The associated density function is denoted $f(t)$. The seller has the possibility to get the product tested by the certifier.

The certifier has a testing technology. When it is used to test the product, it prints out a certificate (C) with probability

$$\Pr(C \mid t) = G(t), \tag{1}$$

conditional on t. $G(t)$ is also assumed to be continuous, differentiable and strictly increasing on $[a, b]$ with first derivative denoted $g(t)$. Tested by this technology, the higher a product's quality is the higher its probability of receiving a certificate will be. Naturally the probability of no certificate (NC) is $\Pr(NC \mid t) = 1 - G(t)$. This setup requires function $G(t)$ to be bounded below by 0 and above by 1. For convenience, we assume $G(a) = 0$ and $G(b) = 1$, i.e., it is not possible for the lowest type to pass the test while the highest type always passes when tested.[8] It is also assumed that the certifier does not manipulate the test result produced by the technology. The certifier can charge a certification fee P for the test and the cost associated with testing is normalized to zero.

Both buyers observe whether a product possesses a certificate or not and bid simultaneously based on their beliefs. They, however, cannot distinguish the event that the product was not tested from the event that the product failed the test. That is, they observe if a product has a certificate, $\theta : \theta \in \{C, NC\}$, but not what the seller did.

3.2 Timing

Stage 1 The certifier announces its certification fee, P, for the test.

Stage 2 At the beginning, the seller learns his type t (chosen by nature according

[7] When product quality is negative, consumption of such goods harms the buyers.
[8] This assumption does not change our results qualitatively.

to $F(\cdot)$) and the announced certification fee, P; the seller then decides whether or not to get the product tested by paying the certifier the certification fee.

Stage 3 If the seller chooses to test, then the certifier employs the testing technology and the seller receives a certificate with probability $G(t)$, receives no certificate with probability $1 - G(t)$.

Stage 4 Both buyers observe P and if the product has a certificate or not.

Stage 5 Buyers bid independently and simultaneously for the product. The product is sold to the buyer who bids higher than the other at the price of the winning bid. Buyers get the product equally likely in case of a tie. When both bids are zero, the product is not sold.

3.3 Strategies

The certifier's strategy is simply the choice of certification fee, $P \in \mathbb{R}_+$.

The seller's strategy specifies his decision for all combinations of own quality type and certification fee level. Namely, it is a function $\rho(P, t)$, from $\mathbb{R}_+ \times [a, b]$ to $\{TS, NTS\}$, that maps the vector (P, t) into a set of two elements, *to test* or *not to test*.

A strategy for a buyer is a function $\beta(P, \theta)$, from $\mathbb{R}_+ \times \{C, NC\}$ to $\mathbb{R}_+$, that maps the announced certification fee and the product's certification status to a bid for that product. Buyers' beliefs are denoted by $\mu(t \mid C, P)$ for a certified product and $\mu(t \mid NC, P)$ for a non-certified product. Since buyers have identical information, when beliefs are Bayesian updatable they are identical. Note that competition will make them both bid up to their common belief. Therefore, no subscripts are used for individual buyers.

3.4 Payoffs

All players are assumed to be risk neutral. Hence, they maximize their payoffs in expected terms.

A buyer's payoff function, in the following three types of outcomes, reads

$$U(t, \beta) = \begin{cases} t - \beta(P, NC) & \text{when the buyer gets a non-certified product,} \\ t - \beta(P, C) & \text{when the buyer gets a certified product,} \\ 0 & \text{when the buyer does not get the product.} \end{cases}$$

The seller receives buyers' bids for a non-certified product when the product is not tested. If the seller chooses to test, he has a probability of $G(t)$ getting a certificate and receiving buyers' bids for a certified product. In other cases $(1 - G(t))$, he does not get the certificate and receives bids for a non-certified product. Taking the certification fee into account, the seller's payoff is

$$V(\rho, t, P, \beta) = \begin{cases} \beta(P, NC) & \text{not to test,} \\ [1 - G(t)]\beta(P, NC) + G(t)\beta(P, C) - P & \text{to test.} \end{cases}$$

The certifier's payoff is the product of the certification fee and the demand for the certification service, i.e., $\Pi(P, \rho) = P \cdot \Pr(\text{the event that the seller tests})$, or

$$\Pi(P, \rho) = P \cdot \int_T dF(t), \quad \text{where } T = \{t \mid \rho(P, t) = TS\}.$$

3.5 Equilibrium notion

The equilibrium notion employed in this paper is Perfect Bayesian Equilibrium. As we argued before competition between the buyers will force them bid identically up to their common belief, we have

$$\beta^*(P, \theta) = \begin{cases} \mu(t \mid \theta, P) & \text{if } \mu(t \mid \theta, P) \geq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Bayesian perfectness requires their expectations should be consistent with equilibrium outcome. Hence, for both buyers, when their beliefs are Bayesian updatable,

$$\mu(t \mid \theta, P) = E(t \mid \theta, P), \forall \theta \in \{C, NC\}, \forall P \in \mathbb{R}_+, \tag{3}$$

where E is the mathematical expectation operator. We also need that the seller not to have incentive in deviating from equilibrium strategy after knowing his quality type. The seller's strategy choice should be, for each type, his best response to the announced certification fee and buyers' biding strategies. Therefore, for any given combination of certification fee P and buyers bidding function β, we need

$$V(\rho^*, t, P, \beta) \geq V(\rho', t, P, \beta), \forall t \in [a, b], \text{ where } \rho' = \{TS, NTS\} \setminus \{\rho^*\}. \tag{4}$$

The certifier's fee should then be chosen to maximize his expected payoff,

$$P^* = \arg\max \left\{ P \cdot \int_{\{t \mid \rho(P,t) = TS\}} dF(t) \right\}. \tag{5}$$

Formally we define the equilibrium notion as the following.

Definition 1. *A strategy profile* $\{P^*, \rho^*(P,t), \beta^*(P,\theta)\}$ *and buyers' belief* $\mu(t \mid \theta, P)$, *constitute a* Perfect Bayesian Equilibrium *of the game, if and only if conditions (2), (3), (4) and (5) hold.*

3.6 Discussion

The testing technology (1) essentially only requires whenever two products get tested, the product that is of the higher quality has a higher probability than the other to pass. It doesn't specify any functional form.

4 Monopoly: bypassing

In the situation depicted in section 3, without certification service information asymmetry leads to market breakdown when the prior expectation of product quality is below zero, $E(t) < 0$. When $E(t) > 0$, however, the product is traded with probability one. From social welfare point of view, there is over-trading since there are cases trading results in a loss to the society.[9]

With perfect testing technology, for example, as in Lizzeri (1999), it is found that a monopoly certifier will only certify non-negative seller types; hence, only those certified types will be traded in equilibrium. This is an efficient outcome since all positive types are traded while none of the negative types will be. It is also shown that the mere existence of this perfect testing possibility grants the certifier the power to take away the entire market surplus leaving the seller a payoff of zero. Consequently, the monopolist's interest is coincident with social welfare.[10] This explains why the monopolist's profit maximizing conduct is also socially optimal.

When the testing technology is imperfect, however, the game changes dramatically with respect to both the monopoly certifier's power and the market outcome. Although with perfect testing technology the certifier can always guarantee itself the demand for certification service by offering to the seller that it will reveal the exact quality type of a tested product, when testing technology is imperfect the certifier may even be completely bypassed.

Proposition 1. *Any of the following strategy profiles, such that,*

[9] The lowest type a is assumed to be less than 0. Therefore, some negative types will be traded. When $a \geq 0$, full trading is efficient.

[10] Note that buyers always end up with zero payoff because they engage in Bertrand bidding competition.

1. *for all levels of the certification fee, all seller types choose not to test,*

2. *for all levels of certification fee, buyers bid for a non-certified product either the ex ante expected quality when it is positive or zero when non-positive, bid for a certified product either the belief for a certified product when it is positive or zero when non-positive,*

3. *the certifier charges any non-negative fee,*

4. *and the buyers' belief being that the quality of a certified product is* no higher *than the ex ante expected quality,*

constitutes an equilibrium. That is,

$$
\begin{aligned}
P^* &= P \in \mathbb{R}_+ \\
\rho^*(P,t) &= NTS,\ \forall t \in [a,b],\ \forall P \in \mathbb{R}_+ \\
\beta^*(P,NC) &= \max\{E(t),0\},\ \forall P \in \mathbb{R}_+ \\
\beta^*(P,C) &= \max\{\mu(t \mid C,P),0\},\ \forall P \in \mathbb{R}_+ \\
\mu(t \mid NC,P) &= E(t),\ \forall P \in \mathbb{R}_+ \\
\mu(t \mid C,P) &= \mu \in (a,E(t)],\ \forall P \in \mathbb{R}_+.
\end{aligned}
$$

Proof. See Appendix. □

One direct implication of Proposition 1 is the following remark.

Remark 1. *When testing technology is imperfect, it's possible for the seller to bypass the monopoly certifier.*

The main underlying reason for this result is the strictly positive probability that lower types may pass the test. This leaves the buyers the scope of forming the beliefs that are required for the equilibria in Proposition 1. In the perfect testing technology case, such beliefs cannot be supported; consequently, bypassing is not possible.

This difference between perfect and imperfect testing technology is not only of theoretical interest but also of practical importance. Consider "a" seller in the literal sense. Before nature's draw, there are collective interests among seller types. We can think of a monopoly seller or an industry in aggregation. From this perspective, when $E(t) \leq 0$, it is not in the seller's interest to bypass the certification service because there would then be no trading. When $E(t) > 0$, however, the seller makes maximal profit $E(t)$ without the certification service. Given that the testing technology is imperfect, it's at least possible for the seller to bypass the certifier.

We are aware that buyers' belief in Proposition 1 seems irregular. It essentially says that a certificate does not serve a signal of high quality even though buyers know that when tested higher types are more likely to obtain a certificate than lower types. First of all, when the certification service is not used, the beliefs stated in Proposition 1 are not exactly irrational. Second, the reason we present Proposition 1 in this paper is to show the difference in feasible equilibria when testing technology is perfect versus when it is imperfect. Although we can put more restrictions on buyers' beliefs by adopting other equilibrium notions, this possibility result signifies the decrease of certifier's power caused by imperfectness in testing technology.

5 Monopoly: separating equilibrium

In the following we search out those equilibria in which there is a positive measure of seller types paying for the test. This is of particularly importance when $E(t) \leq 0$ since in this case the market would break down if there were no certification service available. To focus on this issue and to simplify the analysis, we assume the prior expected product quality to be negative.[11]

Assumption. *The prior expected product quality is less than zero, i.e., $E(t) \leq 0$.*

As an example, consider the labor market for IT specialists. If there are no other signals available and the average potential worker does not qualify, then a certificate for such expertise would be crucial both to job applicants and to employers. Yet, we need to find out for a given imperfect testing technology what a certificate could mean and how the market for the certification service performs.

We solve the game by investigating first the subgames induced by different certification fees. Not surprisingly, when the certification fee is set too high, it does not pay for the seller to get the product tested. The following proposition states.

Proposition 2. *In subgames induced by the certifier's fee setting P, it is true that:*

1. *if the certifier charges a fee higher than the highest type, then any strategy profile such that all seller types choosing not to test, buyers bidding zero for a non-certified, bidding for a certified product the belief for such a product when it is positive or zero when non-positive, and buyers' beliefs for a certified product*

[11] Again, this assumption does not change the result on separating equilibrium qualitatively.

being no higher than b, constitutes an equilibrium in the subgame induced by P; that is, in subgames where $P > b$,

$$\begin{aligned}
\rho^*(t \mid P > b) &= NTS,\ \forall t \in [a, b] \\
\beta^*(NC \mid P > b) &= 0 \\
\beta^*(C \mid P > b) &= \max\{\mu(t \mid C, P > b), 0\} \\
\mu(t \mid NC, P > b) &= E(t) \\
\mu(t \mid C, P > b) &= \mu \in (a, b];
\end{aligned}$$

2. *if the certifier charges a fee equal to the highest type, there is only one equilibrium in the subgame other than bypassing, in which only the highest seller type chooses to test and buyers bid the value of the highest type for a certified product, zero for a non-certified product and buyers' beliefs being the ex ante expectation for a non-certified product and b for a certified product; that is, in the subgame where $P = b$,*

$$\begin{aligned}
&\rho^*(t = b \mid P = b) = TS \text{ and } \rho^*(t \mid P = b) = NTS,\ \forall t \in [a, b) \\
&\beta^*(C \mid P = b) = b \text{ and } \beta^*(NC \mid P = b) = 0 \\
&\mu(C \mid P = b) = b \text{ and } \mu(NC \mid P = b) = E(t).
\end{aligned}$$

Proof. See the Appendix. □

This result can be interpreted as the following. When the price for test is too high, there is intuitively not much demand for it. As a preparation for solving the whole game, we establish the following corollary with respect to the certifier's profit. The result is immediate from Proposition 2.

Corollary 1. *The certifier makes zero profit by setting $P \geq b$, or $P = 0$.*

5.1 Separating equilibrium

We now turn to the more interesting subgames induced by intermediate certification fees. Before proceeding to the result, the following definition is useful in simplifying notation.

Definition 2. *Denote*

$$\Omega(m, n) = \frac{\int_m^n tG(t)dF(t)}{\int_m^n G(t)dF(t)} \text{ for } a \leq m < n \leq b.$$

Function $\Omega(m,n)$ gives type expectation of a product with a certificate if and only if all types from the interval $(m,n]$ (or (m,n), $[m,n)$, $[m,n]$) choose to test.

Further we introduce the following tie-breaking rule.

Assumption. *When a seller type is indifferent between to test and not to test, we assume he chooses to test.*

Proposition 3 (**Separating**). *In each subgame induced by $0 < P < b$, there is a* unique *subgame equilibrium other than bypassing the certifier completely. Moreover, the set of seller types, which strictly prefer testing, is of the form $(x,b]$ and type x is indifferent between testing and not testing, where x solves $G(x)\Omega(x,b) = P$. Buyers bid $\beta(P,C) = \Omega(x,b)$ for a certified product and $\beta(P,NC) = 0$ for a non-certified product. That is,*

$$\text{the seller's strategies:} \begin{cases} \rho^*(t \mid P) = TS, \forall t \in [x,b], \\ \rho^*(t \mid P) = NTS, \forall t \in [a,x), \end{cases}$$

$$\text{buyer's strategies:} \begin{cases} \beta^*(C \mid P) = \Omega(x,b), \\ \beta^*(NC \mid P) = 0, \end{cases}$$

$$\text{and buyer's expectation:} \begin{cases} \mu(t,C \mid P) = \Omega(x,b), \\ \mu(t,NC \mid P) < 0. \end{cases}$$

constitute the equilibrium in the subgame induced by $P \in (0,b)$.

Proof. See Appendix. □

This result states that for each positive certification fee that is less than the highest quality type, there is a unique subgame equilibrium in which those relatively high types choose to test by paying the certification fee while relatively low types choose not to.[12] Since only those higher types choose to test, after taking the imperfectness in the testing technology into account, buyers still bid more for a product that has a certificate. This bidding difference justifies the fee that high seller types pay for the test. The probability of a type passing the test is critical to the type's willingness to pay. Even high types have a certain probability failing a test. But the nature of the testing technology ensures that in expected terms higher types are better off by paying for the test while lower types are better off by choosing not to test.

For ease of exposition and motivated by the proof of Proposition 3 in Appendix A.3, we introduce the next definition.

[12]Note that bypassing is still possible but in this section we focus on the cases when the certification service is used.

Definition 3. *Denote* $\kappa(P) = x$ *such that* $G(x)\Omega(x, b) = P$ *where* $0 < P < b$. *For a given* P, $\kappa(P)$ *gives the unique type who is indifferent between to test and not to test in the equilibrium identified in Proposition 3.*

Proposition 3 states that in equilibrium all types higher than $\kappa(P)$ prefer paying for the test and playing the certification lottery over not to test. The difference for any type t between these two options can be represented by function $\Gamma(t)$, [13]

$$\Gamma(t) = G(t)\Omega\left(\kappa(P), b\right) - P.$$

While $\Gamma(\kappa(P)) = 0$,

$$\begin{aligned}\Gamma(t \mid t > \kappa(P)) &= G(t \mid t > \kappa(P))\Omega(\kappa(P), b) - P \\ &> G(\kappa(P))\Omega(\kappa(P), b) - P = \Gamma(\kappa(P)) = 0.\end{aligned}$$

This explains that the set of the seller types who pay for the test is always connected. Whenever a certain type finds it worthwhile paying for the test, any type above would find it so as well. For the same fee, a higher type gets a better lottery than a lower type. On the other hand, this guarantees the existence of the separating equilibrium by preventing lower types from applying the test. A certification service provides a device by which relatively high seller types can separate themselves from relatively low types. They also need to pool together to induce buyers to form a quality expectation that is positive. In the case of perfect testing technology, however, pooling is not necessarily needed since a certifier can certify a seller's true type. From the perspective of the seller, we have the following remark.

Remark 2. 1. *When there is no testing technology, seller types' interests are all pooled together without choice;*

2. *when there is a perfect testing technology, an individual seller type has the opportunity to perfectly identify itself unilaterally;*

3. *when there is an imperfect testing technology, seller types depend on each other to a certain degree.*

Recall that in the case of perfect testing technology the certifier is able to make all tested types indifferent between testing and not testing and take away the entire market surplus. The certifier chooses a minimum quality standard, say $\kappa' = 0$, and charges $P' = E(t \mid t \geq 0)$ for the test. It turns out that types above 0 are all indifferent between testing and not testing. Note that even though each seller

[13] See also Equation (17) in A.3.

type is left with zero surplus, this is the unique equilibrium when perfect testing technology is available in the monopoly certifier case.[14]

Suppose that a certifier with an imperfect technology wants to employ such a strategy. The certifier claims that all types higher than κ' will pass the test while all types below will not. Since the certifier is unable to make sure that every low type does not pass and every high type passes, the expected quality of a certified product is not assured to be at $E(t \mid t \geq \kappa')$. Therefore, buyers will not bid as much as $E(t \mid t \geq \kappa')$ and neither will the seller types pay as much for the test. So it is clear that when testing technology is imperfect, a monopoly certifier cannot take away the entire market surplus. Indeed most of the testing seller types derive strictly positive payoff in a separating equilibrium. The following remark summaries.

Remark 3. *When imperfect certification service is used in equilibrium, the monopoly certifier's power in taking up market surplus against the seller is limited compared to the case in which a perfect testing technology is available.*

5.2 Value of a certificate

It is worth noting how buyers form their expectations towards a certified product. Without equilibrium analysis a certificate does not give a definitive meaning in terms of product quality. Proposition 3, however, says only types higher than or equal to $\kappa(P)$ go to the certifier in equilibrium at the cost of a positive fee. By successfully attracting a positive measure of seller types, the certification service practically blocks away types lower than $\kappa(P)$ in the original population and filters the remaining into a new population of those with a certificate. The new population is distributed on $[\kappa(P), b]$ with density $\frac{G(t)f(t)}{\int_{\kappa(P)}^{b} G(t)dF(t)}$ where $f(t)$ is the density function of the original distribution. Thus buyers form their expectations of a certified product as

$$\frac{\int_{\kappa(P)}^{b} tG(t)dF(t)}{\int_{\kappa(P)}^{b} G(t)dF(t)} = \Omega\left(\kappa(P), b\right).$$

First, this observation further emphasizes the idea that buyers are only able to attribute a value to a certificate for equilibrium outcomes but not for off-equilibrium incidences. Second, in an equilibrium of the form stated in Proposition 3, the value of a certificate directly depends both on the population of the seller types who choose to test and on the nature of the testing technology. This implies that to be able

[14] For a formal reasoning, the reader is referred to Lizzeri (1999). This situation resembles the observation that in the unique subgame perfect equilibrium of a 2-player *Ultimatum game*, the proposer gets all and the other gets nothing even though she can reject.

to assess a certificate, a buyer first needs to understand what types of products are likely to choose to test and how difficult it is to pass such a test. Third, note that the value of the certificate $\Omega(\kappa(P), b)$ for a given type distribution and a given testing technology is a function of the certification fee P. Hence, when the certification fee changes, the value of the certificate also changes.

Compared to the case in which a perfect testing technology is available, the dependence on the test takers' population is crucial in imperfect testing. In the former case, a certifier can always identify the type when a product is tested. The meaning of such a test can be made independent of the seller's type distribution. In our imperfect testing case, the certifier has to rely on a positive measure of seller types to make the certificate meaningful. This dependence is responsible for the limited ability of the certifier both in ensuring demand for the test (Remark 1) and in taking up market surplus against the seller (Remark 3).

5.3 Free certification

There is one subgame yet to be discussed, the one induced by $P = 0$. It is of additional importance because we are also interested in the case when tests are provided for free to the seller, for instance, through a public policy program.

Proposition 4. *In the subgame induced by $P = 0$, buyers make positive bids for a certified product if and only if $\Omega(a, b) > 0$.*

Proof. See Appendix. □

Free certification produces two contrasting outcomes with respect to trading probabilities. It gives the maximum probability of $\int_a^b G(t)dF(t)$ when $\Omega(a, b) > 0$ since all seller types have already chosen to test and there is no other way to increase the probability of having a certified product. If $\Omega(a, b) < 0$, the product will for sure not be traded. However, neither of these two is necessarily desirable compared to the socially optimal level discussed in subsection 6.3 below.

6 Monopoly: market performance

6.1 Equilibrium of the game

After having investigated all subgames, we are now ready to solve the game in its entirety. At the first stage, the certifier chooses the certification fee for the test, $P \in \mathbb{R}_+$. Since we put aside bypassing equilibria, the next result follows.

Proposition 5. *In equilibrium, a monopoly certifier sets* P *to maximize profit* $\Pi(P) = P[1 - F(\kappa(P))]$. *That is,*

$$P^* = \arg\max_{P \in (0,b)} P[1 - F(\kappa(P))]. \tag{6}$$

It can also be represented as to choose the indifferent type x*, such that it maximizes the certifier's profit. Formally,*

$$x^* = \arg\max_{x \in (a,b)} G(x)\Omega(x,b)[1 - F(x)]. \tag{7}$$

Proof. See Appendix. □

The monopoly certifier's trade-off resembles that of many other monopoly producers who face a downward sloping demand curve. Demand decreases when the fee (price) increases. The difference, however, is that while the negative slope of the demand function of consumer products is normally a result of consumers' descending willingness to pay for the unit-by-unit-identical product, here the value of the certificate that is being offered is actually evolving along with participating seller types. The value of a certificate deteriorates in the participation of lower seller types. When a certifier lowers its certification fee, it lowers the value of its certificate too.

6.2 An example

To have a better understanding of the equilibrium outcome, we present a fully specified numerical example.

Example 1. Suppose seller types are uniformly distributed on the interval $[-2, 1]$, that is, $F(t) = \frac{t+2}{3}$. The testing technology $G(t)$ follows a power function, $G(t) = \left(\frac{t+2}{3}\right)^2$ on $[-2, 1]$. Under this model specification, as stated in Equation (22), the monopoly certifier solves the following problem,

$$\max_{-2<x<1} \left(1 - \frac{x+2}{3}\right) \left(\frac{x+2}{3}\right)^2 \frac{\int_x^1 t \frac{1}{3} \left(\frac{t+2}{3}\right)^2 dt}{\int_x^1 \frac{1}{3} \left(\frac{t+2}{3}\right)^2 dt}.$$

The solution to this problem is $x = 0.3154$. This means the fee the certifier charges is

$$P = G(x)\Omega(x,1) = \left(\frac{0.3154 + 2}{3}\right)^2 \frac{\int_{0.3154}^1 t \frac{1}{3} \left(\frac{t+2}{3}\right)^2 dt}{\int_{0.3154}^1 \frac{1}{3} \left(\frac{t+2}{3}\right)^2 dt} = 0.4092.$$

It turns out that seller types in $[0.3154, 1]$ choose to test while the rest choose not to. Buyers bid

$$\beta(C \mid P = 0.4092) = \Omega(0.3154, 1) = \frac{\int_{0.3154}^{1} t \frac{1}{3} \left(\frac{t+2}{3}\right)^2 dt}{\int_{0.3154}^{1} \frac{1}{3} \left(\frac{t+2}{3}\right)^2 dt} = 0.6870$$

for a certified product and 0 for a non-certified. The expected profit the certifier makes is

$$\Pi(0.4092) = P(1 - F(x))) = 0.4092 \int_{0.3154}^{1} \frac{1}{3} dt = 0.0934,$$

which is less than the amount it would have made,

$$\Pi' = \int_{0}^{1} \frac{1}{3} t dt = 0.1667,$$

if a perfect testing technology were available.[15] This point can indeed be generalized.

Remark 4. *A monopoly certifier with an imperfect testing technology makes a smaller profit than a monopoly certifier with a perfect testing technology under otherwise identical circumstances.*

The explanation is the following. With perfect testing technology, a certifier is able to take away the entire trading surplus in the market leaving nothing to the seller. Consequently, the certifier will seek to reach the highest possible market surplus. In contrast, with imperfect testing technology, the surplus generated in the product market is shared between the certifier and the seller.[16] From the perspective of the certifier, with perfect testing technology it achieves first best outcome; while in the case of imperfect testing technology, not only the certifier's share is less than 1 but also the total level of generated surplus can be well below maximum.

An interesting question concerns the type distribution of a certified product in equilibrium. The type distribution of a certified product has the support of $[0.3154, 1]$. Its density function is a transformation of part of the original density function via the testing technology. Denote $f^c(t)$ the new probability density function of a certified product; $f^c(t)$ can be written as the following.

$$f^c(t) = \frac{G(t)f(t)}{\int_x^b G(t)dF(t)} = \frac{\frac{1}{3}\left(\frac{t+2}{3}\right)^2}{\int_{0.3154}^{1} \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt} = 0.20566(t+2)^2.$$

Figure 1 gives a graphical representation of the original distribution, the testing technology and the transformed type distribution of a certified product.

[15] The profit under perfect testing technology is found when the certifier only certifies types above zero and charges $E(t \mid t \geq 0)$.

[16] Note that the set of seller types who strictly prefer paying for the test obtain positive expected payoffs. See subsection 5.1.

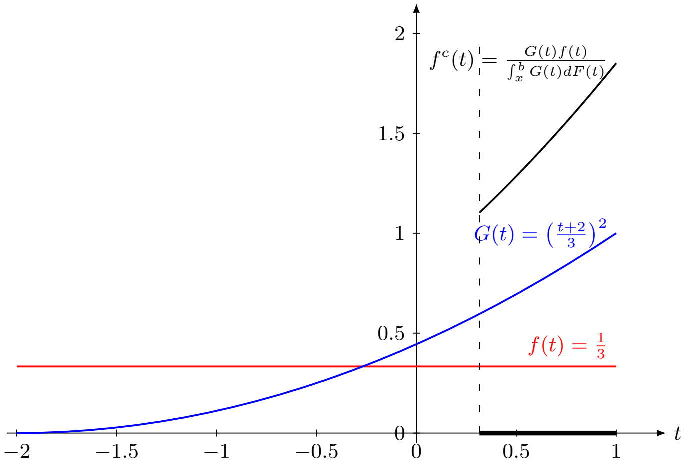

Figure 1: A case of an uniformly distributed type population ($f(t) = \frac{1}{3}$) and a power testing technology ($G(t) = \left(\frac{t+2}{3}\right)^2$); types to the right of the dashed line, $[0.3154, 1]$, pay for the test in equilibrium; the curve in the upper right represents the type density function of a certified product.

6.3 Welfare

An important issue in markets with asymmetric information is market performance in terms of social welfare. The next result gives the condition for welfare maximization.

Proposition 6. *In the separating equilibrium of subgames induced by* $0 < P < b$, *market surplus is represented by* $\int_{\kappa(P)}^{b} tG(t)dF(t)$. *It is maximized when* $\kappa(P^{**}) = 0$, *i.e., when type* 0 *is made indifferent between testing and not testing. Therefore, the welfare maximizing certification fee is* $P^{**} = G(0)\Omega(0, b)$.

Proof. See Appendix. □

The intuition is the following. For a product to be traded in a separating equilibrium, it has to obtain a certificate. Note that trading of positive types increases while trading of negative types decreases social welfare. So the ideal outcome is that all positive types obtain a certificate while all negative types are uncertified. But given the nature of the imperfect testing technology, this is not achievable. Also note that once a give type decides to test, the probability of getting a certificate is governed by the testing technology. The second best is then to set the certification fee to a level such that it is low enough for all positive types to pay for the test while it is still high enough to discourage negative types from using the test. Hence, the optimal certification fee should make type 0 the indifferent type. Note that $G(0)\Omega(0,b)$ is strictly positive, we emphasize the result as a corollary to Proposition 6.

Corollary 2. *The social welfare maximizing certification fee P^{**} is strictly positive.*

Apparently, free certification under imperfect testing technology is not an optimal policy. Because of the inability of the testing technology in blocking negative types from getting a certificate, we need a positive certification fee to function as a self-selection mechanism.

We can also see the difference between social welfare and the certifier's profit in a comparison of the following two expressions.

$$\begin{aligned}
\text{Social welfare} \quad &: \quad \int_{\kappa(P)}^{b} tG(t)dF(t) \\
\text{Certifier's profit} \quad &: \quad P[1-F(\kappa(P))] \\
&= \quad [1-F(\kappa(P))]G(\kappa(P))\Omega(\kappa(P),b) \\
&= \quad \left\{ \frac{\int_{\kappa(P)}^{b} G(\kappa(P))dF(t)}{\int_{\kappa(P)}^{b} G(t)dF(t)} \right\} \int_{\kappa(P)}^{b} tG(t)dF(t). \qquad (8)
\end{aligned}$$

They differ by the part in the curly brackets in equation (8). Note that $G(t \mid t > \kappa(P)) > G(\kappa(P))$, the part in the curly brackets is less than 1. Hence, not all of the total market surplus is taken by the certifier. Part of it is shared by the seller. But for a certifier equipped with a perfect testing technology, $G(t \mid t \geq \kappa')$ could be set to 1 and $G(t \mid t < \kappa')$ to 0. The part in the curly brackets hence vanishes and the monopoly certifier's profit is equal to the entire social surplus. When such a certifier maximizes its profit it as well maximizes social welfare. This comparison tells us that the inability of taking up all market surplus leads to a lower level of social welfare, i.e., inefficiency.

Boom (2001) shows that in a market with a monopolistic rating agency there can be over or under supply of rating services in equilibrium compared to socially optimal level. In the next proposition we establish the necessary condition for profit maximizing conduct to be welfare maximizing. When this condition does not hold, market either oversupplies or undersupplies certification service depending on model specification.

Proposition 7. *A necessary condition for the profit maximizing certifier to set the welfare maximizing certification fee $P^{**} = G(0)\Omega(0,b)$ is,*

$$\frac{f(0)}{1-F(0)} = \frac{g(0)}{G(0)} + \frac{G(0)f(0)}{\int_0^b G(t)f(t)dt}. \qquad (9)$$

Moreover, when $P[1 - F(\kappa(P))]$ is concave for $P \in (0, b)$, there is oversupply (undersupply) of certification service if

$$\frac{f(0)}{1 - F(0)} > (<) \frac{g(0)}{G(0)} + \frac{G(0)f(0)}{\int_0^b G(t)f(t)dt}. \tag{10}$$

Proof. See the Appendix. □

This necessary condition requires the Hazard rate of the original type distribution when evaluated at type 0 has to be equal to the sum of a value related to the testing technology ($G(t)$) and certified product's density at type 0. When condition (9) doesn't hold, socially optimal certification fee will not be achieved by profit maximizing monopoly certifier.

Further, with additional information of certifier's profit function concavity, we can identify conditions for over and under supply of certification service. When

$$\frac{f(0)}{1 - F(0)} < \frac{g(0)}{G(0)} + \frac{G(0)f(0)}{\int_0^b G(t)f(t)dt}, \tag{11}$$

the first derivative of profit is positive at type 0. Therefore, the certifier will have an incentive to raise the certification fee from the socially optimal level $P^{**} = G(0)\Omega(0, b)$ and the indifferent type will be strictly higher than type 0. Because there are strictly positive types find the certification fee too high and do not apply the test, there is under utilization of the certification service. Social welfare could be improved by lowering the certification fee. Similarly, when the reverse of condition (11) holds, the indifferent type will be strictly lower than 0 and some negative types will be traded. Hence there will be oversupply of certification service.

6.4 Example 1 continued

In the above numerical example, the indifferent type is 0.3154. Social welfare would be higher if types in $[0, 0.3154]$ applied the test. Hence, the certification fee 0.4092 is too high. By lowering the fee, more seller types will use the certification service and the product will have a higher probability to be traded. To be exact, the socially optimal fee is

$$P^{**} = G(0)\Omega(0, 1) = \left(\frac{2}{3}\right)^2 \frac{\int_0^1 t \frac{1}{3} \left(\frac{t+2}{3}\right)^2 dt}{\int_0^1 \frac{1}{3} \left(\frac{t+2}{3}\right)^2 dt} = 0.2515.$$

So that types in $[0, 1]$ choose to test while types in $[-2, 0]$ choose not to.

	Social welfare $\int_x^b tG(t)dF(t)$	Trading prob. $\int_x^b G(t)dF(t)$
Perfect testing	$\int_0^1 \frac{1}{3}tdt = \frac{1}{6} = 0.1667$	$\int_0^1 \frac{1}{3}dt = \frac{1}{3} = 0.3333$
Imperfect (Social)	$\int_0^1 \frac{1}{3}t\left(\frac{t+2}{3}\right)^2 dt = 0.1327$	$\int_0^1 \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt = 0.2346$
Imperfect (Profit)	$\int_{0.3154}^1 \frac{1}{3}t\left(\frac{t+2}{3}\right)^2 dt = 0.1237$	$\int_{0.3154}^1 \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt = 0.1801$

Table 1: Welfare under (im)perfect testing in example 1.

In Table 1 we compare social welfare and the product's trading probability in example 1 under three different scenarios: perfect testing technology, imperfect testing technology used to maximize social welfare and imperfect testing technology used to maximize the certifier's profit. According to the original type distribution, the mean of all positive types is 1/6 which is the entire surplus that can be generated from trading. Since with perfect testing technology, all positive types get a certificate, the probability of trading is 1/3. With imperfect testing technology, under welfare maximization all positive types should at least be tested. For the given imperfect testing technology $G(t) = \left(\frac{t+2}{3}\right)^2$, the probability that the product gets a certificate is only $\int_0^1 \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt = 0.2346$. The surplus generated is $\int_0^1 \frac{1}{3}t\left(\frac{t+2}{3}\right)^2 dt = 0.1327$. When the certifier maximizes profit, certification fee is higher and less types apply the test. The probability that the product gets a certificate now is $\int_{0.3154}^1 \frac{1}{3}\left(\frac{t+2}{3}\right)^2 dt = 0.1801$. The generated surplus is $\int_{0.3154}^1 \frac{1}{3}t\left(\frac{t+2}{3}\right)^2 dt = 0.1237$ which is less than the optimal level. So the efficiency of the market is reduced both by the imperfectness in testing technology and by the certifier's profit maximizing conduct.[17]

Generally, profit maximizing monopoly certifier does not set the certification fee to the socially optimal level. But even when the service is run by the public sector and the certification fee is optimally set such that all positive types apply the test and all negative types do not, inefficiency remains because some positive types will fail the test and will not be traded. However, compared to the market breakdown outcome without certification service, there at least will be some trading in a separating equilibrium. The next remark summarizes.

Remark 5. *An imperfect testing technology solves the asymmetric information problem imperfectly. The market is not as efficient as it is with perfect testing technology but it does improve buyers' information on product quality in equilibrium.*

[17]Note that in perfect testing case, the certifier's profit is coincident with social welfare. One may argue the efficiency loss is entirely caused by testing technology imperfectness.

7 Duopoly

In this section we investigate a market with two certifiers. The main purpose of this section is to provide a new perspective for the study of competing certifiers. To this aim, we are interested in market behavior with given certification fees. The seller now can choose which certifier to go for a test or not to test at all. We do not consider the possibility that a seller type applies both tests. Hence, the seller's decision ρ maps $\mathbb{R}_+^2 \times [a,b]$ to $\{TS_1, TS_2, NTS\}$. TS_1 is to test at Certifier 1 and TS_2 is to test at Certifier 2. When a seller type fails a test, the type is pooled with those who do not test. For buyers, β is now a function from $\mathbb{R}_+^2 \times \{C_1, C_2, NC\}$ to $\mathbb{R}_+$, which specifies their bids for a product conditional on which certificate it has or none at all. Here, C_1 stands for a certificate from Certifier 1 and C_2 a certificate from Certifier 2. As a tie-breaking rule, in the analysis of equilibrium strategies, when a seller type is indifferent between two options, he makes the same decision as the type slightly higher than he is.

7.1 Segmentation in identical tests

We consider a case in which these two certifiers employ identical testing technologies. Formally, we have $G_1(t) = G_2(t) = G(t)$ for all $t \in [a,b]$. This setup is to say these two certifiers are providing identical tests and they are identical except that they charge different certification fees. The next result reveals that the usual intuition of Bertrand competition between certifiers need not hold. Even with different certification fees, both certifiers can attract positive measures of seller types in equilibrium.

Proposition 8 (**Segmentation**). *Assume two certifiers charge different certification fees and, without loss of generality, the certifier who charges the higher fee is named Certifier* 1 *and the one charges the lower fee, Certifier* 2, $0 < P_2 < P_1 < b$. *If there exist* x_1 *and* x_2 *such that* $a < x_2 < x_1 < b$ *and*

$$P_1 - P_2 = G(x_1)[\Omega(x_1, b) - \Omega(x_2, x_1)] \tag{12}$$

$$P_2 = G(x_2)\Omega(x_2, x_1), \tag{13}$$

then x_1 *and* x_2 *identify a subgame equilibrium in which types in* $(x_1, b]$ *strictly prefer testing at Certifier* 1, *type* x_1 *is indifferent between testing at either of these two certifiers, types in* (x_2, x_1) *strictly prefer testing at Certifier* 2, *type* x_2 *is indifferent between testing at Certifier* 2 *and not to test at all, types below* x_2 *strictly prefer not to test, buyers bid* $\Omega(x_1, b)$ *for a product with Certificate* 1, $\Omega(x_2, x_1)$ *for a product* *with Certificate* 2 *and* 0 *for a non-certified product. That is,*

$$\begin{aligned}
\rho^*(t \mid P_1, P_2) &= TS_1, \forall t \in [x_1, b] \\
\rho^*(t \mid P_1, P_2) &= TS_2, \forall t \in [x_2, x_1) \\
\rho^*(t \mid P_1, P_2) &= NTS, \forall t \in [a, x_2) \\
\beta^*(C_1 \mid P_1, P_2) &= \mu(C_1 \mid P_1, P_2) = \Omega(x_1, b) \\
\beta^*(C_2 \mid P_1, P_2) &= \mu(C_2 \mid P_1, P_2) = \Omega(x_2, x_1) \\
\beta^*(NC \mid P_1, P_2) &= 0,\ \mu(NC \mid P_1, P_2) < 0.
\end{aligned}$$

Proof. See appendix. □

When the equilibrium identified in Proposition 8 exists, for instance in our example in subsection 7.2, we call such equilibrium *segmentation* equilibrium. The existence of segmentation equilibrium suggests that it is possible for both certifiers to attract positive measures of seller types while charging different fees. Since the testing technologies are identical, they are providing supposedly identical certification service. One may expect that the lower fee certifier takes up entire market demand for the certification service and competition would drive the certification fee to marginal cost as in normal Bertrand competition. In the current setup, this means free certification service.[18] Proposition 8, however, shows this line of reasoning need not hold. When segmentation equilibrium exists, certifiers need not engage in Bertrand competition because lowering one's certification fee does not necessarily increase the demand for its certification service nor its profit. Being a higher fee certifier does not mean having zero demand either.

This result can be understood in light of the endogeneity of a certificate's value. (Subsection 5.2) When the certifiers charge different fees, their certificates have different values in a segmentation equilibrium. Hence, although they have identical testing processes, their end products (certificates) are differentiated.

In the monopoly certifier case, a certification service provides a device that higher types can differentiate themselves from lower types by paying for the test. With two certifiers providing imperfect certification services, those really high types choose the higher fee certifier to differentiate themselves from moderate types.

Remark 6.
1. *A higher certification fee can serve as a signal of higher product quality.*
2. *Even with identical imperfect testing technology, duopoly certifiers need not to engage in Bertrand Competition.*

[18]Proposition 4 finds free certification is generally not socially optimal.

7.2 An example in Duopoly

We work through an example to verify the existence of segmentation equilibrium.

Example 2. Suppose seller types are distributed on the interval $[-1, 1]$ following a power function $F(t) = \left(\frac{t+1}{2}\right)^{\frac{1}{2}}$. The testing technology $G(t)$ is represented by this power distribution function as well, $G(t) = F(t) = \left(\frac{t+1}{2}\right)^{\frac{1}{2}}$ on $[-1, 1]$.

The type expectation function $\Omega(m, n)$ is, after simple algebra, simply $\frac{m+n}{2}$. Equations (12) and (13) then read

$$\begin{aligned} P_1 - P_2 &= \left(\frac{x_1+1}{2}\right)^{\frac{1}{2}} \frac{1-x_2}{2} \text{ and} \\ P_2 &= \left(\frac{x_2+1}{2}\right)^{\frac{1}{2}} \frac{x_1+x_2}{2}. \end{aligned}$$

Suppose Certifier 1 charges $P_1 = 0.6$ and Certifier 2 charges $P_2 = 0.1$. In this case, the above system obtains a unique solution, $x_1 = 0.4742, x_2 = -0.1648$. Seller types in $[0.4742, 1]$ choose Certifier 1, types in $[-0.1648, 0.4742)$ choose Certifier 2, types in $[-1, -0.1648)$ choose not to test. Type 0.4742 is indeed indifferent between choosing either of these two certifiers and type -0.1648 is indifferent between choosing Certifier 2 or not to test at all. Buyers in this case bid $\Omega(0.4742, 1) = (0.4742 + 1)/2 = 0.7371$ for a product with Certificate 1, bid $\Omega(-0.1648, 0.4742) = (-0.1648 + 0.4742)/2 = 0.1547$ for a product with Certificate 2 and bid zero for a non-certified product.

The profits the certifiers make are

$$\Pi_1(P_1 = 0.6, P_2 = 0.1) = P_1\left(1 - F(x_1)\right) = 0.084873$$

and

$$\Pi_2(P_2 = 0.1, P_1 = 0.6) = P_2\left(F(x_1) - F(x_2)\right) = 0.021233.$$

So in this example the higher fee certifier earns a higher profit than the lower fee certifier.

In the perfect testing case studied in Lizzeri (1999), competition of certifiers will drive the certification fee to zero. When testing technology is imperfect, even if both certifiers provide identical testing technology, the current analysis shows fee differentiation is possible and Bertrand Competition is not guaranteed. The point is that when certifiers charge different fees, there can be subgame equilibria in which high seller types choose the high fee certifier to signal their type. Hence certifiers need not to lower their certification fee to the marginal cost level. In example 2, each certifier has a positive profit and lowering one's certification fee doesn't necessarily increase one's demand nor profit.

Remark 7. *Although imperfect testing technology limits certifiers' power in collecting generated surplus from the seller, it does help to soften competition among certifiers.*

7.3 An alternative explanation to auditing fee differences

The significant fee differentiation between major and non-major auditing firms has long been documented in the accounting literature (e.g., Simunic (1980)). See also more recent evidence like Hay et al. (2006).[19] It is also known that in Initial Public Offerings and debt financing, firms audited by major auditors generally receive more favorable bids than those audited by other auditors. Evidences include Teoh and Wong (1993) and Mansi et al. (2004) among others. The empirical observation here is, in other words, the positive correlation between auditing fees and bids received.

DeAngelo (1981), Titman and Trueman (1986) and in a context similar to our paper, Hvide (2005), suggest that the differences in auditors' auditing qualities or standards are responsible for this observation.[20] Yet, as acknowledged in Hay et al. (2006), differences in auditing qualities are hard to identify. Here we suggest a new perspective to this question, namely identical imperfect testing technology. We show in Example 2 that even two identical testing technologies can support fee differentiation in equilibrium and those who choose the higher fee certifier receive higher bids from the buyers. Applied to the auditing context, those major auditing firms (Certifier 1 in Proposition 8) may have exactly the same ability in identifying audited companies' financial soundness as other auditing firms (Certifier 2 in Proposition 8). If segmentation equilibrium is supported, by paying a higher audition fee, a company of higher quality receives higher bids in equilibrium. Audited by a non-major auditing firm, however, signals a lower quality. Note also that moderate quality companies will not try major auditing firms since those are too expensive and they are very likely to get unfavorable auditing reports. They try non-major firms nevertheless since the fee is low enough to justify their relatively small probability of getting favorable auditing reports. To apply the above analysis, we only need to assume that auditing processes are imperfect, that is, auditing firms are not able to know exactly the financial situation of each audited firm and yet are able to ensure better companies have a higher probability receiving favorable financial reports.

[19]Major auditing firms here refer to the few largest auditing firms. The exact number varies from time to time.

[20]Additional references on this topic can be found in Hvide (2005).

That major auditing firms make more profits than the rest is also predicted in Example 2. Though we have argued that different certification fees P_1, P_2 are possible in equilibrium, we leave solving the entire duopoly game to future research.

8 Conclusion

In this paper, we propose a general model of imperfect testing technology in certification services. The main assumption of our suggested model is that whenever two products get tested the higher quality product is more likely to pass than the lower quality one. The model also admits continuous quality types and strategic certifiers.

The analysis provided in this paper aims to improve our understanding of imperfect certification. It's not always clear what a certificate means in real life. Yet, we have seen a large number of successful certification services that are of practical uses. This paper takes a formal theoretical approach and proves that when a certification service can ensure that higher quality products stand a better chance obtaining a certificate than lower quality products, such certification service can reduce information asymmetry and facilitate trading.

Monopoly certifiers with imperfect testing technologies are not as powerful as they would be if perfect testing technologies were available. According to the analysis, a certifier with an imperfect technology can be completely bypassed. This is in sharp contrast to the case of perfect testing technology.

A separating equilibrium is also supported in which only high quality seller types (products) utilize the certification service. By paying the certification fee a seller type in principle obtains the right to play a lottery. The lottery, however, is type dependent and is in favor of higher types since higher types are more likely to get a certificate for the same certification fee. The value of a certificate is determined jointly by the type distribution and the nature of the testing technology. Welfare accounting shows that the monopolistic certifier's profit maximizing conduct can lead to under or over supply of certification service depending on model specification. The welfare maximizing certification fee is always positive and such that it makes all positive types choose to test. Hence, free certification is not recommended under imperfect testing technology.

When there are two certifiers with identical testing technologies offering certification services in the market, intuition suggests Bertrand competition of the certifiers. While this is true in the perfect testing case studied in Lizzeri (1999), the arguments for Bertrand competition are not valid in imperfect testing cases. Segmentation equilibrium in which higher seller types choose the more expensive certification service and not so high types choose the less expensive service can be supported. In this case, keeping on lowering one's certification fee is not necessarily the best response. In the context of auditing industry, we show that to explain the fee differentiation between major and non-major auditing firms we do not have to assume differences in auditing processes.

A Appendix

A.1 Proof of Proposition 1

Proof. If no seller types choose to get the product tested, the type population of a non-certified product is exactly the original one. Hence, it is optimal for the buyers to bid $\max\{E(t), 0\}$ for a non-certified product. As long as the buyers believe the type of a certified product $\mu(t \mid C, P) \leq E(t)$, that is, it is not above the population mean, any bid $\beta(P, C) = \max\{\mu(t \mid C, P), 0\}$ for a certified product is one of the best responses (Condition 2).

Because a certificate is an off-equilibrium incidence and any type except type a could get a certificate with a strictly positive probability, buyers' beliefs for a certified product can be supported (Condition 3).[21,22]

If buyers' bids for a certified product are no higher than those for a non-certified product, no seller types choose to test. Note also that a single type choosing to test does not convince the buyers to bid higher, so the seller will not pay for the test after learning his own type (Condition 4).

Given the strategies of the seller and the buyers, the certifier's action is irrelevant (Condition 5). □

A.2 Proof of Proposition 2

Proof. With respect to the certification fee P, we have the following two cases.

$P > b$: It is obvious that in no cases buyers will bid above b. All seller types will make a loss by paying for the test. Since $E(t) \leq 0$, buyers bid zero for a non-certified and up to their belief for a certified product. So any of the stated strategy pair constitutes an equilibrium in these subgames. Note that buyers's out of equilibrium belief $a < \mu(t \mid C, P > b) \leq b$ can be supported.

$P = b$: Note that any combination of seller types other than type b alone choosing to test will result buyers' belief for a certified product being less than b, $\mu(t \mid C) < b$. In turn their bids $\beta(C \mid P) < b$. Choosing to test makes a loss for all seller types in such a situation.

[21] Given that there is a positive probability for low types to pass, buyers' belief are not irrational. For perfect Bayesian equilibrium, any not exactly impossible off-equilibrium belief will do. In other words, there is no prior to be updated.

[22] Here buyers can hold different beliefs so long as they satisfy the specified conditions, i.e., their beliefs for a non-certified product are both no higher than the ex ante type expectation.

When type b alone chooses to test, however, we have $\mu(t \mid C) = b$. Because type b for sure gets the certificate by choosing to test, type b is indifferent between testing

$$\beta(C \mid P) - P = b - b = 0,$$

and not testing (also 0). Types other than b has a strictly positive possibility of getting no certificate. Consequently, if choose to test, seller types $t < b$ will receive a negative payoff $G(t)b - b < 0$. The only equilibrium other than bypassing when $P = b$ is then the one in which type b alone chooses to test and all others not to. The buyers then bid b for a certified product and 0 for a non-certified product in this equilibrium. Since type b alone is of zero measure, buyers' belief for a non-certified product remains to be the product's prior expectation $E(t)$ which is less than zero.

□

A.3 Proof of Proposition 3

Proof. The logic of the proof is the following. First, we investigate the properties of equilibrium strategies in subgames induced by $P \in (0, b)$ with some seller types choosing to test, when such equilibrium exists. Second, we prove the existence by constructing strategies that fulfill all such properties. The uniqueness of the equilibrium is then shown by examination of an equivalent mathematical system.

Step 1 is to show that in such equilibria buyers bid more for a certified product and the lowest seller type does not choose to test in equilibrium.
In the subgames induced by $0 < P < b$, suppose there exist a set of seller types who choose to test by paying the testing fee P in equilibrium. Denote such a set $\Psi(P)$. That is,

$$\Psi(P) \equiv \{t \mid \rho^*(t \mid P) = TS\}.$$

For all seller types in $\Psi(P)$, the expected payoff from testing has to be no less than what they could get by not to test. We have, $\forall t \in \Psi(P)$,

$$G(t)\beta(C \mid P) + (1 - G(t))\beta(NC \mid P) - P \geq \beta(NC \mid P). \tag{14}$$

After rearranging, $\forall t \in \Psi(P)$,

$$G(t)[\beta(C \mid P) - \beta(NC \mid P)] \geq P. \tag{15}$$

Since $P > 0$ by assumption, $\forall t \in \Psi(P)$

$$G(t)[\beta(C \mid P) - \beta(NC \mid P)] > 0.$$

Note that $\forall t \in [a, b], G(t) \geq 0$, so *both* $G(t \mid t \in \Psi(P))$ and $\beta(C \mid P) - \beta(NC \mid P)$ have to be strictly larger than zero. That is,

$$a \notin \Psi(P) \wedge \beta(C \mid P) > \beta(NC \mid P). \tag{16}$$

So we showed that when there exist a set of seller types who choose to test by paying a strictly positive fee in equilibrium, buyers bid more for a certified product and the lowest seller type a does not test.

Step 2 is to prove when buyers bid more for a certified product the set of seller types that pay for the test exists and is of the form $[x, b]$.
Let's denote $\Gamma(t)$ the difference in expected payoffs for type t between to test and not to.

$$\Gamma(t) \equiv G(t)[\beta(C \mid P) - \beta(NC \mid P)] - P. \tag{17}$$

Apparently, $t \in \Psi(P)$ if and only if $\Gamma(t) \geq 0$. Note that for any given P and β such that $0 < P < b$ and $\beta(C \mid P) > \beta(NC \mid P)$, $\Gamma(t)$ is continuous and strictly increasing in t; $\Gamma(b) \geq \Gamma(t)\ \forall t \in [a, b]$. Hence, if any types choose to test, type b must be one of them, $b \in \Psi(P)$.

1. Suppose type b is the only element of $\Psi(P)$, that is $\Psi = \{b\}$. From Proposition 2, $\beta(C \mid P) = b$ and $\beta(NC \mid P) = 0$. Therefore, combined with $G(b) = 1$ and $P < b$, we have $\Gamma(b) = G(b)b - P > 0$.
   Solving the equation $G(\bar{t})b - P = 0$, we have $\bar{t} = G^{-1}(P/b)$ where G^{-1} is the inverse of G. Because G(t) is strictly increasing, for the types $t \in (G^{-1}(P/b), b)$, their expected payoff of testing $G(t)b - P$ is strictly larger than zero. These types will also choose to test. Hence we prove that when $0 < P < b$, the supposition that $\Psi(P)$ has only one element is false.

2. Now we know $\Psi(P)$, when it exists, contains more elements than just type b alone. Note also $G(t)$ is strictly increasing and $\beta(C \mid P) > \beta(NC \mid P)$. Therefore, if a type t' other than b is in $\Psi(P)$, that is, if the expression (15) holds for t', it also must hold with strict inequality for any $t > t'$. Hence, all t such that $t > t'$ should be in $\Psi(P)$ as well. Moreover, these types strictly prefer testing. In equilibrium, the set of seller types strictly prefer testing must be of the form $(x, b]$ or $[x, b]$ for some $x < b$.

3. For type b, we have

$$\Gamma(b) = G(b)[\beta(C \mid P) - \beta(NC \mid P)] - P > 0.$$

This inequality holds strictly because type b obtains a higher payoff than type $\inf \Psi(P)$. For type a, $G(a) = 0$,

$$\Gamma(a) = -P < 0.$$

By the continuity and monotonicity of function $\Gamma(t)$, there is a unique solution for $\Gamma(t) = 0$ in the domain of (a, b). Suppose $x = \Gamma^{-1}(0)$, for type x, it is indifferent between to test and not to test. For $t > x$, $\Gamma(t) > 0$. Consequently, when buyers bid more for a certified product the set of seller types that pay for the test exists in each subgame induced by $0 < P < b$ and, by the tie-breaking rule, is of the form $[x, b]$.

Step 3 is to construct the required buyers' optimal bids.
In this part we search out compatible buyers' strategies, $\beta(\cdot \mid P)$ that will satisfy

$$\beta(C \mid P) > \beta(NC \mid P) \geq 0.$$

Buyers bid positively for a certified product ($\beta(C \mid P) > 0$), only when their beliefs for a certified product is positive ($\mu(t \mid C) > 0$). In equilibrium, $\mu(t \mid C)$ requires to be consistent with rational expectation,

$$\mu(t \mid C) = E(t \mid C).$$

Further, by the following identity

$$\Pr(C)E(t \mid C) + (1 - \Pr(C))E(t \mid NC) \equiv E(t) < 0, \tag{18}$$

it cannot be true that both conditional expectations are non-negative. Hence, to have $E(t \mid C) > 0$, $E(t \mid NC)$ has to be less than zero. In turn, $\mu(t \mid NC) < 0$ and $\beta(NC \mid P) = 0$. Since the set of seller types that choose to test is of the form $[x, b]$, the buyers' Bayesian updated belief should be,

$$E(t \mid C) = \frac{\int_x^b tG(t)dF(t)}{\int_x^b G(t)dF(t)} = \Omega(x, b). \tag{19}$$

The bid for a certified product is, therefore, $\beta(C \mid P) = E(t \mid C) = \Omega(x, b)$. To find indifferent type x, we need to solve

$$G(x)[\Omega(x,b) - 0] = P.$$

The existence and uniqueness of the solution is established in the next step. Note that if $G(x)\Omega(x,b) = P$ holds, then $\Omega(x,b) = \frac{P}{G(x)}$. Since both P and $G(t), \forall t \in (a,b]$ are larger than zero, $\Omega(x,b)$ is also large than zero. Hence we constructed feasible buyers' strategies and their beliefs. For $0 < P < b$, buyers bid

$$\beta(C \mid P) = \mu(t \mid C) = E(t \mid C) = \Omega(x,b)$$

and $\beta(NC \mid P) = 0$ with belief $\mu(t \mid NC) < 0$. These bidding strategies are compatible to the seller's strategy.

Step 4 is to prove the existence and uniqueness of the indifferent type x for each $0 < P < b$.

The existence and uniqueness of the equilibrium in the subgames boils down to the existence and uniqueness of the solution to $\Gamma(t) = 0$ or

$$G(x)\Omega(x,b) = P. \tag{20}$$

Note that $\Omega(x,b)$ is bounded, it is clear that

$$\begin{aligned} \lim_{x \to a} G(x)\Omega(x,b) &= 0 \text{ and} \\ \lim_{x \to b} G(x)\Omega(x,b) &= b. \end{aligned}$$

Note also function $\Omega(x,b)$ and $G(x)\Omega(x,b)$ are continuous,[23] $G(x)\Omega(x,b) = P$ obtains at least one solution when $0 < P < b$.

To prove the uniqueness, we first derive the derivative of the function $\Omega(x,b)$,

$$\frac{d\Omega(x,b)}{dx} = \frac{G(x)f(x)\int_x^b (t-x)G(t)dF(t)}{\left(\int_x^b G(t)dF(t)\right)^2}.$$

It's easy to verify that all parts in the right hand side are positive. Hence $\frac{d\Omega(x,b)}{dx} > 0$ and $\Omega(x,b)$ increases in x.

According to the value of $\Omega(a,b)$, we discuss two cases.

1. When $\Omega(a,b) \geq 0$, then $\Omega(x,b) \geq 0, \forall x \in (a,b]$. Since the derivative of the

[23] The continuity of $\Omega(x,b)$ follows from the theorem that the quotient of two continuous functions is continuous. That the divisor $\int_x^b G(t)dF(t)$ is non-zero for $x \in (a,b)$ is checked.

function $G(x)\Omega(x,b)$ is as the following:

$$\frac{d\left(G(x)\Omega(x,b)\right)}{dx} = g(x)\Omega(x,b) + G(x)\frac{d\Omega(x,b)}{dx}. \tag{21}$$

All parts are positive and $G(x)\Omega(x,b)$ increases monotonically form 0 to b. Hence Equation 20 only obtains one solution when $0 < P < b$.

2. When $\Omega(a,b) < 0$, because of continuity and monotonicity of $\Omega(x,b)$, we first find $\overline{x}$ such that $\Omega(\overline{x},b) = 0$. For any $x < \overline{x}$, $\Omega(x,b) < 0$ hence Equation (20) has no solution. Within the interval of $[\overline{x},b]$, $G(x)\Omega(x,b)$ increases monotonically form 0 to b. Hence Equation (20) only obtains one solution in $[\overline{x},b]$ when $0 < P < b$.

This proves the existence and uniqueness of the indifferent type x for each $0 < P < b$. Together with above steps, all conditions required by equilibrium notion (Definition 1) for the subgames are satisfied and we have established uniqueness. □

A.4 Proof of Proposition 4

Proof. 1. If $\Omega(a,b) > 0$, it's easy to verify that all types choose to test and buyers bid $\Omega(a,b)$ for a certified product and zero for a non-certified product is an equilibrium.

2. On the other hand, if buyers make positive bids for a certified product, all types above a will choose to test. This is because there is simply no cost involved in testing for the seller and there is a certain probability receiving positive bids. Hence, to test is the dominant strategy except for the lowest type. Suppose $\Omega(a,b) \leq 0$, then buyers' belief for a certified product is non-positive and consequently will bid zero for a certified product. This contradicts the supposition that buyers make positive bids. Hence when buyers make positive bids, $\Omega(a,b) > 0$.

□

A.5 Proof of Proposition 5

Proof. According to Corollary 1, if $P \geq b$ or $P = 0$ the seller's profit will be zero. Note as well that according to the proof of the uniqueness of the subgame equilibrium when $0 < P < b$, $G(t)\Omega(t,b)$ is a continuous and strictly increasing function in (a,b) or $(\overline{x},b)$ where $\overline{x}$ is find by solving $\Omega(\overline{x},b) = 0$ when $\Omega(a,b) < 0$.[24] Hence, its

[24] See A.3, especially Step 4 and Equation (21).

inverse function $\kappa(P)$ from $(0, b)$ to (a, b) or $(\overline{x}, b)$ is also strictly increasing in $(0, b)$. Consequently, the certifier can also maximize his profit by optimally choosing the indifferent type x. The certification fee P is then $G(x)\Omega(x, b)$. From Proposition 3, the demand for certification service will be $1 - F(x)$. The product of these two components give the profit,[25]

$$\Pi(x) = (1 - F(x))G(x)\Omega(x, b), x \in (a, b). \tag{22}$$

Since the extreme points in Corollary 1 are dominated, the maximum *is obtained* inside the interval. The certifier's best response to the equilibrium strategies of the seller and the buyers is hence P^* defined in Equation (6). This, together with Proposition 3, concludes the proof. $\square$

A.6 Proof of Proposition 6

Proof. Because buyer always bid up to the expected value of a certified product, they do not derive positive gains. Social welfare is then the sum of the payoff of the certifier and the payoff of the seller. Moreover, the sum is exactly what buyers pay for the product in equilibrium, because this is the only source for the revenues of both the certifier and the seller.

Since buyers bid zero for a non-certified product, trading only takes place when the product has a certificate. The total surplus is then, for a given certification fee, the result of multiplying buyers' bid for a certified product and the probability of the product getting a certificate,

$$\Omega(\kappa(P), b) \int_{\kappa(P)}^{b} G(t)dF(t) = \int_{\kappa(P)}^{b} tG(t)dF(t).$$

Taking derivative of this expression gives us,

$$\frac{d\left(\int_{\kappa(P)}^{b} tG(t)dF(t)\right)}{d\left(\kappa(P)\right)} = -\kappa(P)G(\kappa(P))f(\kappa(P)). \tag{23}$$

It is then obvious that the right hand side of equation (23) is strictly negative when $\kappa(P) > 0$, strictly positive when $\kappa(P) < 0$ and equal to zero when $\kappa(P) = 0$. Maximization of $\int_{\kappa(P)}^{b} tG(t)dF(t)$ with $a < \kappa(P) < b$ requires $\kappa(P) = 0$. The welfare maximizing certification fee is hence $P^{**} = G(0)\Omega(0, b)$. $\square$

[25] Note that when $\Omega(x, b) < 0$ when $x \in (a, \overline{x})$, $\Pi(x) < 0$ on this interval too. This allows us to represent the problem as Equation (22) without explicitly write the case for $(\overline{x}, b)$.

A.7 Proof of Proposition 7

Proof. In the proof of Proposition 5 we show that the certifier can set the indifferent type x to maximize profit. The first order derivative of $\Pi(x) = G(x)\Omega(x,b)[1-F(x)]$ is

$$\begin{aligned} & g(x)\Omega(x,b)[1-F(x)] + G(x)[1-F(x)]\frac{d\Omega(x,b)}{dx} - G(x)\Omega(x,b)f(x) \\ = \; & g(x)\Omega(x,b)\left([1-F(x)]\left(1+\frac{G(x)}{g(x)\Omega(x,b)}\frac{d\Omega(x,b)}{dx}\right) - \frac{G(x)}{g(x)}f(x)\right). \end{aligned} \tag{24}$$

Since $g(x) > 0$ and $\Omega(x,b) > 0$, when $0 < P < 0$ a necessary condition for profit maximization is

$$\begin{aligned} & [1-F(x)]\left(1+\frac{G(x)}{g(x)\Omega(x,b)}\frac{d\Omega(x,b)}{dx}\right) - \frac{G(x)}{g(x)}f(x) = 0 \\ \Rightarrow \; & [1-F(x)]\left(1+\frac{G(x)}{g(x)\Omega(x,b)}\frac{d\Omega(x,b)}{dx}\right) = \frac{G(x)}{g(x)}f(x) \\ \Rightarrow \; & \frac{f(x)}{1-F(x)} = \frac{g(x)}{G(x)} + \frac{d\Omega(x,b)}{dx}\frac{1}{\Omega(x,b)}. \end{aligned} \tag{25}$$

Hence if profit maximizing x is socially optimal, i.e., $x^* = 0$, the next condition has to hold,

$$\frac{f(0)}{1-F(0)} = \frac{g(0)}{G(0)} + \frac{1}{\Omega(0,b)}\frac{d\Omega(x,b)}{dx}\Big|_{x=0}.$$

Note that

$$\begin{aligned} \frac{d\Omega(x,b)}{dx} &= \frac{d}{dx}\left(\frac{\int_x^b tG(t)f(t)dt}{\int_x^b G(t)f(t)dt}\right) \\ &= \frac{G(x)f(x)\int_x^b tG(t)f(t)dt - xG(x)f(x)\int_x^b G(t)f(t)dt}{\left(\int_x^b G(t)f(t)dt\right)^2} \end{aligned}$$

hence

$$\frac{d\Omega(x,b)}{dx}\Big|_{x=0} = \frac{G(0)f(0)\int_0^b tG(t)f(t)dt}{\left(\int_0^b G(t)f(t)dt\right)^2}.$$

Consequently,

$$\begin{aligned} \frac{f(0)}{1-F(0)} &= \frac{g(0)}{G(0)} + \frac{1}{\Omega(0,b)}\frac{d\Omega(x,b)}{dx}\Big|_{x=0} \\ &= \frac{g(0)}{G(0)} + \left(\frac{\int_0^b G(t)f(t)dt}{\int_0^b tG(t)f(t)dt}\right)\frac{G(0)f(0)\int_0^b tG(t)f(t)dt}{\left(\int_0^b G(t)f(t)dt\right)^2} \\ &= \frac{g(0)}{G(0)} + \frac{G(0)f(0)}{\int_0^b G(t)f(t)dt}. \end{aligned}$$

This proves the first part of Proposition 7. With the additional condition of profit function concavity, we know the second derivative is negative and the first order condition (25) becomes sufficient for profit maximization. However, we are interested in the value of the first derivative (24) at $x = 0$. When it is larger than 0, the monopoly certifier will increase P in order to increase x and because of the profit function concavity the profit maximizing x^* is larger than 0. Consequently, some positive types find it too expensive to test and the certification service is under supplied. Hence, the condition for undersupply is

$$\begin{aligned}
& g(x)\Omega(x,b)\left([1-F(x)]\left(1+\frac{G(x)}{g(x)\Omega(x,b)}\frac{d\Omega(x,b)}{dx}\right)-\frac{G(x)}{g(x)}f(x)\right)\Big|_{x=0} > 0 \\
\Rightarrow \quad & \left([1-F(x)]\left(1+\frac{G(x)}{g(x)\Omega(x,b)}\frac{d\Omega(x,b)}{dx}\right)-\frac{G(x)}{g(x)}f(x)\right)\Big|_{x=0} > 0 \\
\Rightarrow \quad & \frac{g(0)}{G(0)}+\frac{G(0)f(0)}{\int_0^b G(t)f(t)dt} > \frac{f(0)}{1-F(0)}.
\end{aligned}$$

Likewise, when

$$\frac{f(0)}{1-F(0)} > \frac{g(0)}{G(0)}+\frac{G(0)f(0)}{\int_0^b G(t)f(t)dt}$$

there is oversupply of certification service. So we proved the second part of Proposition 7. □

A.8 Proof of Proposition 8

Proof. The following is to prove when $a < x_2 < x_1 < b$ solve the system of equations (12) and (13), we claim the strategies profile in Proposition 8 constitutes a perfect Bayesian equilibrium. This is done in the following steps.

1. First, for given $0 < P_2 < P_1 < b$ when $a < x_2 < x_1 < b$ solves

$$\begin{aligned}
P_1 - P_2 &= G(x_1)[\ \Omega(x_1,b) - \Omega(x_2,x_1)] \\
P_2 &= G(x_2)\Omega(x_2,x_1),
\end{aligned}$$

   we have $\Omega(x_1,b) > \Omega(x_2,x_1) > 0$. This is because $G(t) > 0, \forall t > a$.

2. Suppose types in $[x_1, b]$ choose Certifier 1, types in $[x_2, x_1)$ choose Certifier 2 and types in $[a, x_2)$ chooses not to test, then buyers expectation for a product certified by Certifier 1 $\mu(C_1 \mid P_1, P_2) = E(C_1 \mid P_1, P_2)$ is $\Omega(x_1, b)$ and for a product certified by Certifier 2 $\mu(C_2 \mid P_1, P_2) = E(C_2 \mid P_1, P_2)$ is $\Omega(x_2, x_1)$. Because the prior expectation of the product is negative, the expectation for a none certified product $\mu(NC \mid P_1, P_2)$ is less than zero.

3. Then buyers bids are $\beta(C_1 \mid P_1, P_2) = \Omega(x_1, b)$ for a product certified by Certifier 1, $\beta(C_2 \mid P_1, P_2) = \Omega(x_2, x_1)$ for a product certified by Certifier 2 and 0 for a non-certified product.

4. Since $P_1 - P_2 = G(x_1)[\ \Omega(x_1, b) - \Omega(x_2, x_1)]$, $P_2 = G(x_2)\Omega(x_2, x_1)$ and $G(t)$ strictly increases in t, we have for all $x_1 < t \leq b$,

$$\begin{aligned} G(t)[\ \Omega(x_1,b) - \Omega(x_2,x_1)] &> P_1 - P_2 \\ G(t)\Omega(x_2,x_1) &> P_2 \\ \Longrightarrow G(t)\ \Omega(x_1,b) - P_1 &> G(t)\ \Omega(x_2,x_1) - P_2 > 0; \end{aligned}$$

for all $x_2 < t < x_1$,

$$\begin{aligned} G(t)[\ \Omega(x_1,b) - \Omega(x_2,x_1)] &< P_1 - P_2 \\ G(t)\Omega(x_2,x_1) &> P_2 \\ \Longrightarrow G(t)\ \Omega(x_2,x_1) - P_2 &> G(t)\ \Omega(x_1,b) - P_1 \\ G(t)\ \Omega(x_2,x_1) - P_2 &> 0; \end{aligned}$$

for all $a \leq t < x_2$,

$$\begin{aligned} G(t)[\ \Omega(x_1,b) - \Omega(x_2,x_1)] &< P_1 - P_2 \\ G(t)\Omega(x_2,x_1) &< P_2 \\ \Longrightarrow 0 > G(t)\ \Omega(x_2,x_1) &> P_2.G(t)\ \Omega(x_1,b) - P_1. \end{aligned}$$

Hence we compared the expected payoffs for different choices for types in $[a, b]$. Employing also the tie break rule, we conclude that it is true that types in $[x_1, b]$ choose Certifier 1, types in $[x_2, x_1)$ choose Certifier 2 and types in $[a, x_2)$ choose not to test.

5. In summary, if there exist such x_1, x_2 that satisfy $a < x_2 < x_1 < b$ and solve the system of equations (12) and (13), the above construction proves that the strategy combinations in Proposition 8 constitute an equilibrium for the given P_1, P_2.

□

References

Akerlof, G. A. (1970). Market for lemons: Quality uncertainty and the market mechanism. *Quarterly Journal of Economics*, 84(3):488–500.

Albano, G. L. and Lizzeri, A. (2001). Strategic certification and provision of quality. *International Economic Review*, 42(1):267–283.

Biglaiser, G. (1993). Middlemen as experts. *RAND Journal of Economics*, 24:212–223.

Biglaiser, G. and Friedman, J. W. (1994). Middlemen as guarantors of quality. *International Journal of Industrial Organization*, 12:509–531.

Boom, A. (2001). A monopolistic credit rating agency. Revised Version of Diskussionsbeiträge des Fachbereichs Wirtschaftswissenschaft der Freien Universität Berlin; 1999/27: Volkswirtschaftliche Reihe.

De, S. and Nabar, P. (1991). Economic implications of imperfect quality certification. *Economics Letters*, 37(4):333–337.

DeAngelo, L. (1981). Audit size and audit quality. *Journal of Accounting and Economics*, 3:183–200.

Grossman, S. J. (1981). The informational role of warranties and private disclosure about product quality. *Journal of Law and Economics*, 24(3):461–483.

Hay, D., Knechel, W. R., and Wong, N. (2006). Audit fees: A meta-analysis of the effect of supply and demand attributes. *Contemporary Accounting Research*, 23:141–191.

Heinkel, R. (1981). Uncertain product quality: The market for lemons with an imperfect testing technology. *The Bell Journal of Economics*, 12(2):625–636.

Hvide, H. K. (2005). A theory of certification with an application to the market for auditing services. *The B. E. Journal of Theoretical Economics*.

Klein, B. and Leffler, K. B. (1981). The role of market forces in assuring contractual performance. *Journal of Political Economy*, 89(4):615–641.

Lizzeri, A. (1999). Information revelation and certification intermediaries. *RAND Journal of Economics*, 30(2):214–231.

Mansi, S. A., Maxwell, W. F., and Miller, D. P. (2004). Does auditor quality and tenure matter to investors? evidence from the bond market. *Journal of Accounting Research*, 42:755–793.

Mason, C. F. and Sterbenz, F. P. (1994). Imperfect product testing and market size. *International Economic Review*, 35(1):61–86.

Simunic, D. (1980). The pricing of audit services: Theory and evidence. *Journal of Accounting Research*, 18:161–190.

Strausz, R. (2005). Honest certification and the threat of capture. *International Journal of Industrial Organization*, 23:45–62.

Teoh, S. H. and Wong, T. J. (1993). Perceived auditor quality and the earnings response coefficient. *The Accounting Review*, pages 346–366.

Titman, S. and Trueman, B. (1986). Information quality and the valuation of new issues. *Journal of Accounting and Economics*, 8:159–172.