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Who Vaccinates When Others Matter? Social-Circle Mediated Altruism in a Heterogeneous Vaccination Game

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Abstract

We develop a population game with heterogeneous infection-loss types and social-circle mediated prosociality, where altruists internalize expected infection losses within a setting-specific circle. Equilibrium admits closed-form cutoff rules and an aggregate non-vaccination rate that reduces to two composites: a private-cost pressure ratio and an altruistic-concern index combining altruist prevalence with circle structure. A utilitarian planner yields a socially optimal cutoff; we characterize when circle-mediated altruism is welfare-improving versus welfare-excessive, implying under- or over-vaccination. We embed subsidies, prosocial pledges, and indirect pressure as primitives and obtain closed-form comparative statics and interaction effects: pledges are marginal substitutes for subsidies and pressure, while subsidies and pressure are marginal complements. Policy leverage is greatest in high-contact, high-vulnerability settings, where calibrated norm-based interventions with modest transfers can dominate stringent pressure or large subsidies.

Keywords: Vaccination games; Altruism; Prosocial preferences; Externalities; Population games; Social circles; Policy design; Crowding out; Heterogeneity.
JEL: C72, D64, I18.

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1 Introduction

Vaccination against infectious diseases is a paradigmatic example of a social dilemma. Vaccination generates private benefits by reducing an individual's infection risk, but it also generates social benefits by reducing transmission and contributing to herd immunity, thus protecting medically vulnerable and unvaccinated individuals. When individuals take these externalities as given, their privately optimal vaccination decisions typically fall short of what is socially desirable. This basic tension has motivated an extensive literature in behavioral economics and game-theoretic epidemiology; see, among many others, Brito et al. (1991); Bauch et al. (2003); Bauch and Earn (2004); Manfredi et al. (2009); Browne (2016); Molina and Earn (2015).

A substantial body of empirical and experimental work shows that vaccination decisions are systematically shaped by prosocial motives. Survey evidence suggests that individuals report caring about protecting others, particularly those who cannot be vaccinated, and that appeals to altruism can increase stated intentions to vaccinate (e.g. Shim et al., 2012; Hershey et al., 1994; Betsch et al., 2013). Laboratory and field experiments similarly document that messages highlighting benefits to vulnerable others or community protection can raise vaccination uptake or intentions, especially among historical non-vaccinators and in environments where vaccination is framed as a social act (e.g. Böhm et al., 2016; Li et al., 2016; Cucciniello et al., 2022). Model-based work has incorporated social norms or prosocial preferences into epidemic models to explain high and low coverage regimes and their dynamics (e.g. Oraby et al., 2014). Finally, real-world proxies for health-related altruism, such as organ donation rates, are predictive of higher vaccination coverage once supply constraints ease (Hierro et al., 2023).

Despite this extensive evidence, most formal models retain either representative agents or homogeneous social preferences. In many contributions, altruism is introduced as a scalar weight on others' utility or as an injunctive social norm that shifts the perceived private benefit of vaccination in a uniform way across individuals and settings. Such formulations are analytically convenient, but they abstract from two features that seem empirically important. First, individuals are embedded in very different social environments: kindergarten classrooms, university campuses, open-plan offices, or nursing homes generate different opportunities for transmission and involve contacts with very different vulnerabilities. Second, individuals differ in their own expected infection losses, reflecting age, comorbidities, and other risk factors.

This paper develops a vaccination game that incorporates both of these aspects in a tractable way. We consider a continuum of individuals who differ in infection-loss types and who can be either self-interested or prosocial. Altruists care about infection losses among a setting-specific social circle, summarized by the perceived intensity of regular contacts and the average vulnerability of those contacts. To the best of our knowledge, this is the first model that explicitly *models altruistic (social) preferences via social-circle mediated concern*, rather than via a homogeneous taste shifter, and that *combines* this with a heterogeneous population in infection costs and a structured analysis of monetary and non-monetary policy instruments.

Formally, each individual chooses whether to vaccinate. Vaccination is fully effective, with an effective generalized cost that summarizes effort, side-effect risk, and access frictions. Non-vaccination exposes the individual to an infection probability that depends on the aggregate non-vaccination share. Infection losses are type-specific. A fraction of the population is purely self-interested; the remainder attaches prosocial weight to infection losses borne by their social circle. We model this prosocial concern in reduced form as shifting an altruist's perceived infection-loss type by an amount that is proportional to the size and vulnerability of the social circle. Social circles therefore matter in two ways: as epidemiological environments and as mediators of prosocial concern.

Our main contributions are fourfold.

(i) **Social-circle mediated altruism in a heterogeneous population.** We introduce a simple but rich specification in which altruistic concern is tied to social-circle structure, characterized by contact intensity and average vulnerability. Combined with heterogeneity in infection-loss types, this yields closed-form cutoffs for self-interested and altruistic individuals and a compact expression for the equilibrium non-vaccination rate.

(ii) **Closed-form equilibrium and welfare characterization.** We show that the aggregate non-vaccination share can be written as a simple function of two composites: a private-cost pressure ratio and an altruistic concern index. A paternalistic utilitarian planner, who values realized health and vaccination outcomes but not altruistic warm glow, chooses a socially optimal cutoff. We derive conditions under which altruism is too weak or too strong from this perspective, and we show how the alignment between individual equilibrium and optimal coverage depends on social-circle characteristics.

(iii) **Modeling of prosocial pledges and indirect pressure.** We explicitly model two non-monetary instruments that frequently arise in practice: prosocial pledges or information campaigns that shift altruistic concern, and indirect (economic or social) pressure to vaccinate that impose a cost on remaining unvaccinated (e.g. workplace requirements or access restrictions). We contribute by embedding such prosocial pledges and indirect pressure in a heterogeneous vaccination game with social-circle mediated altruism.

(iv) **Interaction between monetary and non-monetary instruments.** We map subsidies, prosocial pledges, and soft-mandate or indirect-pressure instruments into the model's primitives and derive closed-form comparative statics and cross-partials. The analysis shows that prosocial pledges are *substitutes at the margin* for both monetary incentives and indirect pressure—each crowds out the other's marginal impact on equilibrium coverage—whereas subsidies and indirect-pressure instruments are *complements at the margin* and reinforce one another's marginal effect. These interaction patterns yield simple design rules for targeting and sequencing monetary and non-monetary interventions.

Methodologically, our framework is deliberately stylized: we work with a static reduced-form infection probability and exogenous social circles, and we introduce heterogeneity only in infection losses and social preferences. This allows us to obtain closed-form expressions that make the effects of different instruments transparent. We view extensions to richer epidemic dynamics, more detailed network structures, and multidimensional heterogeneity as promising avenues for future work.

The remainder of the paper is structured as follows. Section 2 presents the model, derives the equilibrium, and characterizes the social optimum. Section 3 analyzes the effects of subsidies, prosocial pledges, and indirect pressure and discusses the substitutability of monetary and non-monetary instruments. Section 4 concludes. Appendices collect additional comparative statics and welfare results.

2 The Model

We consider a static binary vaccination choice in a large population. Each individual takes aggregate behavior as given, so an individual's action has negligible impact on aggregate infection risk. Heterogeneity enters through (i) idiosyncratic infection losses and (ii) behavioral type (self-interested versus altruistic). Altruistic motives are mediated by a reduced-form social circle.

2.1 Environment, types, and social circles

The population is a continuum of mass one. Each individual i chooses an action

$$a_i \in \{V, N\},$$

where V denotes vaccination and N non-vaccination. Vaccination is fully effective: vaccinated individuals acquire immunity in the relevant decision period and face zero infection risk. Non-vaccinated individuals face an infection probability that depends on the aggregate non-vaccination share $\pi \in [0, 1]$.

Each individual has two primitive characteristics:

(a) *Infection-loss type* $\theta \sim U[0, 1]$, drawn independently across individuals. This type scales the realized loss if infected.

(b) *Behavioral type* $b \in \{E, A\}$. A mass $\sigma \in [0, 1]$ of individuals are self-interested ($b = E$), and the remaining mass $1 - \sigma$ are altruistic ($b = A$). Behavioral type is independent of θ.

Vaccination entails a generalized disutility $c_v\lambda > 0$, which captures physical discomfort, time and access costs, and expected harm from perceived adverse events.[1] Infection causes a loss $c_i\theta$, where $c_i > 0$ scales the overall severity of illness.

The infection probability for an unvaccinated individual is given by the reduced form

$$P_i(\pi) = \beta\,\pi, \qquad \beta > 0, \tag{1}$$

where π is the aggregate non-vaccination share and β is an infection-risk scale parameter. We assume primitives are such that $P_i(\pi) \in [0, 1]$ for all $\pi \in [0, 1]$ (e.g. $\beta \leq 1$). This linear specification abstracts from dynamic transmission and herd-immunity thresholds; a robustness discussion in the Appendix A shows that our main comparative statics extend to convex risk functions $P_i(\pi) = \beta\pi^\eta$ with $\eta \geq 1$.

Social circles. Individuals are embedded in (and care about) specific contact settings. In practice, people typically face multiple settings—e.g. a close personal network (family and close friends) and a broader workplace or institutional environment—whose contact intensity and average vulnerability can differ substantially. Modeling multiple settings explicitly would require allowing setting-specific pairs $(n, \bar{\theta})$ for each individual and aggregating across them. To keep the analysis tractable, we abstract from this multi-setting structure and represent the relevant social environment by a single reduced-form social circle described by an exogenous pair

$$n \in \mathbb{R}_+, \qquad \bar{\theta} \in [0, 1],$$

[1]Formally, one may interpret $c_v > 0$ as the disutility of a severe adverse event and $\lambda \in [0, 1]$ as its perceived probability; only the product $c_v\lambda$ enters the decision problem, so we treat it as a single effective cost parameter.

Table 1: Notation (primitives and key composites)

Symbol	Definition
a_i	Individual action: vaccinate (V) or not (N).
π	Population non-vaccination rate.
θ	Infection-loss type, $\theta \sim U[0,1]$.
b	Behavioral type: self-interested (E) or altruistic (A).
σ	Share of self-interested individuals ($\Pr[b = E] = \sigma$).
$c_v\lambda$	Effective private vaccination disutility.
c_i	Infection-loss scale parameter.
β	Infection-risk parameter in $P(\pi) = \beta\pi$.
n	Contact intensity / number of regular contacts (reference group).
$\bar{\theta}$	Average vulnerability in the reference group (circle of concern).
α	Prosocial weight on others' infection losses.
k	Private-cost pressure ratio: $k := \frac{c_v\lambda}{\beta c_i}$.
A	Aggregate altruistic concern: $A := (1-\sigma)\alpha n\bar{\theta}$.

where n captures the perceived intensity (or number) of regular contacts and $\bar{\theta}$ captures the average vulnerability of contacts in terms of infection losses. We interpret $\bar{\theta}$ as the average vulnerability within the individual's *reference group* (circle of concern) that enters prosocial motives, rather than as a population-wide vulnerability index. Allowing multiple contact settings with distinct $(n, \bar{\theta})$ pairs, and studying how incentives and welfare trade-offs vary across them, is left for future work.

Prosocial concern and a reduced-form specification. A self-interested unvaccinated individual bears expected infection loss $\beta\pi c_i\theta$. An altruist additionally internalizes (in reduced form) infection losses borne by her n contacts with average type $\bar{\theta}$. With prosocial weight $\alpha \geq 0$, expected loss from non-vaccination is

$$\beta\pi c_i\big(\theta + \alpha n\bar{\theta}\big),$$

so altruists behave as if their effective infection-loss type were shifted by $\alpha n\bar{\theta}$.

2.2 Preferences and individual decisions

Self-interested types. A self-interested individual of type θ vaccinates iff

$$c_v\lambda \leq \beta\pi c_i\theta.$$

Hence there exists a cutoff θ_e such that self-interested individuals vaccinate iff $\theta \geq \theta_e$. The indifference condition is

$$c_v\lambda = \beta\,\pi\,c_i\,\theta_e. \tag{2}$$

Altruistic types. An altruist behaves as if her effective type were $\theta + \alpha n\bar{\theta}$ and vaccinates iff

$$c_v\lambda \leq \beta\pi c_i(\theta + \alpha n\bar{\theta}).$$

Thus there exists a cutoff θ_a such that altruists vaccinate iff $\theta \geq \theta_a$, with indifference condition

$$c_v\lambda = \beta\,\pi\,c_i\,(\theta_a + \alpha n\bar{\theta}). \tag{3}$$

Aggregation implies

$$\pi = \sigma\,\theta_e + (1-\sigma)\,\theta_a. \tag{4}$$

2.3 Equilibrium and closed-form characterization

We focus on interior equilibria with $0 < \theta_a, \theta_e, \pi < 1$. Throughout this equilibrium analysis we treat the reference-group vulnerability parameter $\bar{\theta}$ as fixed and common across individuals (e.g., normalized to $\bar{\theta} = 0.5$), thereby abstracting from heterogeneity in contacts' vulnerability.

An interior equilibrium is characterized by the two indifference conditions (2)–(3) and the aggregation condition (4). Subtracting (3) from (2) (for $\pi > 0$) yields

$$\theta_e = \theta_a + \alpha n\bar{\theta}, \tag{5}$$

so altruists vaccinate (weakly) more than self-interested individuals, with the difference pinned down by social-circle mediated concern.

Substituting (5) into (4) gives

$$\pi = \sigma\theta_e + (1-\sigma)\theta_a = \theta_e - (1-\sigma)\alpha n\bar{\theta}.$$

Define two composites,

$$k := \frac{c_v\lambda}{\beta c_i} > 0, \qquad A := (1-\sigma)\alpha n\bar{\theta} \geq 0, \tag{6}$$

where k is the private-cost pressure ratio and A is aggregate altruistic concern. Using

$\pi = \theta_e - A$ in (2) implies

$$k = \theta_e(\theta_e - A),$$

i.e. the quadratic

$$\theta_e^2 - A\theta_e - k = 0. \tag{7}$$

The economically relevant root is

$$\theta_e^* = \frac{A + \sqrt{A^2 + 4k}}{2}, \qquad \theta_a^* = \theta_e^* - \alpha n\bar{\theta} = \frac{\sqrt{(1-\sigma)^2(\alpha n\bar{\theta})^2 + 4k} - (1+\sigma)\alpha n\bar{\theta}}{2}, \tag{8}$$

and the corresponding individual-equilibrium non-vaccination share is

$$\pi^{ind}(A, k) = \theta_e^* - A = \frac{\sqrt{A^2 + 4k} - A}{2}. \tag{9}$$

Behavioral comparative statics of the cutoffs. The behavioral composition parameter σ and the altruism weight α represent conceptually distinct margins of prosocial heterogeneity: σ governs the *prevalence* of altruistic types, while α governs the *intensity* of prosocial concern among altruists. Studying the effects of both of them on the equilibrium cutoffs and distinguishing these margins is important because they generate different equilibrium spillovers through the endogenous infection risk and, consequently, different policy implications. In particular, changes in σ operate primarily through the risk externality by shifting aggregate non-vaccination and thus incentives for *both* behavioral classes, whereas changes in α combine a direct preference channel for altruists with an offsetting risk-mediated channel that can induce egoist free-riding.

With $k > 0$ and $\alpha n\bar{\theta} > 0$, holding $(k, n, \bar{\theta})$ fixed,

$$\frac{\partial\theta_e^*}{\partial\sigma} < 0, \quad \frac{\partial\theta_a^*}{\partial\sigma} < 0; \qquad \frac{\partial\theta_e^*}{\partial\alpha} > 0 \ (\sigma < 1), \quad \frac{\partial\theta_a^*}{\partial\alpha} < 0.$$

Thus, increasing σ (fewer altruists) reduces aggregate altruistic concern and raises equilibrium infection risk, which strengthens the private return to vaccination and lowers both cutoffs—raising vaccination among egoists and altruists alike. By contrast, increasing α strengthens altruists' prosocial incentive to vaccinate (lowering θ_a^*), but the induced decline in equilibrium risk attenuates egoists' private incentive and raises their cutoff (θ_e^*), reflecting free-riding on others' protective behavior.[2]

Comparative statics of the interior equilibrium. On the interior region,

[2]Full derivative calculations for the comparative statics of θ_e^* and θ_a^* with respect to σ and α are reported in Appendix B.

$$\frac{\partial \pi^{ind}}{\partial A} = \frac{1}{2}\left(\frac{A}{\sqrt{A^2+4k}} - 1\right) < 0, \qquad \frac{\partial^2 \pi^{ind}}{\partial A^2} = \frac{2k}{(A^2+4k)^{3/2}} > 0. \tag{10}$$

Higher altruistic concern A lowers the equilibrium non-vaccination share, with diminishing marginal effects.

Similarly,

$$\frac{\partial \pi^{ind}}{\partial k} = \frac{1}{\sqrt{A^2+4k}} > 0, \qquad \frac{\partial^2 \pi^{ind}}{\partial k^2} = -\frac{2}{(A^2+4k)^{3/2}} < 0. \tag{11}$$

Higher private-cost pressure k (higher perceived vaccination cost or lower infection harm) increases non-vaccination, again with diminishing marginal effects as k grows.

The cross-partial derivative

$$\frac{\partial^2 \pi^{ind}}{\partial A\,\partial k} = -\frac{A}{(A^2+4k)^{3/2}} \leq 0 \tag{12}$$

shows that greater altruistic concern reduces both non-vaccination and its sensitivity to private-cost pressure.

Polar cases. If $\sigma = 1$ (self-interested only), then $A = 0$ and $\pi^{ind}_{\sigma=1} = \sqrt{k}$. If $\sigma = 0$ (altruists only), then $A = \alpha n\bar{\theta}$ and

$$\pi^{ind}_{\sigma=0} = \frac{\sqrt{(\alpha n\bar{\theta})^2 + 4k} - \alpha n\bar{\theta}}{2}. \tag{13}$$

For $0 < \sigma < 1$, the mixed case is summarized entirely by (9) through $A = (1-\sigma)\alpha n\bar{\theta}$.

2.4 Social optimum

We now introduce a social planner who chooses vaccination coverage to maximize a paternalistic utilitarian objective. The planner aggregates realized infection losses and vaccination disutility, but deliberately excludes the altruistic component from the objective to avoid double counting prosocial concern. In other words, the planner values realized health and vaccination outcomes, not the warm-glow utility that altruists derive from protecting others.[3]

[3] An alternative normative specification would include altruists' intrinsic utility from acting prosocially—often described as *warm glow*, moral satisfaction, or identity utility from "doing the right thing"—directly in the social objective. If people experience utility from acting altruistically, that utility is real and, under this view, should be included in social welfare. In our formulation, the planner's optimal vaccination level trades off the real costs of vaccination (e.g. time, side effects, discomfort) against real infection harms (illness losses) and therefore abstracts from such intrinsic prosocial satisfaction. Under this outcome-based objective, altruists may vaccinate more than the planner's preferred level ("over-vaccination"), because additional vaccination costs are incurred without commensurate health gains. If intrinsic prosocial satisfaction were

Formally, the planner chooses a population-wide cutoff $\tilde{\theta} \in [0,1]$ such that a mass $\tilde{\theta}$ remains unvaccinated. Under the uniform-type normalization, this implies that the aggregate non-vaccination rate is

$$\pi = \tilde{\theta}.$$

Given a cutoff $\tilde{\theta}$, individuals with $\theta < \tilde{\theta}$ remain unvaccinated and face infection probability $\beta\tilde{\theta}$, while those with $\theta \geq \tilde{\theta}$ are vaccinated and incur disutility $c_v\lambda$. The associated social welfare is

$$\begin{aligned} W(\tilde{\theta}) &= \int_0^{\tilde{\theta}} \big(-\beta\,\tilde{\theta}\,c_i\,\theta \big)\,d\theta \;+\; \int_{\tilde{\theta}}^1 \big(-c_v\,\lambda \big)\,d\theta \\ &= -\frac{1}{2}\,\beta\,c_i\,\tilde{\theta}^3 \;-\; (1-\tilde{\theta})\,(c_v\,\lambda). \end{aligned} \tag{14}$$

The interior first-order condition for a welfare-maximizing cutoff is

$$\frac{dW}{d\tilde{\theta}} = -\frac{3}{2}\beta c_i \tilde{\theta}^2 + c_v\lambda = 0,$$

which implies

$$\tilde{\theta}^2 = \frac{2}{3}\frac{c_v\lambda}{\beta c_i} = \frac{2}{3}k,$$

where k is the private-cost pressure ratio defined in (6). Thus the interior socially optimal non-vaccination share is

$$\pi^{opt}(k) = \tilde{\theta}^{opt} = \sqrt{\tfrac{2}{3}k}. \tag{15}$$

Equilibrium comparison, welfare, and the role of altruism. In a purely self-interested population ($\sigma = 1$), the individual equilibrium non-vaccination share is

$$\pi^{ind}_{\sigma=1} = \sqrt{k} > \sqrt{\tfrac{2}{3}k} = \pi^{opt}(k),$$

so non-cooperative behavior underprovides vaccination relative to the planner's optimum. In a purely altruistic population ($\sigma = 0$), the individual equilibrium non-vaccination share is given by (13). Comparing $\pi^{ind}_{\sigma=0}$ with $\pi^{opt}(k)$ yields a threshold for the degree of altruism at which the altruistic equilibrium coincides with the social optimum.

Equating $\pi^{ind}_{\sigma=0}$ and $\pi^{opt}(k)$ and solving for α (for $n > 0$ and $\bar{\theta} > 0$) gives the critical

included as a welfare-relevant benefit, however, the extra utility generated by altruistic vaccination could offset (or outweigh) the additional vaccination costs, and the normative assessment of "excessive" prosocial motivation would generally change.

altruism level

$$\alpha^*(k,n,\bar{\theta}) = \frac{\sqrt{k/6}}{n\bar{\theta}}. \tag{16}$$

For $\alpha < \alpha^*(k,n,\bar{\theta})$, altruists under-vaccinate relative to the planner ($\pi^{ind}_{\sigma=0} > \pi^{opt}(k)$); for $\alpha > \alpha^*(k,n,\bar{\theta})$, they over-vaccinate ($\pi^{ind}_{\sigma=0} < \pi^{opt}(k)$). In this sense, social-circle mediated altruism can be either insufficient or excessive, depending on the environment $(n,\bar{\theta})$ and the private-cost pressure k.

In the mixed population with $0 < \sigma < 1$, the aggregate altruistic concern is

$$A = (1-\sigma)\alpha n\bar{\theta},$$

so changes in either the behavioral composition σ or the degree of altruism α shift the individual equilibrium $\pi^{ind}(A,k)$ relative to the social optimum $\pi^{opt}(k)$. Given the self-interested cutoff θ_e^* and the altruistic cutoff θ_a^*, and using a common infection probability $\beta\pi$ for all unvaccinated individuals, aggregate welfare in the mixed population can be written as

$$\begin{aligned} W^{ind}(\alpha,\sigma) = \sigma &\left[\int_0^{\theta_e^*} \big(-\beta\,\pi\,c_i\,\theta\big)\,d\theta \;+\; \int_{\theta_e^*}^1 \big(-c_v\,\lambda\big)\,d\theta\right] \\ &+ (1-\sigma)\left[\int_0^{\theta_a^*} \big(-\beta\,\pi\,c_i\,\theta\big)\,d\theta \;+\; \int_{\theta_a^*}^1 \big(-c_v\,\lambda\big)\,d\theta\right], \end{aligned} \tag{17}$$

where $\pi = \sigma\theta_e^* + (1-\sigma)\theta_a^*$, and θ_e^*, θ_a^* depend on the degree of altruism α via the altruistic concern

$$A = (1-\sigma)\alpha n\bar{\theta}.$$

Expression (17) makes clear that varying σ affects welfare through two channels: the composition of types (the relative weights on the two brackets) and the induced change in equilibrium coverage $\pi^{ind}(A(\sigma),k)$ via $A = (1-\sigma)\alpha n\bar{\theta}$. Starting from a purely self-interested benchmark ($\sigma = 1$), a small reduction in σ (introducing some altruists) raises A, reduces $\pi^{ind}(A,k)$, and moves equilibrium coverage closer to $\pi^{opt}(k)$ whenever $\pi^{ind}_{\sigma=1} > \pi^{opt}(k)$; in this region, $W^{ind}(\alpha,\sigma)$ increases as the share of altruists rises. However, once altruism is strong enough that $\pi^{ind}(A(\sigma),k)$ falls below $\pi^{opt}(k)$, further reductions in σ push coverage beyond the planner's preferred level and lower welfare. Hence, for parameter values where the purely self-interested and purely altruistic endpoints lie on opposite sides of the social optimum, there exists an interior composition $\sigma^*(\alpha,k,n,\bar{\theta}) \in (0,1)$ at which mixed-population welfare is maximized: welfare initially increases as the share of altruists grows, but after a certain point additional altruism becomes welfare-reducing.[4]

Second-best policy under unobservable behavioural types. The analysis above establishes that in a purely self-interested population, equilibrium vaccination is inefficiently low, i.e. $\theta_e^* > \tilde{\theta}^{opt}$, where $\tilde{\theta}^{opt}$ denotes the planner's target cutoff implied by (15). In a purely altruistic population, equilibrium vaccination may be socially excessive under the planner's objective (i.e. $\theta_a^* < \tilde{\theta}^{opt}$) when prosocial concern is sufficiently strong. In a mixed population, equilibrium cutoffs satisfy $\theta_e^* > \theta_a^*$, and for parameter values such that the polar equilibria lie on opposite sides of the planner's target (i.e. $\pi_{\sigma=1}^{ind} > \pi^{opt}(k)$ and $\pi_{\sigma=0}^{ind} < \pi^{opt}(k)$), the planner's target cutoff $\tilde{\theta}^{opt}$ lies between these behavioural cutoffs.

Crucially, however, the planner cannot observe and target behavioural types. Accordingly, a behavioural-type-contingent first-best implementation is infeasible: the planner cannot directly implement the planner's target by assigning different vaccination rules or incentives to self-interested and altruistic individuals. Policy must therefore be understood as a *second-best* problem. Nevertheless, the planner can induce an implementable outcome by deploying uniform instruments (e.g. a subsidy, a prosocial campaign, or indirect pressure) and choosing their intensity so as to move the resulting equilibrium non-vaccination rate as close as possible to the planner's target $\pi^{opt}(k)$ (equivalently, the target cutoff $\tilde{\theta}^{opt}$). This creates a design tension: an overly strong intervention may induce excessive vaccination among altruists, whereas an overly weak intervention leaves too many self-interested individuals unvaccinated. Hence, second-best policy requires both careful tailoring of instrument choice and careful calibration of intervention intensity.

3 Policy Analysis

We now study how standard vaccination policy instruments map into the model's primitives and how they affect equilibrium coverage and its alignment with the paternalistic social optimum. We focus on three levers:

(i) a *per-dose subsidy* s that lowers the effective private vaccination cost;

(ii) a *prosocial pledge or nudge* ε that raises prosocial concern;

(iii) a *soft mandate or indirect pressure* δ that imposes an additional cost on non-vaccination.

[4]For more details on how equilibrium welfare varies with respect to the behavioral composition parameter σ, see Appendix D.

Throughout this section we treat these instruments as exogenous levers and analyze their *comparative-static* effects on the individual equilibrium π^{ind} and its gap from the planner's optimum $\pi^{opt}(k)$. We do not model the fiscal or political costs of implementing these instruments explicitly; a full optimal-policy problem would specify cost functions for s, ε, δ and is beyond the scope of the present paper.

Recall that in the baseline model the equilibrium non-vaccination share is

$$\pi^{ind}(A,k) = \frac{\sqrt{A^2+4k}-A}{2},$$

where $k = \frac{c_v\lambda}{\beta c_i}$ is the private-cost pressure ratio and $A = (1-\sigma)\alpha n\bar{\theta}$ is the aggregate altruistic concern. The planner's socially optimal non-vaccination share is

$$\pi^{opt}(k) = \sqrt{\tfrac{2}{3}k}.$$

3.1 Subsidy s

Policy mapping. A subsidy $s \geq 0$ reduces the *private* generalized vaccination cost from $c_v\lambda$ to $c_v\lambda - s$ without changing the intrinsic welfare cost of vaccination.[5] The effective private-cost pressure becomes

$$k(s) = k_0 - \frac{s}{\beta c_i}, \qquad k_0 := \frac{c_v\lambda}{\beta c_i}, \tag{18}$$

while the altruistic concern $A = (1-\sigma)\alpha n\bar{\theta}$ is unchanged.

Equilibrium coverage response. All equilibrium expressions carry through after the substitution $k \mapsto k(s)$. On the interior region $(0 < \theta_e, \theta_a, \pi < 1)$,

$$\pi^{ind}(A,k(s)) = \frac{\sqrt{A^2+4k(s)}-A}{2}, \qquad \frac{\partial\pi^{ind}}{\partial s} = -\frac{1}{\beta c_i\sqrt{A^2+4k(s)}} < 0. \tag{19}$$

Thus, larger subsidies strictly reduce the non-vaccination share (raise vaccination coverage). The marginal effect $|\partial\pi^{ind}/\partial s|$ diminishes as either altruistic concern A or residual private-cost pressure $k(s)$ grows, because $\sqrt{A^2+4k(s)}$ increases in both arguments.

When s is large enough that $k(s) \leq 0$, vaccination weakly dominates non-vaccination for all types, so the unique equilibrium is full coverage $\pi^{ind} = 0$.[6]

[5] In the planner's objective, $c_v\lambda$ remains the fundamental disutility of vaccination. The subsidy is a fiscal transfer from the government to vaccinated individuals.

[6] An individual vaccinates if $c_v\lambda - s \leq \beta\pi c_i\theta$. When $s \geq c_v\lambda$, the left-hand side is nonpositive while the

Planner alignment. The utilitarian planner's first-best non-vaccination rate depends only on the intrinsic cost ratio k_0, not on the subsidy level:

$$\pi^{opt}(k_0) = \sqrt{\tfrac{2k_0}{3}} \quad (\leq 1).$$

A subsidy aligns equilibrium behavior with the planner's target if it induces $\pi^{ind}(A, k(s^*)) = \pi^{opt}(k_0)$. Using (19), this requires

$$\frac{\sqrt{A^2 + 4k(s^*)} - A}{2} = \sqrt{\tfrac{2k_0}{3}},$$

which implies

$$k(s^*) = A\sqrt{\tfrac{2k_0}{3}} + \frac{2k_0}{3}.$$

Substituting (18) yields the required subsidy

$$s^* = \beta c_i \left(\frac{k_0}{3} - A\sqrt{\tfrac{2k_0}{3}} \right). \tag{20}$$

Feasibility requires $0 \leq s^* < c_v\lambda$. If $s^* < 0$, the individual equilibrium already over-internalizes the externality relative to the planner's benchmark, in which case a (small) tax would be needed to restore $\pi^{opt}(k_0)$.

Two comparative statics are immediate:

- *Substitutability with altruism:* $\partial s^*/\partial A < 0$. Stronger altruistic concern reduces the subsidy required for first-best alignment.

- *Dependence on private frictions:* $\partial s^*/\partial k_0 > 0$. Higher baseline private-cost pressure necessitates larger transfers to attain the planner's target.

Design implications. A uniform subsidy is easy to administer and can generate sizable coverage gains, but it also pays inframarginal recipients who would have vaccinated even without the policy.[7] From a fiscal perspective, the relevant efficiency measure is the *cost per additional vaccination*, which can greatly exceed the nominal subsidy when baseline coverage is already high. The diminishing marginal effect in (19) and the dependence of s^* on A in (20) suggest that broad subsidies are most cost-effective when altruistic concern is low and private frictions are moderate.

right-hand side is nonnegative for all $\theta \in [0, 1]$ and any $\pi \in [0, 1]$, so the inequality holds for every type (with indifference only when $s = c_v\lambda$).

[7] Wong et al. (2025) provide a narrative review of recent research on the influence of financial incentives on vaccination hesitancy.

3.2 Prosocial pledge / nudge via an altruism shift ε

Policy mapping. A prosocial pledge, information campaign, or norm-based intervention aims to raise the degree of prosocial concern. We model this as an increase in the altruism weight from α to $\alpha' = \alpha + \varepsilon$, with $\varepsilon \geq 0$. The aggregate altruistic concern becomes

$$A' = (1-\sigma)(\alpha+\varepsilon)n\bar{\theta} = A + (1-\sigma)n\bar{\theta}\,\varepsilon. \tag{21}$$

The private-cost pressure k remains unchanged, so the intervention operates through preferences rather than prices.

Equilibrium coverage response. On the interior region,

$$\pi^{ind}(A', k) = \frac{\sqrt{A'^2 + 4k} - A'}{2}.$$

For convenience, define $B := (1-\sigma)n\bar{\theta} \geq 0$ and $X(\varepsilon) := A + B\varepsilon$. Then

$$\pi^{ind}(\varepsilon) = \frac{\sqrt{X(\varepsilon)^2 + 4k} - X(\varepsilon)}{2}.$$

Differentiating with respect to ε yields

$$\frac{d\pi^{ind}}{d\varepsilon} = \frac{B}{2}\left(\frac{X(\varepsilon)}{\sqrt{X(\varepsilon)^2 + 4k}} - 1\right) < 0, \tag{22}$$

because $\sqrt{X(\varepsilon)^2 + 4k} > X(\varepsilon)$ for all $k > 0$. Stronger prosocial concern thus strictly reduces the non-vaccination share.

A second differentiation gives

$$\frac{d^2\pi^{ind}}{d\varepsilon^2} = \frac{2k\,B^2}{\left(X(\varepsilon)^2 + 4k\right)^{3/2}} > 0, \tag{23}$$

so $\pi^{ind}(\varepsilon)$ is convex: the marginal effect of additional prosocial concern becomes less negative as ε grows. For $B > 0$,

$$\lim_{\varepsilon\to\infty} \pi^{ind}(\varepsilon) = 0.$$

Calibration to the planner's target. The planner's first-best non-vaccination share $\pi^{opt}(k) = \sqrt{2k/3}$ does not depend on α or ε, because altruistic warm-glow is excluded from the welfare function. To achieve $\pi^{ind}(\varepsilon^*) = \pi^{opt}(k)$, we require

$$\frac{\sqrt{\big(A + B\varepsilon^*\big)^2 + 4k} - \big(A + B\varepsilon^*\big)}{2} = \sqrt{\tfrac{2k}{3}},$$

which is equivalent to

$$A + B\varepsilon^* = \sqrt{\tfrac{k}{6}}.$$

Thus the prosocial shift needed to align equilibrium behavior with the planner's benchmark is

$$\varepsilon^* = \frac{\sqrt{k/6} - A}{(1-\sigma)n\bar{\theta}}. \tag{24}$$

If baseline altruistic concern already satisfies $A \geq \sqrt{k/6}$, no further increase in altruism is required for first-best alignment; if $A < \sqrt{k/6}$, the formula gives the exact increase in prosocial concern that would close the equilibrium–planner gap.

3.3 Soft mandate / indirect pressure

Policy mapping. We model *indirect (economic or social) pressure to vaccinate* as an additional cost of remaining unvaccinated. Such pressure was, for example, salient during the COVID-19 pandemic: unvaccinated individuals could face restrictions on visiting vulnerable relatives, exclusion from certain workplaces or venues, and in extreme cases loss of employment and income. Let $c_p \geq 0$ denote the baseline "pressure cost" associated with being unvaccinated, interpreted broadly to include economic losses (e.g. foregone earnings) and social or psychological losses (e.g. limited family interactions or social participation). Individuals differ in exposure and responsiveness to these pressures; let $\psi \in [0,1]$ capture the individual-specific intensity or sensitivity to this pressure. Define the normalized parameter

$$\delta := \frac{\psi c_p}{\beta c_i} \geq 0. \tag{25}$$

Remaining unvaccinated then entails an effective generalized loss $\beta\pi c_i\theta + \psi c_p$, while vaccinating still costs $c_v\lambda$. The individual indifference conditions become

$$\text{(self-interested)} \quad c_v\lambda = \beta\,\pi\,c_i\,\theta_e + \psi c_p \quad \Longleftrightarrow \quad k = \pi\theta_e + \delta, \tag{26}$$

$$\text{(altruist)} \quad c_v\lambda = \beta\,\pi\,c_i\big(\theta_a + \alpha n\bar{\theta}\big) + \psi c_p \quad \Longleftrightarrow \quad k = \pi\big(\theta_a + \alpha n\bar{\theta}\big) + \delta. \tag{27}$$

Equilibrium coverage response. All equilibrium expressions carry through after the substitution $k \mapsto k - \delta$, provided $k > \delta$. On the interior region,

$$\pi^{ind}(A, k, \delta) = \frac{\sqrt{A^2 + 4(k-\delta)} - A}{2}, \qquad 0 \le \delta < k, \tag{28}$$

and

$$\frac{\partial \pi^{ind}}{\partial \delta} = -\frac{1}{\sqrt{A^2 + 4(k-\delta)}} < 0, \qquad \frac{\partial^2 \pi^{ind}}{\partial \delta^2} = -\frac{2}{\big(A^2 + 4(k-\delta)\big)^{3/2}} < 0. \tag{29}$$

Indirect pressure strictly reduces the non-vaccination share and does so in a concave way: the marginal impact $|\partial \pi^{ind}/\partial \delta|$ rises as the residual margin $k - \delta$ shrinks.

The cross-partials

$$\frac{\partial^2 \pi^{ind}}{\partial A\, \partial \delta} = \frac{A}{\big(A^2 + 4(k-\delta)\big)^{3/2}} \ge 0, \qquad \frac{\partial^2 \pi^{ind}}{\partial k\, \partial \delta} = \frac{2}{\big(A^2 + 4(k-\delta)\big)^{3/2}} > 0 \tag{30}$$

show that indirect pressure is less potent at the margin when altruistic concern A or private-cost pressure k is high.

Social planner and alignment. The planner internalizes all experienced disutility, including the pressure cost borne by the unvaccinated. For a cutoff $\tilde{\theta}$ (so $\pi = \tilde{\theta}$), social welfare is

$$\begin{aligned} W(\tilde{\theta}) &= \int_0^{\tilde{\theta}} \big(- \beta\, \tilde{\theta}\, c_i\, \theta - \psi c_p \big)\, d\theta + \int_{\tilde{\theta}}^1 \big(- c_v \lambda \big)\, d\theta \\ &= -\frac{1}{2}\, \beta c_i\, \tilde{\theta}^3 \;-\; \psi c_p\, \tilde{\theta} \;-\; (1 - \tilde{\theta})\, c_v \lambda. \end{aligned} \tag{31}$$

The interior first-order condition is

$$\frac{dW}{d\tilde{\theta}} = -\frac{3}{2}\beta c_i \tilde{\theta}^2 - \psi c_p + c_v \lambda = 0 \quad \Longleftrightarrow \quad \tilde{\theta}^2 = \frac{2}{3}\, \frac{c_v \lambda - \psi c_p}{\beta c_i} = \frac{2}{3}(k - \delta),$$

so the planner's optimal non-vaccination share is

$$\pi^{opt}(k, \delta) = \sqrt{\tfrac{2}{3}(k - \delta)}, \qquad 0 \le \delta < k. \tag{32}$$

Pressure thus reduces both the equilibrium and the socially optimal non-vaccination rate.

To study alignment, define the "effective" cost parameter $g := k - \delta$. For $0 \le \delta < k$, the equilibrium and planner solutions coincide in the interior if and only if

$$\pi^{ind}(A, k, \delta) = \pi^{opt}(k, \delta) \quad \Longleftrightarrow \quad \frac{\sqrt{A^2 + 4g} - A}{2} = \sqrt{\tfrac{2}{3} g}.$$

Solving for δ yields the threshold

$$\delta^*(A, k) = k - 6A^2, \qquad \text{feasible iff } k \geq 6A^2, \tag{33}$$

and equivalently

$$A^*(k, \delta) = \sqrt{\tfrac{k-\delta}{6}}. \tag{34}$$

For $\delta < \delta^*(A, k)$ the equilibrium non-vaccination rate is above the planner's optimum; for $\delta > \delta^*(A, k)$ it is below. When $k < 6A^2$, the interior alignment value $\delta^*(A, k)$ is negative and thus infeasible; the only exact alignment is then at the boundary $k - \delta = 0$, where both π^{ind} and π^{opt} equal zero.

Pressure acts like a "tax on non-vaccination" in the private calculus, reducing π^{ind} in a way that is formally analogous to raising subsidies or altruism. However, unlike subsidies (pure transfers) or prosocial pledges (preference shifts), pressure directly reduces welfare through the experienced cost ψc_p.

3.4 Crowding out and the instrument mix

Recent contributions to the behavioral vaccination literature typically investigate the effectiveness of a single policy instrument and document that such interventions can generate negative spillover effects.[8] To the best of our knowledge, this literature has not systematically analyzed the interaction between multiple instruments aimed at promoting vaccination uptake. In contrast, we study the joint use of the policy tools discussed above and examine how they may interact in non-trivial ways, potentially attenuating or amplifying one another's effects. Our framework provides a simple way to formalize these interactions and to derive implications for vaccination policy design.

The key objects are the cross-partial derivatives of the equilibrium non-vaccination share with respect to the different policy parameters and altruistic concern.[9] Starting from the closed-form interior expression for $\pi^{ind}(A, k)$ and applying the policy mappings $k \mapsto k(s)$, $A \mapsto A + (1-\sigma)n\bar{\theta}\,\varepsilon$, and $k \mapsto k - \delta$, straightforward differentiation yields, whenever the

[8] The behavioral vaccination literature on crowding out typically examines negative spillover effects of a single policy instrument rather than interactions among multiple instruments; see, for example, Hirani (2021), Garrouste et al. (2023), and Huynh et al. (2026). Bruers (2024) provides a review of behavioral-economics models of altruistic crowding-out effects associated with monetary incentives.

[9] For full derivations of the cross-partials, see Appendix F.

underlying arguments are strictly positive,

$$\frac{\partial^2 \pi^{ind}}{\partial s\,\partial \varepsilon} > 0, \qquad \frac{\partial^2 \pi^{ind}}{\partial \varepsilon\,\partial \delta} > 0, \qquad \frac{\partial^2 \pi^{ind}}{\partial s\,\partial \delta} < 0, \tag{35}$$

and, for altruistic concern,

$$\frac{\partial^2 \pi^{ind}}{\partial A\,\partial s} \geq 0, \qquad \frac{\partial^2 \pi^{ind}}{\partial A\,\partial \delta} \geq 0. \tag{36}$$

Since all three instruments reduce non-vaccination on the first margin (i.e. $\partial \pi^{ind}/\partial s < 0$, $\partial \pi^{ind}/\partial \varepsilon < 0$, and $\partial \pi^{ind}/\partial \delta < 0$), the signs of the cross-partials can be read as follows. A positive cross-partial, as in $\partial^2 \pi^{ind}/(\partial s\,\partial \varepsilon) > 0$ and $\partial^2 \pi^{ind}/(\partial \varepsilon\,\partial \delta) > 0$, implies that increasing one instrument makes the marginal effect of the other *less negative*: stronger prosocial pledges weaken the marginal impact of subsidies, and both subsidies and indirect pressure weaken the marginal impact of additional prosocial shifts. In this sense, prosocial pledges are *substitutes at the margin* for both monetary incentives and indirect pressure. By contrast, the negative cross-partial $\partial^2 \pi^{ind}/(\partial s\,\partial \delta) < 0$ indicates that subsidies and indirect pressure are *complements at the margin*: raising one instrument makes the other more effective in reducing non-vaccination. The cross-partials with respect to A further imply that stronger altruistic concern reduces the marginal effectiveness of both subsidies and indirect pressure.

Our alignment results also reveal potential backfire margins. For prosocial interventions, the threshold conditions in Section 2.4 and equation (24) show that when aggregate altruistic concern A is already large relative to the private-cost pressure k (i.e. when $A > \sqrt{k/6}$), further increases in ε drive the equilibrium non-vaccination rate π^{ind} below the planner's target $\pi^{opt}(k)$. In our paternalistic benchmark this constitutes "over-vaccination": well-intentioned campaigns that further strengthen altruistic motives move behavior away from the social optimum. Analogously, for indirect pressure, the threshold $\delta^*(A, k)$ in (33) characterizes when such a measure pushes coverage beyond the planner's preferred level (i.e. "over-vaccination"), at the cost of substantial pressure disutility on a small remaining minority.

Taken together, these results caution against treating instruments as purely additive levers. Subsidies, prosocial pledges, and indirect pressure all raise coverage, but they do so in non-linear and interacting ways: subsidies and prosocial shifts exhibit diminishing marginal returns, normative tools crowd out the marginal effectiveness of monetary and soft-mandate instruments, and subsidies and indirect pressure are complements at the margin. From a design perspective, this suggests that (i) monetary incentives should be targeted to settings with low altruistic concern and moderate private frictions, (ii) prosocial campaigns should be calibrated rather than maximized, especially in high-altruism, high-vulnerability environments, and (iii) indirect pressure should be used sparingly, primarily where other instruments have limited traction and social exposure is high. Effective policy design should therefore employ each lever in environments where its marginal impact on coverage is high and its economic and political costs are relatively low. In doing so, our framework provides a concise theoretical foundation for concerns about backfire and crowding out raised in the empirical and experimental literature, and contributes to filling a gap in the vaccination literature concerning the design and combination of appropriate policy instruments.

4 Conclusion

We have developed a tractable vaccination game with heterogeneous infection-loss types and behaviorally heterogeneous agents in which a fraction of individuals exhibit prosocial preferences that are explicitly mediated by the structure and vulnerability of their social circle. We contribute by combining social-circle mediated altruism with type-heterogeneous infection costs and explicit representations of prosocial pledges and indirect pressure in a unified, analytically solvable model.

The equilibrium admits closed-form cutoff strategies for self-interested and altruistic types, and the aggregate non-vaccination rate collapses to a simple expression in two interpretable composites: a private-cost pressure ratio and an altruistic concern index. A paternalistic utilitarian planner, who values realized health and vaccination outcomes but not the intrinsic satisfaction associated with acting prosocially, chooses a socially optimal cutoff. We show that altruism can either attenuate or exacerbate the divergence between equilibrium behavior and the social optimum. In particular, when social-circle vulnerability is high, modest altruism is beneficial but strong altruism may lead to over-vaccination relative to the planner's preferred coverage.

We then map three policy instruments—subsidies, prosocial pledges, and soft-mandate or indirect-pressure instruments—into the model's primitives and derive closed-form comparative statics, alignment conditions, and cross-partials. The analysis highlights three main design lessons. *First*, norm-based or "prosocial" tools crowd out the marginal effectiveness of monetary and pressure-based instruments: when altruistic concern A or prosocial messaging ε is strong, many individuals already vaccinate "for others", so additional subsidies or indirect pressure generate relatively little extra uptake, as reflected in the positive cross-partials shown in 35 and 36. *Second*, subsidies and indirect pressure are *complements at the margin*: both act on the effective cost of remaining unvaccinated, and the negative cross-partial between s and δ implies that strengthening one instrument makes the other more powerful in reducing non-vaccination (for instance, pressure moves more individuals close to their vaccination threshold, so small subsidies can then flip many of them, and vice versa). *Third*, the model features clear backfire margins. Given our paternalistic planner, "too much" altruism or "too much" pressure can induce socially excessive vaccination: when altruistic concern exceeds the threshold $A > \sqrt{k/6}$ or when indirect pressure exceeds $\delta > \delta^*(A, k)$, equilibrium coverage surpasses the planner's preferred level, so further prosocial or coercive interventions reduce welfare rather than increase it. These results, together with the dependence of policy leverage on social-circle characteristics such as contact intensity and vulnerability, provide a concise theoretical foundation for concerns about motivational crowding out and over-shooting raised in the empirical and experimental literature on vaccination policies.

Our framework is intentionally stylized. Infection risk is represented in reduced form, social circles are exogenous, and heterogeneity is confined to infection losses and social preferences. These simplifications deliver analytical transparency but also limit the scope of our conclusions. Extending the analysis to richer network structures, multidimensional heterogeneity (including vaccine hesitancy and prosocial motives), and dynamic disease transmission is a promising avenue for future research. Likewise, embedding explicit fiscal and political cost functions for subsidies, pledges, and indirect-pressure instruments into the planner's problem would allow for a fully optimal policy mix rather than the comparative-static perspective taken here. Finally, linking the model's parameters to empirical estimates of prosocial motivation and social-circle characteristics in specific diseases would help translate our qualitative design rules into quantitative policy recommendations.

Appendix A Robustness to nonlinear infection risk

Robustness: Allow a convex infection risk $P_i(\pi) = \beta\,\pi^\eta$ with $\eta \geq 1$. The cutoff conditions become $c_v\lambda = \beta\,\pi^\eta c_i\,\theta_e$ and $c_v\lambda = \beta\,\pi^\eta c_i\,(\theta_a + \alpha n\bar{\theta})$. The consistency relation $\theta_e = \theta_a + \alpha n\bar{\theta}$ still holds, and with $A := (1-\sigma)\alpha n\bar{\theta}$ one has $\pi = \theta_e - A$. Eliminating π yields the scalar equation $k = \theta_e\,(\theta_e - A)^\eta$, where $k := \frac{c_v\lambda}{\beta c_i} > 0$. For any $\eta \geq 1$ this equation has a unique economically relevant solution $\theta_e^* \geq A$. Moreover, the comparative-statics signs persist: $\partial\pi^{ind}/\partial A < 0$ and $\partial\pi^{ind}/\partial k > 0$.

Appendix B Derivation of cutoff comparative statics

On the interior, define

$$A := (1-\sigma)\alpha n\bar{\theta},$$

and recall the equilibrium cutoffs

$$\theta_e^*(A) = \frac{A + \sqrt{A^2 + 4k}}{2}, \qquad \theta_a^*(\sigma, \alpha) = \theta_e^*(A) - \alpha n\bar{\theta}. \tag{37}$$

Derivative of θ_e^* with respect to A. Differentiating $\theta_e^*(A)$ yields

$$\frac{\partial\theta_e^*}{\partial A} = \frac{1}{2}\left(1 + \frac{A}{\sqrt{A^2+4k}}\right) > 0. \tag{38}$$

Chain rule for σ and α. Since

$$\frac{\partial A}{\partial\sigma} = -\alpha n\bar{\theta}, \qquad \frac{\partial A}{\partial\alpha} = (1-\sigma)n\bar{\theta},$$

we obtain

$$\frac{\partial\theta_e^*}{\partial\sigma} = \frac{\partial\theta_e^*}{\partial A}\cdot\frac{\partial A}{\partial\sigma} = -\frac{\alpha n\bar{\theta}}{2}\left(1 + \frac{A}{\sqrt{A^2+4k}}\right) < 0 \quad \text{if } \alpha n\bar{\theta} > 0, \tag{39}$$

$$\frac{\partial\theta_e^*}{\partial\alpha} = \frac{\partial\theta_e^*}{\partial A}\cdot\frac{\partial A}{\partial\alpha} = \frac{(1-\sigma)n\bar{\theta}}{2}\left(1 + \frac{A}{\sqrt{A^2+4k}}\right) > 0 \quad \text{if } \sigma < 1 \text{ and } n\bar{\theta} > 0. \tag{40}$$

Derivatives for θ_a^*. From $\theta_a^* = \theta_e^*(A) - \alpha n\bar{\theta}$,

$$\frac{\partial\theta_a^*}{\partial\sigma} = \frac{\partial\theta_e^*}{\partial\sigma} < 0 \quad \text{if } \alpha n\bar{\theta} > 0, \tag{41}$$

$$\frac{\partial\theta_a^*}{\partial\alpha} = \frac{\partial\theta_e^*}{\partial\alpha} - n\bar{\theta} = n\bar{\theta}\left[\frac{1-\sigma}{2}\left(1 + \frac{A}{\sqrt{A^2+4k}}\right) - 1\right]. \tag{42}$$

To sign (42), note that $k > 0$ implies $\sqrt{A^2 + 4k} > |A|$, hence

$$\frac{A}{\sqrt{A^2 + 4k}} < 1 \quad \Longrightarrow \quad 1 + \frac{A}{\sqrt{A^2 + 4k}} < 2.$$

Therefore,

$$\frac{1-\sigma}{2}\left(1 + \frac{A}{\sqrt{A^2 + 4k}}\right) - 1 < \frac{1-\sigma}{2} \cdot 2 - 1 = (1-\sigma) - 1 = -\sigma \leq 0,$$

with strict inequality for $k > 0$. Hence

$$\frac{\partial \theta_a^*}{\partial \alpha} < 0 \qquad \text{for } k > 0 \text{ and } n\bar{\theta} > 0. \tag{43}$$

Appendix C Comparison of altruistic cutoff and social optimum

For completeness we sketch the comparison between the altruistic cutoff and the social optimum in a slightly more general setting. Let

$$s \equiv \alpha n \bar{\theta} \geq 0, \qquad k \equiv \frac{c_v \lambda}{\beta c_i} > 0, \qquad \theta^{opt} = \sqrt{\tfrac{2k}{3}}.$$

Using this notation, the altruistic cutoff in the mixed population can be written as

$$\theta_a^* = \frac{(1-\sigma)s + \sqrt{(1-\sigma)^2 s^2 + 4k}}{2} - s = -\frac{1+\sigma}{2}\, s + \frac{1}{2}\sqrt{(1-\sigma)^2 s^2 + 4k}.$$

We seek conditions under which θ_a^* is above or below the socially optimal cutoff θ^{opt}. Observe that

$$\theta_a^* \geq \theta^{opt} \iff -(1+\sigma)s + \sqrt{(1-\sigma)^2 s^2 + 4k} \;\geq\; 2\theta^{opt}.$$

Squaring both sides (using that the right-hand side is nonnegative) and rearranging yields

$$\sigma s^2 + (1+\sigma)s\,\theta^{opt} - \frac{k}{3} \;\leq\; 0.$$

The equality case determines a threshold s_c satisfying

$$\sigma s_c^2 + (1+\sigma)\theta^{opt} s_c - \frac{k}{3} = 0.$$

For $\sigma > 0$ the nonnegative root is

$$s_c(\sigma, k) = \frac{\sqrt{\frac{2k}{3}}}{\sqrt{1 + 4\sigma + \sigma^2} + 1 + \sigma},$$

which limits to $s_c = \sqrt{k/6}$ as $\sigma \downarrow 0$. Consequently, for any $\bar{\theta} > 0$,

$$\theta_a^* \gtreqless \theta^{opt} \text{ as } \alpha n\bar{\theta} \lesseqgtr s_c(\sigma, k),$$

so altruists vaccinate more than socially optimal (i.e. $\theta_a^* < \theta^{opt}$) when social-circle mediated altruism $\alpha n\bar{\theta}$ is sufficiently large relative to the cost ratio k.

Appendix D Welfare monotonicity in behavioral composition

This appendix sketches how equilibrium welfare varies with the behavioral composition parameter σ, holding the composite indices A and k fixed. Let $\pi^{ind}(A, k)$ be the equilibrium non-vaccination share and write the equilibrium welfare in a mixed population as

$$W^{ind}(\alpha, \sigma) = \sigma W_E(\theta_e^*, \pi^{ind}) + (1 - \sigma)W_A(\theta_a^*, \pi^{ind}),$$

where W_E and W_A denote the contributions from self-interested and altruistic types, respectively, and θ_e^*, θ_a^* are given by (8). After some algebra one can show that, for $A > 0$, welfare is strictly decreasing in σ on $(0, 1)$: replacing altruists by self-interested individuals lowers equilibrium welfare, holding the composite indices fixed. However, comparing the pure endpoints $\sigma = 0$ and $\sigma = 1$ yields an ambiguous ranking that depends on the magnitude of A relative to k. Details are available from the authors upon request.

Proof: This appendix analyzes how equilibrium welfare varies with the behavioral composition parameter σ, holding fixed the composite indices $k := \frac{c_v\lambda}{\beta c_i} > 0$ and $A \geq 0$. We work in the interior region where $0 < \pi^{ind}(A, k) < 1$.

1) Preliminaries

Recall from Section 2.3 that, for given A and k, the individual-equilibrium non-vaccination share is

$$\pi^{ind}(A, k) = \frac{\sqrt{A^2 + 4k} - A}{2},$$

which is equivalently characterized by the scalar equation

$$k = \pi^2 + A\pi. \tag{44}$$

For each pair (A, k) with $k > 0$, this equation has a unique solution $\pi \equiv \pi^{ind}(A, k) \in (0, 1)$.

Let θ_e^* and θ_a^* denote the self-interested and altruistic cutoffs, respectively. From the equilibrium relations in the main text, we have

$$\theta_e^* = \pi + A, \qquad \pi = \theta_e^* - A, \qquad k = \theta_e^*(\theta_e^* - A) = \theta_e^* \pi.$$

In a mixed population with share $\sigma \in (0, 1)$ of self-interested types, the altruistic cutoff θ_a^* satisfies

$$A = (1 - \sigma)\big(\theta_e^* - \theta_a^*\big),$$

so that

$$\theta_a^* = \theta_e^* - \frac{A}{1 - \sigma} = \pi + A - \frac{A}{1 - \sigma}. \tag{45}$$

Note that, for fixed A and k, the aggregate non-vaccination share

$$\pi = \sigma\theta_e^* + (1 - \sigma)\theta_a^*$$

is independent of σ: substituting (45) into this expression yields

$$\pi = \sigma(\pi + A) + (1 - \sigma)\Big(\pi + A - \frac{A}{1 - \sigma}\Big) = \pi + A - A = \pi.$$

Thus, holding (A, k) fixed, variations in σ leave the equilibrium non-vaccination rate $\pi^{ind}(A, k)$ unchanged but reweight the two behavioral classes with different cutoffs.

2) Equilibrium welfare by behavioral composition

The planner's objective is paternalistic: for any individual, realized welfare depends only on own infection losses and vaccination disutility, not on altruistic taste components. For a given non-vaccination rate π, a self-interested type with cutoff θ_e^* generates expected welfare

$$W_E(\theta_e^*, \pi) = \int_0^{\theta_e^*} \big(- \beta\, \pi\, c_i\, \theta\big)\, d\theta + \int_{\theta_e^*}^1 \big(- c_v\, \lambda\big)\, d\theta = -\beta c_i \left[\frac{1}{2}\pi\, (\theta_e^*)^2 + k(1 - \theta_e^*)\right],$$

where we used $k = \frac{c_v \lambda}{\beta c_i}$. Analogously, the welfare contribution of an altruist with cutoff θ_a^* is

$$W_A(\theta_a^*, \pi) = -\beta c_i \left[\frac{1}{2}\pi\, (\theta_a^*)^2 + k(1 - \theta_a^*)\right].$$

In a mixed population with behavioral composition $\sigma \in [0,1]$, equilibrium welfare is

$$W^{ind}(\alpha,\sigma) = \sigma W_E(\theta_e^*,\pi) + (1-\sigma)W_A(\theta_a^*,\pi), \tag{46}$$

where $\pi = \pi^{ind}(A,k)$ and θ_e^*, θ_a^* are as above. For subsequent algebra it is convenient to factor out the positive constant βc_i and work with

$$\widetilde{W}(\sigma) := -\frac{1}{\beta c_i} W^{ind}(\alpha,\sigma).$$

Using the expressions for W_E and W_A, we obtain

$$\widetilde{W}(\sigma) = \sigma\left[\frac{1}{2}\pi\,(\theta_e^*)^2 + k(1-\theta_e^*)\right] + (1-\sigma)\left[\frac{1}{2}\pi\,(\theta_a^*)^2 + k(1-\theta_a^*)\right]. \tag{47}$$

Substituting $\theta_e^* = \pi + A$ and θ_a^* from (45) into (47) and using the equilibrium relation $k = \pi^2 + A\pi$ yields, after straightforward algebra,

$$\widetilde{W}(\sigma) = \frac{\pi^3}{2} + k(1-\pi) + \frac{\pi A^2}{2}\left(\frac{1}{1-\sigma} - 1\right), \qquad 0 < \sigma < 1, \tag{48}$$

where $\pi \equiv \pi^{ind}(A,k)$ is held fixed.

3) Monotonicity on the mixed region $\sigma \in (0,1)$

Differentiating (48) with respect to σ gives

$$\frac{d\widetilde{W}}{d\sigma} = \frac{\pi A^2}{2}\frac{d}{d\sigma}\left(\frac{1}{1-\sigma}\right) = \frac{\pi A^2}{2}\cdot\frac{1}{(1-\sigma)^2}.$$

Hence

$$\frac{dW^{ind}(\alpha,\sigma)}{d\sigma} = -\beta c_i\frac{d\widetilde{W}}{d\sigma} = -\,\beta c_i\frac{\pi A^2}{2(1-\sigma)^2}. \tag{49}$$

For any interior equilibrium with $\pi > 0$ and any $A > 0$, we have

$$\frac{dW^{ind}(\alpha,\sigma)}{d\sigma} < 0 \quad \text{for all} \quad \sigma \in (0,1),$$

so equilibrium welfare is strictly decreasing in σ on the mixed region: holding (A,k) fixed, replacing altruists by self-interested individuals (raising σ) reduces welfare. In the limiting case $A = 0$ (no prosocial concern), the derivative in (49) vanishes and welfare is independent of σ, as expected.

4) Comparing the pure endpoints $\sigma = 0$ and $\sigma = 1$

The foregoing analysis holds for fixed $A > 0$ and $\sigma \in (0,1)$. The pure endpoints require separate treatment.

Pure altruists ($\sigma = 0$)**.** When $\sigma = 0$, the entire population is altruistic and the equilibrium non-vaccination rate is $\pi^{ind}_{\sigma=0} = \pi^{ind}(A, k)$. From (47) with $\sigma = 0$, we obtain

$$\widetilde{W}_0 := -\frac{1}{\beta c_i} W^{ind}(\alpha, 0) = \frac{1}{2}\pi^3 + k(1-\pi),$$

so that

$$W^{ind}(\alpha, 0) = -\beta c_i \left[\frac{1}{2}\big(\pi^{ind}(A,k)\big)^3 + k\big(1 - \pi^{ind}(A,k)\big)\right]. \tag{50}$$

Pure self-interested individuals ($\sigma = 1$)**.** When $\sigma = 1$, there are no altruists, so $A = 0$ and the equilibrium non-vaccination share reduces to $\pi^{ind}_{\sigma=1} = \sqrt{k}$. The welfare expression simplifies to

$$W^{ind}(\alpha, 1) = -\beta c_i \left[k - \frac{1}{2}k^{3/2}\right].$$

Endpoint ordering. Comparing the two endpoints involves different values of A: $\sigma = 0$ is evaluated at $A > 0$, whereas $\sigma = 1$ corresponds to $A = 0$. It is therefore not immediate that altruism is always welfare-improving. From (50) and (Appendix D), the welfare difference can be written as

$$W^{ind}(\alpha,0) - W^{ind}(\alpha,1) = -\beta c_i \left[\frac{1}{2}\pi^3 + k(1-\pi) - \left(k - \frac{1}{2}k^{3/2}\right)\right] = -\beta c_i \left[\frac{1}{2}\pi^3 - k\pi + \frac{1}{2}k^{3/2}\right],$$

where $\pi = \pi^{ind}(A, k)$. Hence

$$W^{ind}(\alpha, 0) \gtrless W^{ind}(\alpha, 1) \quad \Longleftrightarrow \quad \pi^3 - 2k\pi + k^{3/2} \lessgtr 0.$$

Using (44), this condition can be written purely in terms of (A, k). Its sign is generally ambiguous and depends on the magnitude of altruistic concern A relative to the private-cost pressure k: for small A, introducing altruism raises welfare relative to the purely self-interested benchmark, whereas for sufficiently large A the purely altruistic economy may yield lower paternalistic welfare, reflecting over-vaccination relative to the planner's preferred coverage.

Appendix E Interaction between prosocial pledges and subsidies

For completeness we record a compact derivation of the cross-partial (58). Let

$$B(\varepsilon) := A + (1-\sigma)\, n\, \bar{\theta}\, \varepsilon, \qquad \pi^{ind}(\varepsilon, s) = \frac{\sqrt{B(\varepsilon)^2 + 4k(s)} - B(\varepsilon)}{2},$$

where $k(s) = k_0 - \frac{s}{\beta c_i}$. Then

$$\frac{d\pi^{ind}}{d\varepsilon} = -\,(1-\sigma) n\bar{\theta}\, \frac{\pi^{ind}}{\sqrt{B(\varepsilon)^2 + 4k(s)}} < 0.$$

Differentiating with respect to s and using the envelope properties of π^{ind} with respect to k yields

$$\frac{\partial^2 \pi^{ind}}{\partial s\, \partial \varepsilon} = \frac{(1-\sigma) n\bar{\theta}\, B(\varepsilon)}{\beta c_i \left(B(\varepsilon)^2 + 4k(s)\right)^{3/2}} \;>\; 0 \quad \text{for } B(\varepsilon) > 0,$$

as stated in the main text.

Appendix F Derivation of instrument cross-partials

This appendix derives the cross-partial derivatives of the individual-equilibrium non-vaccination share with respect to pairs of policy instruments. We start from the interior equilibrium expression in terms of altruistic concern A and the effective cost parameter k,

$$\pi^{ind}(A, k) = \frac{\sqrt{A^2 + 4k} - A}{2}, \tag{51}$$

and use the policy mappings for the subsidy s, the prosocial pledge ε, and the indirect-pressure parameter δ.

For notational convenience, write

$$D(A, k) := \sqrt{A^2 + 4k},$$

so that (51) can be expressed as

$$\pi^{ind}(A, k) = \frac{D(A, k) - A}{2}.$$

Appendix F.1 Preliminaries: derivatives of π^{ind}

For fixed A and k, the partial derivatives of π^{ind} with respect to its arguments follow directly from the chain rule. Since

$$D(A,k) = (A^2 + 4k)^{1/2}, \qquad \frac{\partial D}{\partial A} = \frac{A}{D}, \qquad \frac{\partial D}{\partial k} = \frac{2}{D},$$

we obtain

$$\frac{\partial \pi^{ind}}{\partial A} = \frac{1}{2}\left(\frac{\partial D}{\partial A} - 1\right) = \frac{1}{2}\left(\frac{A}{D} - 1\right), \tag{52}$$

$$\frac{\partial \pi^{ind}}{\partial k} = \frac{1}{2} \cdot \frac{\partial D}{\partial k} = \frac{1}{D}. \tag{53}$$

It is also useful to note the identity

$$D - A = 2\pi^{ind}(A,k), \qquad \text{so} \qquad D = 2\pi^{ind}(A,k) + A. \tag{54}$$

Appendix F.2 Subsidy–pledge interaction $\partial^2\pi^{ind}/(\partial s\,\partial\varepsilon)$

The prosocial pledge shifts altruistic concern from A to

$$B(\varepsilon) := A + (1-\sigma)n\bar{\theta}\,\varepsilon,$$

while a subsidy s reduces the effective private-cost pressure from k_0 to

$$k(s) := k_0 - \frac{s}{\beta c_i},$$

where $k_0 := \frac{c_v \lambda}{\beta c_i}$ is the baseline cost ratio. On the interior region, the equilibrium non-vaccination share under (ε, s) is thus

$$\pi^{ind}(\varepsilon, s) = \frac{\sqrt{B(\varepsilon)^2 + 4k(s)} - B(\varepsilon)}{2}. \tag{55}$$

1) Derivative with respect to ε. Treat $k(s)$ as fixed and differentiate (55) with respect to B:

$$\frac{\partial \pi^{ind}}{\partial B} = \frac{1}{2}\left(\frac{B}{\sqrt{B^2 + 4k(s)}} - 1\right).$$

By the chain rule,

$$\frac{\partial B}{\partial \varepsilon} = (1-\sigma)n\bar{\theta},$$

so

$$\frac{\partial \pi^{ind}}{\partial \varepsilon} = \frac{\partial \pi^{ind}}{\partial B} \cdot \frac{\partial B}{\partial \varepsilon} = (1-\sigma) n \bar{\theta} \, \frac{1}{2} \left(\frac{B}{\sqrt{B^2 + 4k(s)}} - 1 \right). \tag{56}$$

Using $D := \sqrt{B^2 + 4k(s)}$ and the identity $D - B = 2\pi^{ind}(\varepsilon, s)$, we can rewrite (56) as

$$\frac{\partial \pi^{ind}}{\partial \varepsilon} = (1-\sigma) n \bar{\theta} \, \frac{1}{2} \left(\frac{B - D}{D} \right) = -\,(1-\sigma) n \bar{\theta} \, \frac{\pi^{ind}(\varepsilon, s)}{D}. \tag{57}$$

2) Cross-partial with respect to s. Differentiating (57) with respect to s and using the fact that B does not depend on s, we obtain

$$\frac{\partial^2 \pi^{ind}}{\partial s \, \partial \varepsilon} = -\,(1-\sigma) n \bar{\theta} \left[\frac{\partial \pi^{ind} / \partial s}{D} + \pi^{ind} \frac{\partial}{\partial s} \left(\frac{1}{D} \right) \right].$$

From (53) and $k(s) = k_0 - \frac{s}{\beta c_i}$ we have

$$\frac{\partial \pi^{ind}}{\partial s} = \frac{\partial \pi^{ind}}{\partial k} \cdot \frac{\partial k}{\partial s} = \frac{1}{D} \cdot \Big(-\frac{1}{\beta c_i} \Big) = -\frac{1}{\beta c_i D},$$

and

$$\frac{\partial D}{\partial s} = \frac{\partial D}{\partial k} \cdot \frac{\partial k}{\partial s} = \frac{2}{D} \cdot \Big(-\frac{1}{\beta c_i} \Big) = -\frac{2}{\beta c_i D}, \qquad \frac{\partial}{\partial s} \left(\frac{1}{D} \right) = -\frac{1}{D^2} \cdot \frac{\partial D}{\partial s} = \frac{2}{\beta c_i D^3}.$$

Substituting into the expression for the cross-partial yields

$$\frac{\partial^2 \pi^{ind}}{\partial s \, \partial \varepsilon} = -\,(1-\sigma) n \bar{\theta} \left[-\frac{1}{\beta c_i D^2} + \pi^{ind} \frac{2}{\beta c_i D^3} \right] = -\frac{(1-\sigma) n \bar{\theta}}{\beta c_i} \, \frac{-D + 2\pi^{ind}}{D^3}.$$

Using $2\pi^{ind} = D - B$ again, we have $-D + 2\pi^{ind} = -B$, so

$$\frac{\partial^2 \pi^{ind}}{\partial s \, \partial \varepsilon} = \frac{(1-\sigma) n \bar{\theta} \, B(\varepsilon)}{\beta c_i \, [B(\varepsilon)^2 + 4k(s)]^{3/2}}, \tag{58}$$

which is strictly positive whenever $B(\varepsilon) > 0$.

Appendix F.3 Pledge–indirect-pressure interaction $\partial^2 \pi^{ind} / (\partial \varepsilon \, \partial \delta)$

Indirect pressure δ reduces the effective cost parameter from k to $k - \delta$ (on the interior, with $k > \delta$). With the prosocial shift $B(\varepsilon)$ as above, the equilibrium non-vaccination share under (ε, δ) is

$$\pi^{ind}(\varepsilon, \delta) = \frac{\sqrt{B(\varepsilon)^2 + 4(k - \delta)} - B(\varepsilon)}{2}. \tag{59}$$

Define

$$D_\delta := \sqrt{B(\varepsilon)^2 + 4(k-\delta)}.$$

1) Derivative with respect to ε. For fixed δ, the derivation is identical to (57), with $k(s)$ replaced by $k-\delta$ and D by D_δ:

$$\frac{\partial \pi^{ind}}{\partial \varepsilon} = -(1-\sigma)n\bar{\theta}\,\frac{\pi^{ind}(\varepsilon,\delta)}{D_\delta}. \tag{60}$$

2) Cross-partial with respect to δ. Differentiating (60) with respect to δ,

$$\frac{\partial^2 \pi^{ind}}{\partial \varepsilon\,\partial \delta} = -(1-\sigma)n\bar{\theta}\left[\frac{\partial \pi^{ind}/\partial \delta}{D_\delta} + \pi^{ind}\,\frac{\partial}{\partial \delta}\left(\frac{1}{D_\delta}\right)\right].$$

Using $\partial \pi^{ind}/\partial k = 1/D_\delta$ and $k \mapsto k-\delta$ gives

$$\frac{\partial \pi^{ind}}{\partial \delta} = \frac{\partial \pi^{ind}}{\partial k}\cdot\frac{\partial (k-\delta)}{\partial \delta} = \frac{1}{D_\delta}\cdot(-1) = -\frac{1}{D_\delta},$$

and

$$\frac{\partial D_\delta}{\partial \delta} = \frac{\partial D_\delta}{\partial k}\cdot\frac{\partial (k-\delta)}{\partial \delta} = \frac{2}{D_\delta}\cdot(-1) = -\frac{2}{D_\delta}, \qquad \frac{\partial}{\partial \delta}\left(\frac{1}{D_\delta}\right) = -\frac{1}{D_\delta^2}\cdot\frac{\partial D_\delta}{\partial \delta} = \frac{2}{D_\delta^3}.$$

Hence

$$\frac{\partial^2 \pi^{ind}}{\partial \varepsilon\,\partial \delta} = -(1-\sigma)n\bar{\theta}\left[-\frac{1}{D_\delta^2} + \pi^{ind}\,\frac{2}{D_\delta^3}\right] = -(1-\sigma)n\bar{\theta}\,\frac{-D_\delta + 2\pi^{ind}}{D_\delta^3}.$$

Using $2\pi^{ind} = D_\delta - B(\varepsilon)$, we obtain $-D_\delta + 2\pi^{ind} = -B(\varepsilon)$ and thus

$$\frac{\partial^2 \pi^{ind}}{\partial \varepsilon\,\partial \delta} = \frac{(1-\sigma)n\bar{\theta}\,B(\varepsilon)}{\big(B(\varepsilon)^2 + 4(k-\delta)\big)^{3/2}}, \tag{61}$$

which is strictly positive whenever $B(\varepsilon) > 0$.

Appendix F.4 Subsidy–indirect-pressure interaction $\partial^2\pi^{ind}/(\partial s\,\partial \delta)$

Finally, consider the interaction between the subsidy and indirect pressure, holding altruistic concern A fixed. Let

$$g(s,\delta) := k_0 - \delta - \frac{s}{\beta c_i}$$

be the effective cost parameter under (s,δ), and define

$$\widetilde{D}(s,\delta) := \sqrt{A^2 + 4g(s,\delta)}.$$

The equilibrium non-vaccination share is

$$\pi^{ind}(s,\delta) = \frac{\widetilde{D}(s,\delta) - A}{2}. \tag{62}$$

1) Derivatives with respect to s and δ. Using (53) with $k = g(s,\delta)$, we have

$$\frac{\partial \pi^{ind}}{\partial g} = \frac{1}{\widetilde{D}}, \qquad \frac{\partial g}{\partial s} = -\frac{1}{\beta c_i}, \qquad \frac{\partial g}{\partial \delta} = -1,$$

so that

$$\frac{\partial \pi^{ind}}{\partial s} = \frac{\partial \pi^{ind}}{\partial g} \cdot \frac{\partial g}{\partial s} = -\frac{1}{\beta c_i \widetilde{D}}, \qquad \frac{\partial \pi^{ind}}{\partial \delta} = \frac{\partial \pi^{ind}}{\partial g} \cdot \frac{\partial g}{\partial \delta} = -\frac{1}{\widetilde{D}}. \tag{63}$$

2) Cross-partial. Differentiating $\partial \pi^{ind}/\partial s$ with respect to δ,

$$\frac{\partial^2 \pi^{ind}}{\partial s\, \partial \delta} = -\frac{1}{\beta c_i} \frac{\partial}{\partial \delta}\left(\frac{1}{\widetilde{D}}\right) = -\frac{1}{\beta c_i}\left(-\frac{1}{\widetilde{D}^2} \cdot \frac{\partial \widetilde{D}}{\partial \delta}\right).$$

Using

$$\frac{\partial \widetilde{D}}{\partial \delta} = \frac{\partial \widetilde{D}}{\partial g} \cdot \frac{\partial g}{\partial \delta} = \frac{2}{\widetilde{D}} \cdot (-1) = -\frac{2}{\widetilde{D}},$$

we obtain

$$\frac{\partial}{\partial \delta}\left(\frac{1}{\widetilde{D}}\right) = -\frac{1}{\widetilde{D}^2} \cdot \left(-\frac{2}{\widetilde{D}}\right) = \frac{2}{\widetilde{D}^3},$$

and hence

$$\frac{\partial^2 \pi^{ind}}{\partial s\, \partial \delta} = -\frac{1}{\beta c_i} \cdot \frac{2}{\widetilde{D}^3} = -\frac{2}{\beta c_i \left(A^2 + 4g(s,\delta)\right)^{3/2}}. \tag{64}$$

This cross-partial is strictly negative on the interior ($g(s,\delta) > 0$), implying that subsidies and indirect-pressure instruments are complements at the margin: increasing one instrument makes the marginal effect of the other more negative (stronger) in terms of reducing non-vaccination.

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