Inflation Expectation Uncertainty in a New Keynesian Framework
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Abstract
For monetary policy guiding inflation expectations provides an instrument to achieve price stability. However, expectation uncertainty may undermine monetary policy’s ability to stabilise the economy. This study examines the effects of inflation expectation uncertainty on inflation, inflation expectations and the output gap by means of a structural VAR with stochastic volatility in mean. Inflation expectation uncertainty negatively affects the inflation rate and the output gap, without having a distinct effect on the level of expectations. This result is replicable with a model in which uncertainty is approximated by a cross-sectional survey measure. Furthermore, simulating an uncertainty shock in a DSGE model shows that the demand channel dominates the supply channel of an inflation expectation uncertainty shock.

JEL-Code: E31, E52, C32, C63

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1 Introduction

Inflation expectations are one of the key determinants of inflation. Hence, for monetary policymakers the stabilisation of inflation expectations is an important element of their strategy (e.g., Bernanke, 2007). In this regard, well-anchored inflation expectations are regarded as a sign of credible monetary policy. The stability of inflation expectations hinges on both their level and the associated uncertainty (Chan and Song, 2018).

This study estimates the effects of inflation expectation uncertainty on inflation, inflation expectations and economic output. In the literature, inflation expectation uncertainty has predominantly been understood as *inflation forecast uncertainty* directly measurable from surveys, for instance by means of density forecasts (e.g., Rich and Tracy, 2010). In this strand of literature, uncertainty surrounding inflation forecasts is commonly labelled inflation uncertainty. In contrast, a different strand of the literature models *inflation uncertainty* by the variance or volatility of unpredictable innovations in the inflation rate (e.g., Grier and Perry, 1998). This study adopts a new approach, deriving a measure of inflation expectations from a consumer survey and explicitly defining *inflation expectation uncertainty* as the stochastic volatility of the inflation expectation shock in a vector autoregression.

The notion of inflation uncertainty goes back to Okun (1971) and Friedman (1977). In his Nobel Lecture, Friedman (1977) argued that a high inflation environment leads to increased inflation uncertainty and that inflation uncertainty undermines economic efficiency, and thus reduces economic output.¹ This hypothesis was motivated by the increased evidence of a positively sloped Phillips curve at that time, i.e., higher inflation was associated with higher unemployment. Importantly, Friedman related the concept of uncertainty not only to the inflation rate but also explicitly to anticipation with respect to inflation. Following this conjecture, Levi and Makin (1980) allow for inflation forecast uncertainty in a modified Phillips curve which results in an improved short-run Phillips curve trade-off. The authors find that inflation forecast uncertainty negatively affects the employment growth rate.

In this study, we investigate the effects of inflation expectation uncertainty on expectations, inflation and economic activity within a New Keynesian framework. The contribution of our study is two-fold. First, we adopt a novel approach to measuring uncertainty of inflation expectations and estimating its effect on the macroeconomy. Second, we provide a theoretical foundation of the effects of inflation expectation uncertainty

¹ Ball (1992) formalised the proposed relationship between inflation and inflation uncertainty in a Barro-Gordon (1983) type model.
in a New Keynesian type model, in which uncertainty affects economic activity via the
supply side and the demand side of the economy.

In order to explore the relationship empirically, we employ a structural vector autore-
gressive model with stochastic volatility in mean (SVAR-SV-mean), incorporating the
variables of the New Keynesian Phillips curve (NKPC): expected inflation, the inflation
rate and the output gap. The measure of inflation expectation uncertainty is generated
endogenously within the VAR model. We estimate how a shock to the volatility of the
structural inflation expectation shock affects the variables of the system. In addition to
the SVAR-SV-mean approach we use a VAR with an exogenous measure of uncertainty, i.e.
a survey dispersion measure, to estimate the effects on the variables of the NKPC. This
allows for analysing the impact of different properties of inflation expectation uncertainty.
In order to rationalise the relationship between inflation expectation uncertainty and
inflation expectations, inflation and the output gap, we employ a dynamic stochastic gen-
eral equilibrium (DSGE) model. The DSGE model provides two channels from inflation
expectations and uncertainty to inflation and economic activity. The New Keynesian
Phillips curve channel suggests that an inflation expectation uncertainty shock can be
interpreted as a supply shock. In addition, the New Keynesian model provides a second
channel from inflation expectations to inflation because inflation expectations enter the
IS equation, thereby affecting aggregate demand. From this perspective, an inflation
expectation uncertainty shock is an economic demand shock.

Evidence on the effects of inflation expectation uncertainty is limited. The literature
on inflation forecast uncertainty typically focuses on the measurement and determinants
of inflation expectation uncertainty. Some studies analyse the reverse causal link and
estimate how the level of the inflation rate or the level of inflation expectations affects
inflation forecast uncertainty (e.g., Zarnowitz and Lambros, 1987; Ungar and Zilberfarb,
1993).

Since the Nobel Lecture of Friedman (1977), the literature has proposed – and
tested – different theories about the relationship between inflation, economic output and
the respective associated uncertainty. We will provide a short overview, focusing on the
most relevant aspect for our study, the effects of inflation uncertainty. Cukierman and
Meltzer (1986) propose a reverse causality of the Friedman hypothesis, suggesting that
inflation uncertainty causes a higher level of inflation. Further, in the model of Dotsey
and Sarte (2000) an increase in inflation uncertainty – in contrast to the conjecture
by Friedman – promotes investment and growth via a precautionary savings motive.
Empirical evidence on the causal link or the sign of the relationships is mixed. Overall,
however, results point to the validity of the Cukierman-Meltzer hypothesis and the Friedman hypothesis regarding the effect of inflation uncertainty (e.g., Grier and Perry, 1998; Elder, 2004; Kontonikas, 2004; Bredin and Fountas, 2009; Fountas, 2010; Hartmann and Roestel, 2013; Conrad and Karanasos, 2015). In a recent study, Barnett et al. (2018) propose that the empirical validity of the theories about the relationship between inflation uncertainty and inflation depends on country-characteristics and is time-variable. For instance, the authors find that during periods of economic instability like the Great Recession inflation uncertainty is leading inflation.

This study adds to the literature by estimating the effects of inflation expectation uncertainty on the level of expected inflation, the level of the inflation rate and the output gap. Our approach allows for the joint determination of the uncertainty measure and its macroeconomic effects. Inflation expectations are approximated by survey data from the University of Michigan’s Surveys of Consumers.

Our results reveal that inflation expectation uncertainty does not have a significant impact on the level of inflation expectations. However, inflation expectation uncertainty negatively affects the inflation rate and the output gap. These results are robust with respect to the measure of inflation expectation uncertainty and with respect to the measure of economic activity. Consequently, expectations are an important channel through which uncertainty affects the economy. Moreover, our empirical findings are in accordance with the effects of an inflation expectation uncertainty shock in a DSGE model based on a modified version of the model by Basu and Bundick (2017). Within this modelling framework, the demand channel of an inflation expectation uncertainty shock outweighs the supply channel.

The paper is structured as follows. Section 2 describes the measures of inflation expectations and uncertainty employed in the analysis. Further, the main features of the VAR model with stochastic volatility in mean, the data and the estimation procedure are illustrated, providing the foundation for empirically estimating the effects of an inflation expectation uncertainty shock. Section 3 discusses the empirical results. Section 4 introduces the main features of the DSGE model employed to simulate an inflation expectation uncertainty shock and presents the main results of the simulation. Section 5 concludes.
2 Empirical Strategy

The estimation of the empirical effects of inflation expectation uncertainty on the variables of the NKPC requires constructing an appropriate measure of this type of uncertainty. In this section, we outline the derivation of the inflation expectation uncertainty measure. We choose a suitable measure of expected inflation and subsequently select an empirical model that is able generate uncertainty shocks based on this variable.

2.1 Measure of Inflation Expectations

Following a wide range of literature (e.g., Roberts, 1995; Leduc et al., 2007; Canova and Gambetti, 2010), we employ a direct measure of inflation expectations obtained from a survey in our analysis. The results of Roberts (1995) suggest that inflation dynamics in the US may be well represented by a forward-looking NKPC in which expectations are approximated by data from the Michigan Surveys of Consumers. Further, Coibion et al. (2018) show that survey expectations are a better fit than full-information rational expectations for the New Keynesian Phillips curve. Moreover, survey expectations have been shown to outperform model-based forecasts by, e.g., Ang et al. (2007), Gil-Alana et al. (2012) and Grothe and Meyler (2015).

Some individuals may have better information about future inflation and therefore form more precise expectations. Surveys usually reflect either the expectations of professional forecasters, industry professionals or the perceptions of private households. Research by Ang et al. (2007) points to the accuracy of households’ forecasts of inflation from the Surveys of Consumers by the University of Michigan, which perform well relative to professional forecasts. Fuhrer (1988) points out that even in case survey forecasts are inefficient and subject to measurement errors, they may contain independent information. He shows that consumer sentiment data from the Michigan Survey provide useful information above that which is given in standard macroeconomic variables. Taken together, these studies suggest that survey data contain information about inflation expectations that can be used in empirical analyses.

A further argument for using consumer survey data is provided by Coibion and Gorodnichenko (2015). They argue that small and medium-sized enterprises are influential drivers of price setting in the US, and that the attitudes of these firms are well represented by the sentiments of private households. In their study, consumers’ expectations from the Michigan Survey are more relevant than professional forecasts for inflation dynamics in a Phillips curve framework.
Figure 1: Inflation and Inflation Expectations in the US

Notes: This figure shows the evolution of expected inflation and the inflation rate in the US between January 1983 and March 2017. Inflation is computed from the Consumer Price Index for All Urban Consumers: All Items (bold black line). Expected Inflation is the expected change of prices over the next 12 months from the Michigan Surveys of Consumers (blue line).

Consequently, we employ expectations from the Surveys of Consumers from the University of Michigan in our study, which allows us to conduct the analysis on the basis of monthly data. The Michigan Survey was established in 1946 and is based on monthly interviews with a representative sample of approximately 500 US households, which are asked about different aspects of their personal finances, business conditions and buying conditions, including their perception of past and future price developments. The assessment of households’ inflation expectations is based upon two questions. Consumers are first asked “During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?”, and subsequently “By about what percent do you expect prices to go (up/down), on the average, during the next 12 months?”

Figure 1 shows monthly US inflation rates and average inflation expectations captured by the Michigan Surveys of Consumers between January 1983 and March 2017. For the greater part of the 1990s, expected inflation exceeds actual inflation, while in the first half of the 2000s, expected inflation follows actual inflation relatively closely. However, since the Great Recession consumers tend to expect inflation to be higher than realised inflation. This may be partly related to the observation that the inflation expectations of consumers increased relative to those of professional forecasters between 2009 and 2011. Coibion and Gorodnichenko (2015) ascribe this phenomenon to the development

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2 For further information on the procedure to construct estimates of households’ price expectations see Curtin (1996).
of oil prices to which consumers react more sensitively. In the second half of the 2010s, after prices stagnated, the inflation rate starts to increase again, while expected inflation remains largely on a level of around three percent, which leads the series to converge. Overall, the inflation rate is more volatile than inflation expectations. During the Great Recession, the inflation rate dropped sharply, whereas the decrease in expected inflation was smaller. The two series are correlated to some extent, with a correlation coefficient of 0.67.

2.2 Measure of Inflation Expectation Uncertainty

Despite an increasing interest in gauging uncertainty and its impact on the economy in recent years (e.g., Caggiano et al., 2014; Baker et al., 2016; Carriero et al., 2018; Mumtaz, 2018), evidence on the impact of inflation expectation uncertainty is scarce. The literature on inflation forecast uncertainty typically focuses on the measurement and the determinants of uncertainty (e.g., Ungar and Zilberfarb, 1993; Arnold and Lemmen, 2008; Gnan et al., 2010).

In a recent paper, Chan and Song (2018) develop a novel approach to derive a measure of inflation expectation uncertainty, employing a model-based measure of uncertainty and a market-based measure of inflation expectations. Using an unobserved components model with stochastic volatility, their estimation of inflation expectation uncertainty draws upon the volatility of trend inflation and the realised volatility of market expectations. However, they do not estimate potential effects on the economy.\(^3\)

Related to our study is the literature on inflation uncertainty, which adopts a time series approach to analyse the links between inflation uncertainty, inflation and other macroeconomic variables. Earlier studies employed models from the family of generalised autoregressive conditional heteroskedasticity (GARCH) models (Engle, 1982; Bollerslev, 1986) in which the conditional variance serves as a proxy for uncertainty, modelled as a deterministic function of previous observations and past variances (e.g., Grier and Perry, 2000; Elder, 2004).

More recently, researchers have drawn upon stochastic volatility models in which variance is a random variable that follows a latent stochastic process (e.g., Berument et al., 2009; Chan, 2017). Stochastic volatility models allow for more flexibility than GARCH models in which the conditional variance is a deterministic function. Further, there is some evidence that stochastic volatility models perform better than GARCH

\(^3\) The data used for their analysis is available beginning in 2003. Our approach allows us to work with a sample beginning in the 1980s.
models (Kim et al., 1998; Lemoine and Mougin, 2010; Chan and Grant, 2016; Ftiti and Jawadi, 2019).

Thus, we opt to use a stochastic volatility approach. Following Mumtaz and Zanetti (2013), we employ a structural VAR with stochastic volatility in mean. This approach allows for the joint estimation of uncertainty and its impact on the endogenous variables via the introduction of the stochastic volatility in the VAR equation. This joint determination is usually preferable to using an exogenous uncertainty measure in a VAR (e.g., Carriero et al., 2018). In addition to our baseline specification, as a robustness exercise, we estimate a VAR model in which we include an exogenous uncertainty measure from the literature on forecast uncertainty. Both models are described in more detail in the following sections.

2.3 Empirical Model

As a first step in our analysis, we estimate a model, in which the inflation expectation uncertainty measure is determined endogenously. Specifically, we employ a three variable SVAR with stochastic volatility in mean based on the specification by Mumtaz and Zanetti (2013). In this model, the stochastic volatilities are added as additional regressors to the observation equation. This allows for analysing the impact of the time-varying volatility on the endogenous variables. In our analysis, the SVAR-SV-mean is given by

$$Z_t = c_t + \sum_{j=1}^{p} \beta_j Z_{t-j} + \sum_{i=0}^{q} \gamma_i \tilde{h}_{t-i} + u_t \quad u_t \sim N(0, \Omega_t), \quad (1)$$

where $Z_t$ is a vector of endogenous variables, namely the output gap, the inflation rate and expected inflation. Vector $\tilde{h}_t$ contains the log volatility of the corresponding structural shocks. We employ a triangular decomposition of the conditional variance-covariance matrix such that

$$\Omega_t = A^{-1} H_t A^{-1'}.$$  

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4 Similar, but univariate models have been used by, for instance, Berument et al. (2009) and Lemoine and Mougin (2010) to estimate the impact of inflation uncertainty and growth uncertainty, respectively. The univariate stochastic volatility in mean model has been introduced by Koopman and Hol Uspensky (2002).

5 The variables enter the model in this order. The results are robust to choosing an alternative order.
Decomposition matrix $A$ is lower triangular, and Matrix $H_t$ collects the volatility of the structural shocks on the diagonal:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix}, \quad H_t = \begin{pmatrix} e^{h_{1t}} & 0 & 0 \\ 0 & e^{h_{2t}} & 0 \\ 0 & 0 & e^{h_{3t}} \end{pmatrix}. \quad (3)$$

Based on this decomposition of $\Omega_t$, the relation between the reduced-form shocks $u_t$ and the structural shocks $\varepsilon_t$ is given by

$$Au_t = \varepsilon_t. \quad (4)$$

The log volatilities follow an AR(1) process. The transition equation is given by

$$\tilde{h}_t = \theta \tilde{h}_{t-1} + \eta_t, \quad \eta_t \sim N(0, Q), \quad E(u_t, \eta_t) = 0, \quad (5)$$

with $\theta$ and $Q$ being diagonal matrices. An innovation in $\eta_t$ represents a shock to the volatility of the respective structural shock. Thus, a shock to the volatility of the inflation expectation shock signifies the uncertainty shock of interest in this analysis. Via Equation (1), this uncertainty shocks affects the endogenous variables of the system. The inflation expectation uncertainty shock is hence generated endogenously within the system, and its effect on the variables in $Z_t$ can be directly estimated. Consequently, the specification allows for a dynamic approach to study the impact of an uncertainty shock.

Observation equation (1) and transition equation (5) are the building blocks of a non-linear state space model. This model is estimated by means of Bayesian methods, using a Gibbs sampling algorithm. We will provide a short overview of the steps of the algorithm in this section. Mumtaz and Zanetti (2013) provide an in-depth description. Details on priors and starting values can be found in Appendix A.

We draw the elements of matrix $A$, VAR coefficients $\Gamma = [\beta, \gamma]$, the parameters of transition equation $\tilde{h}_t$, matrix $Q$ and the elements of $H_t$ as follows. Inference is based on the last 10,000 of 100,000 iterations.

i. The system of equations resulting from relation (4) is transformed into a representation with homoskedastic errors. Subsequently, conditional on $H_t$, the elements of $A$ are drawn from a normal distribution (see, e.g., Cogley and Sargent, 2005).

ii. Conditional on the other parameters, the reduced-form VAR parameters $\Gamma$ are drawn by means of the Carter-Kohn algorithm (Carter and Kohn, 1994). Using
the Kalman filter obtains $\Gamma_{T|T}$ and variance $P_{T|T}$. The parameters are drawn from a normal distribution.

iii. Conditional on $\tilde{h}_t$, the parameters of the transition equation are drawn from a normal distribution. The elements of $Q$ are drawn from an inverse Gamma distribution.

iv. Conditional on the VAR parameters, the elements of $H_t$ are drawn. The mixture sampler of Kim et al. (1998) applied for the traditional SVAR with stochastic volatility is not applicable for our model since the log volatilities are regressors in the mean equation. Thus, the log volatilities are drawn by means of a date-by-date independence Metropolis step algorithm in accordance with Carlin et al. (1992), Jacquier et al. (1994) and Cogley and Sargent (2005).

As mentioned above, expected changes in prices over the next twelve months from the Michigan Survey of Consumers serve as a proxy for inflation expectations in our analysis. Data for the other variables are obtained from the FRED database by the Federal Reserve Bank of St. Louis. The measure for inflation is the year over year change of the Consumer Price Index for All Urban Consumers. The output gap is determined as the difference between industrial production and potential output, where potential output is obtained by employing the Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997) to the industrial production series, with a smoothing parameter of 14400 for monthly data. The sample runs from January 1983 to March 2017. We test the variables for non-stationarity and reject the presence of unit roots (Table A1 in the Appendix).

We select the lags for the endogenous regressors in the VAR equation ($p$) based on the Ljung-Box test, rejecting the null hypothesis of autocorrelated residuals at six lags. Furthermore, the mean equation incorporates two lags of the log volatility ($q$) in addition to the contemporaneous volatility, in line with Mumtaz and Zanetti (2013).

2.4 Specification with Exogenous Uncertainty Measure

As an alternative to our baseline model, in a second step, we estimate a VAR model with an exogenous measure of uncertainty derived from the literature on forecast uncertainty. The literature considers different measures of forecast uncertainty: aggregate uncertainty obtained from the variance of the aggregate histogram of forecasts, average individual

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6 As a robustness check, we re-estimate the model with alternative measures of economic activity in Section 3.2.
uncertainty given by respondents’ individual forecast error variance, and disagreement among respondents (e.g., Giordani and Söderlind, 2003).

To what extent inter-personal disagreement is an appropriate proxy for uncertainty is subject to debate in the literature since it does not necessarily reflect the individuals’ intra-personal uncertainty (e.g., Zarnowitz and Lambros, 1987; Giordani and Söderlind, 2003; Boero et al., 2008; Lahiri and Sheng, 2010; Rich and Tracy, 2010). However, the findings of Boero et al. (2015) point to the validity of the conflicting findings of previous studies when taking into account the economic environment of the particular samples. The Michigan Survey does not request density forecasts from their respondents, which is why we rely on the survey’s cross-sectional dispersion measure. In line with, for instance, Mankiw et al. (2004) and Dovern et al. (2012), we use the interquartile range of inflation expectations as a measure of disagreement.

In the alternative model, the three variables of interest enter as endogenous regressors, and the interquartile range of the Michigan Survey enters as an exogenous variable. This measure of disagreement is denoted by $d_t$. We choose the lag length of the endogenous variables according to the Schwarz information criterion. Analogously to Equation (1), the VAR is given by

$$Z_t = c + \sum_{j=1}^{2} \beta_j Z_{t-j} + \phi d_t + \xi_t. \quad (6)$$

The estimation of the VAR is based on Bayesian methods, with an informative independent Normal-Wishart prior and a training sample of 60 months. The posterior simulation algorithm is a Gibbs sampler.

3 Empirical Results

3.1 Baseline Results

Figure 2 illustrates the estimated volatilities of the structural shocks in the model. The volatility of the output gap shock shows the highest volatility of the three shocks. There is a notable increase at the time of the Gulf War in the early 1990s. The shock is less volatile during the first half of the 2000s. At the time of the Great Recession, when the output gap dropped sharply, shock volatility reaches a pronounced peak. In line with the Great Moderation, the volatility of the inflation shock is very low during the 1990s and moderately low during the early 2000s. During the second half of the 2000s, volatility is
Notes: This figure shows the volatility estimates of the respective structural shocks, computed from the baseline SVAR-SV-mean specification. Due to a training period of 60 months, the observation period reduces to the years between 1988 and 2017. The solid line denotes the median of the volatility estimates, the shaded areas indicate 68 percent probability bands. Dashed lines denote the underlying time series.

at its highest and peaks during the financial crisis. Similar to the volatility of the output gap shock, it returns to a lower level thereafter.

The structural shock of interest, expected inflation, is on the whole more volatile than the inflation shock. Volatility is moderately high during the first half of the 1990s and right after 9/11. Analogous to the other series, volatility is the highest during the Great Recession. Notably, during that period, inflation expectation shock volatility peaks half a year earlier than the inflation shock volatility. Similar to its counterpart, in the first half of the 2010s, the structural shock is hardly volatile. The volatility of all three shocks increases towards the end of the observation period.

Figure 3 shows the impulse responses to an inflation expectation uncertainty shock, defined as a one standard deviation shock to the volatility of the inflation expectation shock in the SVAR-SV-mean model. The lower right panel displays the response of the volatility, which increases by about 0.27 percent. The response is relatively persistent,
being still almost half as large after five years as on impact.\(^7\)

The output gap decreases in response to the uncertainty shock. The initial response is small, but reaches a trough of nearly -0.15 percent after more than one year. The response is persistent, with the output gap declining well beyond the impulse horizon of five years. The output gap response is the strongest compared to the other variables in the system.

Similarly, the inflation rate only drops slightly on impact. Yet, one year and a half after the shock occurs, CPI inflation decreases by 0.08 percent. The median estimate suggests a decline of 0.05 percent at the 5-year horizon. However, the probability bands include the zero line from the 2-year horizon on. Thus, the response may be less persistent than displayed.

In contrast to the other variables, expected inflation increases in response to an inflation expectation uncertainty shock. However, the increase is very small. Furthermore, only the first two periods are precisely estimated. For the remaining horizon, the probability bands include zero. This result does not suggest a significant effect of inflation expectation

\(^7\) The persistence is driven by the estimate of the corresponding parameter of matrix \(\theta\) in transition equation (5), which is close to one.
uncertainty on the level of inflation expectations. Consequently, the baseline results indicate that inflation expectation uncertainty decreases economic activity and the inflation rate, without a distinct impact on the level of expectations.

3.2 Alternative Measures of Economic Activity

Since our results may be sensitive to the output gap variable, we construct two alternatives. First, we use a different filtering method to decompose the economic activity series into a trend component and a cyclical component. Second, instead of the output gap we use a measure of economic growth. Figure A1 in the appendix illustrates the dynamics of these time series. The overall pattern appears similar, but the magnitude of the output gap derived from the HP filter is substantially lower than that of the other two series.

Constructing a measure of output gap requires obtaining a trend from a time series of economic activity. In the baseline specification, we employ a standard HP filter. However, since the HP filtering method is not without flaws (e.g., Harvey and Jaeger, 1993; Cogley and Nason, 1995; Hamilton, 2018), we re-estimate the model with an output gap derived by the method proposed by Hamilton (2018). This filtering method is based on an OLS regression of a time series $y$ at period $t + h$:

$$y_{t+h} = \beta_0 + \sum_{i=1}^{p} \beta_i y_{t+i-1} + v_{t+h}, \quad (7)$$

where $p$ denotes the lag length. Hamilton (2018) proposes a lag order of $p = 4$ for quarterly data. Since we use monthly data, we choose $p = 12$, corresponding to one year. Further, we set the length of the forecast period $h$ to 24 as suggested by Hamilton (2018). The cyclical component can be obtained from the residuals:

$$\hat{v}_{t+h} = y_{t+h} - \hat{\beta}_0 - \sum_{i=1}^{p} \hat{\beta}_i y_{t+i-1}. \quad (8)$$

As a result, the industrial production measure can be disaggregated into a cyclical and a trend component. Accordingly, an output gap measure can be constructed.

Figure 4 shows the impulse responses to a one standard deviation shock to the volatility of the inflation expectation shock in the SVAR-SV-mean for the specification in which the output gap is derived based on the Hamilton filter. These results are very similar to the baseline results in Figure 3. An obvious difference is the magnitude of the response of the

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8 For instance, the HP filter suffers from an endpoint bias and may generate spurious effects.
output gap. While the inflation expectation shock volatility increases by approximately 0.24 percent, which is not considerably lower compared to the baseline specification, the output gap drops by nearly 0.8 percent at a horizon of one and a half years. The response is very persistent. At the end of the 5-year horizon, the impulse response suggests a decline by 0.62 percent. The difference to the baseline result may stem from the notable differences in magnitude of the underlying output measures (Figure A1). The decline of the inflation rate is slightly stronger compared to the baseline response. Furthermore, the response seems to be more precisely estimated. Expected inflation does not significantly react to the shock in this specification.

Furthermore, there are some general drawbacks to quantifying the unobservable output gap, calculated by a solely statistical approach of filtering a time series. That is why, in a second robustness check, we replace the measure of the output gap by the growth rate of the underlying measure of industrial production.\(^9\) Accordingly, the annual change of industrial production, the inflation rate and expected inflation enter as endogenous variables into the modified specification. Figure 5 displays the corresponding impulse responses.

\(^9\) This measure has been used, \textit{inter alia}, in studies that analyse the relationship between inflation uncertainty and economic activity (e.g., Grier and Perry, 2000; Hartmann and Roestel, 2013).
**Figure 5:** Impulse Responses to an Inflation Expectation Uncertainty Shock – Alternative Economic Activity Measure

**Notes:** This figure shows the responses of the endogenous variables output growth, the inflation rate and expected inflation, to a one standard error shock to the volatility of the inflation expectations shock. The solid line denotes the median response, the shaded areas indicate 68 percent probability bands.

The increase of the shock volatility of expected inflation is slightly smaller than in the previous estimations. It increases by about 0.23 percent on impact. Output growth declines by more than the output gap derived by means of the HP filter. Event though the magnitude of the time series itself is comparable to that of the output gap derived by means of the Hamilton filter, the decrease of output growth is not as strong as the one of the output gap in Figure 4. More than one year after the shock, the impulse response displays a trough of approximately -0.30 percent.

The decline of the inflation rate is similar to the responses above, but slightly stronger. Inflation falls by 0.11 percent two years after the shock occurs. The response is more persistent and more precisely estimated than the ones before. Corresponding to the results of the specification with the Hamilton filter, the impulse response of expected inflation does not indicate a significant effect of expectation uncertainty. The estimation of this specification leads to error bands that include zero already on impact.

Overall, the additional estimations conducted support the results of the baseline
estimation. Inflation expectation uncertainty negatively affects economic activity and the inflation rate. In contrast, the findings do not suggest that inflation expectation uncertainty has a sizeable effect on inflation expectations.

3.3 Exogenous Measure of Uncertainty

We further check the robustness of our results by employing a different empirical model, in which we include an exogenous measure of inflation expectation uncertainty. This alternative measure of uncertainty is given by the interquartile range of inflation expectations from the Michigan Survey.

Figure 6 shows the interquartile range of expected inflation and the estimated volatility of the inflation expectations shock generated by the baseline SVAR-SV-mean, between the years 1988 to 2017. Overall, the two series exhibit a similar pattern. However, the dispersion of the Michigan Survey inflation expectations exhibits more fluctuation. The correlation coefficient between the series is 0.68.

Uncertainty is relatively low at the end of the 1980s, but noticeably increases at the beginning of the 1990s. The period between the mid-1990s and the beginning of the 2000s is generally characterised by lower uncertainty. Both series start to increase in 2000. Whereas shock volatility displays a distinct peak right after 9/11, the average level of dispersion is at a higher level over several years. The financial crisis in 2008 causes a

![Figure 6: Inflation Expectation Uncertainty Measures](image)

*Notes:* This figure shows the measures of inflation expectation uncertainty used in this study, between 1988 and 2017. Due to using a training sample of 60 months, the observation period of the volatility of the inflation expectations shock starts later than the sample period. The bold black line denotes the volatility of the inflation expectations shock, computed from the baseline SVAR-SV-mean specification (left scale). The blue line denotes the interquartile range of inflation expectations from the Michigan Consumer Survey (right scale).
considerable increase in uncertainty. However, beginning at the end of 2011 dispersion and shock volatility return to lower levels.

The impulse responses to a shock corresponding to the standard deviation of the interquartile range variable from model (6) are displayed in Figure 7. These results support the findings of the previous estimations. Due to the characteristics of the different models, persistence is, however, not as pronounced as in the baseline specification.\textsuperscript{10}

In line with the results of the SVAR-SV-mean, the uncertainty shock most strongly affects the output gap in this framework. On impact, the output gap drops by more than 0.10 percent. Subsequently, however, the response diminishes rather quickly. The effects run out after three years. This is different from the baseline results, which showed only a marginal effect on impact, but a greater decrease at a horizon of 16 months. Consistent with Figures 3 to Figures 5, inflation drops slightly on impact. However, the impact effect is not precisely estimated. A trough of -0.02 percent occurs at the 1-year horizon.

\textbf{Figure 7: Impulse Responses to a Disagreement Shock}

\begin{figure}[h!]
\centering
\includegraphics[width=\textwidth]{figure7}
\caption{Impulse Responses to a Disagreement Shock}
\end{figure}

\textit{Notes:} This figure shows the response of the macroeconomic variables to an inflation expectation uncertainty shock, approximated by a shock to the exogenous uncertainty measures in Equation (6). This inflation expectation uncertainty is given by the interquartile range of survey inflation expectations from the Michigan Surveys of Consumers. The bold lines represent the median responses, the shaded areas indicate 68 percent probability bands.

\textsuperscript{10} The AR(1) parameter of matrix $\theta$ in the inflation expectation shock volatility equation in (5) is a determining driver of persistence. Equation (6) does not account for persistence.
The small increase in expected inflation is in line with the previous results. However, the impulse response derived from Equation (6) suggests that inflation expectations decrease slightly in the second year after the shock occurs. Overall, the estimates in this section support the findings of the previous estimations. A disagreement shock leads to a decline in the output gap and inflation.

Thus, the empirical analysis of this study presents evidence that an inflation expectation uncertainty shock is contractionary. The uncertainty shock negatively affects economic activity and the inflation rate. This results is robust to using different measures of economic activity and to using a different measure of uncertainty.

4 Inflation Expectation Uncertainty in a New Keynesian Model

In this section, we employ a DSGE model to rationalise the relationship between inflation expectation uncertainty and inflation expectations, inflation and the output gap. Our model is based on the representative-agent model with capital accumulation and nominal price rigidity by Basu and Bundick (2017). This model is able to reproduce the empirical finding that an uncertainty shock leads to a decrease in inflation and output. Due to a precautionary savings motive, higher uncertainty induces households to increase savings and reduce consumption. Furthermore, higher uncertainty causes households to supply more labour. This precautionary labour supply induces lower marginal costs of firms. Due to price rigidity, the markup of firms increases.

In order to adapt the Basu-Bundick model to our purposes, we introduce an additional shock, i.e. an inflation expectation uncertainty shock. The model is suitable for our analysis because it allows for two transmission channels of such an uncertainty shock. As in all New Keynesian type models, inflation expectations affect economic demand and economic supply (e.g., Clarida et al., 1999). The first channel affects demand via the nominal interest rate. Similar to a time preference shock, an inflation expectation uncertainty shock affects private consumption. Because prices are sticky, output falls. This reduction in overall demand has a negative impact on inflation. In addition, inflation expectation uncertainty enters the New Keynesian Phillips curve. Via this channel, an inflation expectation uncertainty shock leads to an increase of inflation and a fall in output.

The overall effect of an inflation expectation uncertainty shock depends on the characteristics of the economy. As a consequence, the sign and the magnitude of the effect of
an inflation expectation uncertainty shock is independent from monetary policy within
this model. Hence, this model offers an additional explanation for the effects of inflation
expectation uncertainty shocks.

The main building blocks of the adapted model are illustrated in the following. They
are based upon optimising households, optimising firms and a central bank. The repre-
sentative household maximises utility with regard to consumption \( C_t \), labour \( N_t \), a
one-period riskless bond \( B_t \) and equity shares \( S_t \) for all periods

\[
V_t = \max \left[ a_t (C^t_t (1 - N_t)^{1 - \eta})^{(1 - \sigma)/\theta_V} + \beta \left( E_t V_{t+1}^{1-\sigma} \right)^{\theta_V/(1 - \sigma)} \right]^{\theta_V/(1 - \sigma)} \tag{9}
\]

subject to the intertemporal household budget constraint

\[
C_t + \frac{P_t^E}{P_t} S_{t+1} + \frac{1}{R_t^R} B_{t+1} \leq \frac{W_t}{P_t} N_t + \left( \frac{D_t^E}{P_t} + \frac{P_t^E}{P_t} \right) S_t + B_t. \tag{10}
\]

The coefficient on current utility \( a_t \) denotes a shock to the discount rate of households \( \beta \),
i.e. a preference shock. Parameter \( \theta_V \) describes a household’s preference for the resolution
of uncertainty, \( \sigma \) is the parameter for risk aversion and parameter \( \eta \) relates to the Frisch
elasticity of labour supply. Following Basu and Bundick (2017), we employ Epstein-Zin
preferences (Epstein and Zin, 1989) to disentangle risk aversion of households from their
degree of intertemporal substitution. Further, \( P_t \) is the price of the consumption good,
\( P_t^E \) is the equity price and \( D_t^E \) is the dividend of an equity share. \( R_t^R \) is the risk-free rate,
\( W_t \) is the nominal wage.

The first-order conditions imply a stochastic discount factor \( (M) \) between \( t \) and \( t + 1 \):

\[
M_{t+1} = \left( \frac{a_{t+1}}{a_t} \right) \left( \frac{C^\eta_{t+1} (1 - N_{t+1})^{1 - \eta}}{C^\eta_t (1 - N_t)^{1 - \eta}} \right)^{(1 - \sigma)/\theta_V} \left( \frac{C_t}{C_{t+1}} \right) \left( \frac{V_{t+1}^{1-\sigma}}{E_t V_{t+1}^{1-\sigma}} \right)^{1 - 1/\theta_V} \tag{11}
\]

The optimisation behaviour of households obtains four first order conditions, one of which
is the Euler equation for a one-period riskless bond

\[
1 = R_t^R E_t [M_{t+1}]. \tag{12}
\]

Further, monetary policy sets the nominal interest rate \( (R_t) \) following the rule

\[
R_t = R (\Pi_t - \Pi)^{\rho_x} \Delta \eta_t^{\rho_x}, \tag{13}
\]

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where $\Pi_t$ is the inflation rate, $\Delta y_t$ is output growth, and $\rho_\pi$ and $\rho_y$ are the weights of inflation and output, respectively. An Euler equation for the zero net supply risk-free bond is added as an additional equilibrium condition. This condition links inflation expectations to the interest rate and the stochastic discount factor:

$$1 = R_t \mathbb{E}_t \left[ M_{t+1} \frac{1}{\Pi_{t+1}} \right].$$

We modify this equation, allowing for expected inflation to be subject to shocks via stochastic process $\kappa_t$:

$$R_t \mathbb{E}_t [M_{t+1}] = \mathbb{E}_t \left[ \Pi_{t+1} \frac{\kappa_t}{\kappa_{t-1}} \right].$$

In line with Basu and Bundick (2017), we derive the price equation assuming that intermediate goods-producers maximise discounted cash flows using the stochastic discount factor of households. Constraints are given by the production function and the capital accumulation equation including quadratic adjustment costs (Rotemberg, 1982). The first-order condition for this problem yields the New Keynesian Phillips curve (Mumtaz and Zanetti, 2013). This equation implies that due to price adjustment costs ($\phi_P$), producers set prices as a markup ($\mu$) over marginal cost ($\Xi_t$). The equation is given by

$$\phi_P \left( \frac{\Pi_t}{\Pi} - 1 \right) \left( \frac{\Pi_t}{\Pi} \right) = (1 - \theta_\mu) + \theta_\mu \bar{\Xi}_t
+ \phi_P \mathbb{E}_t [M_{t+1}] \left( \mathbb{E}_t \left[ \Pi_{t+1} \frac{\kappa_t}{\kappa_{t-1}} \right] - 1 \right) \left( \frac{Y_{t+1} \mathbb{E}_t \left[ \Pi_{t+1} \frac{\kappa_t}{\kappa_{t-1}} \right]}{Y_t} - 1 \right),$$

where $\theta_\mu$ describes the elasticity of substitution between intermediate goods. Analogous to Equation (15), we have modified the price setting equation of intermediate goods firms by adding stochastic process $\kappa_t$.

The shocks in our specification are governed by the following processes:

$$\kappa_t = (1 - \rho_\kappa) \kappa + \rho_\kappa \kappa_{t-1} + \sigma_\kappa^\kappa \varepsilon_\kappa^\kappa, \quad \varepsilon_\kappa^\kappa \sim N(0, 1),$$

$$\sigma_t^\kappa = (1 - \rho_{\sigma_\kappa}) \sigma_\kappa + \rho_{\sigma_\kappa} \sigma_{t-1}^\kappa + \sigma_{\sigma_\kappa}^\kappa \varepsilon_{\sigma_\kappa}^\kappa, \quad \varepsilon_{\sigma_\kappa}^\kappa \sim N(0, 1),$$

where $\varepsilon_\kappa^\kappa$ represents innovations to the level of $\kappa_t$, while $\varepsilon_{\sigma_\kappa}^\kappa$ represents innovations to the volatility of $\kappa_t$. Hence, $\varepsilon_{\sigma_\kappa}^\kappa$ is a second-moment shock and defined as an inflation expectation uncertainty shock in our setup.

Basu and Bundick (2017) model a demand uncertainty shock – via a second-moment...
shock to $a_t$ – that affects consumption and saving decisions. Precautionary saving reduces consumption and increases saving. This leads to a negative effect on labour.

Similarly, via Equations (11) and (15), we allow for an inflation expectation uncertainty shock to affect the path of consumption and leisure, and thus the demand side of the model. In addition, the inflation expectation uncertainty shock affects the price setting of intermediate goods firms and thus the supply side of the model via Equation (16).

To solve and simulate the model we choose parameter values in line with Basu and Bundick (2017). In addition, we calibrate the parameters of the shock processes for inflation expectation uncertainty to obtain impulse responses with the same scaling as in the empirical analysis. Hence, we set both $\rho_\kappa$ and $\rho_{\sigma_\kappa}$ to 0.8 and $\sigma_{\sigma_\kappa}$ to 0.1. Simulating the effects of a second-moment shock requires a third-order approximation. We solve and simulate the model using a third-order approximation around the steady state in Dynare (Adjemian et al., 2018).

Figure 8 shows the responses of output, inflation and the interest rate to a simulated inflation expectation uncertainty shock. The response of the inflation rate is negative in the short run. This indicates that the demand channel dominates the Phillips curve channel, at least in the short run. Moreover, in the long run, an inflation expectation uncertainty shock has a substantially smaller but positive effect on output, inflation and
5 Conclusions

This study analyses inflation expectation uncertainty and the effect this uncertainty has on economic activity. A structural VAR model with stochastic volatility in mean allows us to assess the impact of changes in the volatility of an expectation shock on inflation expectations, the inflation rate and the output gap. Our results indicate that inflation expectation uncertainty negatively affects the inflation rate and the output gap. Accordingly, expectations are an important channel through which uncertainty affects the economy. The results are robust with respect to the measure of inflation expectation uncertainty and to the measure of economic activity.

Moreover, we use a DSGE model to rationalise our findings. The model contains an aggregate demand channel and an aggregate supply channel of inflation expectation uncertainty shocks. In response to an uncertainty shock, inflation, the output gap and the interest rate decrease. Therefore, within this framework, the effects of an inflation expectation uncertainty shock are dominated by demand side factors.

For monetary policy, our results have important implications. Changes in inflation expectation uncertainty can have economically significant effects on inflation and real economic activity even without substantial changes in inflation expectations. Besides focusing on stabilising the level of inflation expectations to control the future path of inflation, it is hence important to reduce the uncertainty about inflation expectations. This requires monetary authorities to better understand how expectations are formed and what the main determinants of expectation uncertainty are.

This study has focused on the link from uncertainty to expectations, inflation and economic activity. However, inflation expectation uncertainty itself may be affected by the level of the inflation rate or the level of inflation expectations. To account for possible feedback effects, the analysis could be extended by allowing for the endogenous variables to enter the transition equation of the log volatility of the structural shocks. This, computationally more complex task, would be an interesting avenue for future research.
References


A Gibbs Sampling Algorithm

The estimation of the SVAR-SV-mean model is based on the following prior distributions and starting values.

**VAR coefficients**

First, a simpler version of baseline model (1) without stochastic volatility in mean is estimated via OLS, based on a training sample comprising the first $T_0 = 60$ periods. Based on this procedure, preliminary estimates of the stochastic volatilities are obtained.

Then, a VAR including these estimated volatilities as exogenous regressors as in Equation (1) is estimated via OLS, based on the same training sample. This obtains the prior on the reduced-form coefficients: The parameters are given as $\Gamma_0 = (X_0'X_0)^{-1}(X_0'Y_0)$. The VAR residuals are then given by $\hat{u}_t = Y_t - X_0'\Gamma_0$. This gives variance covariance matrix $\Sigma_0 = (\hat{u}_t'\hat{u}_t)/T_0$ and coefficient covariance matrix $p_{0|0} = \Sigma_0 \otimes (X_0'X_0)^{-1}$. The initial state of the parameter matrix is obtained as $\Gamma_{0|0} = vec(\Gamma_0)'$, with corresponding initial state covariance $P_{0|0}$.

**Elements of $A$**

From the OLS estimation of the simpler VAR model, we obtain variance-covariance matrix, $\hat{\Sigma}_0$ from which we derive lower triangular matrix $\hat{A}$, where the diagonal elements are normalised to one. The variance of the off-diagonal elements $\hat{a}_{ij}$ set to $v(\hat{a}_{ij}) = |10 * \hat{a}_{ij}|$. Accordingly, the prior for the elements of $A$ is normal: $a_{0i} \sim N(\hat{a}_{ij}, v(\hat{a}_{ij}))$.

**Elements of $H_i$**

We assume a normally distributed prior for the diagonal elements of $H_i$: $\hat{h}_i \sim N(ln \mu_0, I_3)$, where $\mu_0$ denotes the diagonal variance-covariance elements of $\hat{\Sigma}_0$.

**Parameters of the transition equation**

We assume a normal prior for $\theta$: $\theta_{0} \sim N(0.95, 0.1)$. The elements of the covariance matrix of transition equation (5) are inverse Gamma: $Q_i \sim IG(g_0, v_0)$. The scale parameter $g_0$ is equal to $\frac{0.1}{2}$, the degrees of freedom $v_0$ are set to $\frac{5}{2}$. 

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### B Additional Table

**Table A1: Unit Root Tests: Augmented Dickey-Fuller Test**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Test statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Gap</td>
<td>-6.4926</td>
<td>0.0000</td>
</tr>
<tr>
<td>Expected Inflation</td>
<td>-6.4196</td>
<td>0.0043</td>
</tr>
<tr>
<td>Inflation</td>
<td>-3.7505</td>
<td>0.0030</td>
</tr>
</tbody>
</table>

*Notes:* $H_0$: Variable has a unit root. The test equations for inflation and expected inflation include a constant and a trend; the test equation for the output gap does not include additional regressors. The lag length is based on the Schwarz information criterion (maximum lags set to 17).
C Additional Figure

Figure A1: Measures of Economic Activity

Notes: This figure shows the time series used in the analyses to measure economic activity in the US: The output gap derived by means of the HP filter and by the Hamilton filter, respectively, and industrial production growth. The HP filter is applied to the series of industrial production with a smoothing parameter of 14400. The Hamilton filter is applied to the series of industrial production with a lag order $p = 12$ and forecast horizon $h = 24$. 