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Almut Balleer
Georg Duernecker
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Universitätsstr. 150, 44801 Bochum, Germany

Technische Universität Dortmund, Department of Economic and Social Sciences
Vogelpothsweg 87, 44227 Dortmund, Germany

Universität Duisburg-Essen, Department of Economics
Universitätsstr. 12, 45117 Essen, Germany

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Almut Balleer, Georg Duernecker, Susanne Forstner, and Johannes Goensch*

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Abstract

Recent research documents mounting evidence for sizable and persistent biases in individual labor market expectations. This paper incorporates subjective expectations into a general equilibrium labor market model and analytically studies the implications of biased expectations for wage bargaining, vacancy creation, worker flows and labor market policies. Importantly, we find that the specific assumption about the frequency of wage bargaining crucially shapes the propagation mechanism through which expectation biases affect bargained wages and equilibrium outcomes. Moreover, we show that the presence of biased beliefs can qualitatively alter the equilibrium effects of labor market policies. Lastly, when allowing for biased firms' beliefs, we establish that only the difference between firms' and workers' biases matters for the bargained wage but not the size of biases.

JEL-Codes: E24, J64, D84

Keywords: Subjective expectations; labor markets; search and matching; bargaining; policy.

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1 Introduction

A large part of modern labor market research relies on models which incorporate labor market frictions into a macroeconomic framework. These models emanate from the pioneering work of Diamond (1982), Mortensen (1982), and Pissarides (1985) and they have been used to study a wide variety of topics in labor economics.[1] The standard approach in this literature – as in much of macroeconomics – is to adopt the rational expectations paradigm. While this approach has several key advantages, such as analytical tractability, it rests on the strong presumption that individuals hold statistically correct, and thus unbiased, expectations about all future realizations of events.

Recent research has documented the expectations of households about labor market outcomes at the individual level. This includes, for example the expectations of workers about job loss, wage growth, or job finding; see Mueller and Spinnewijn (2023) for a survey. Contrary to the rational expectations hypothesis, these studies provide mounting evidence for sizable and persistent biases in individual labor market expectations. For example, Mueller et al. (2021) use data from the Survey of Consumer Expectations to document that job seekers in the U.S. substantially over-estimate their job finding probability relative to the statistical job finding probability. Similar findings are obtained by Spinnewijn (2015) for the U.K. and by Balleer et al. (2023) for Germany. Along the same lines, Balleer et al. (2021) find that employed workers in the U.S. systematically under-estimate the probability of becoming unemployed, whereas Balleer et al. (2023) find the opposite for German workers who are stubbornly pessimistic about the stability of their job.

The objective of this paper is twofold. First, we incorporate subjective expectations into a general equilibrium frictional labor market model in order to study the implications of biased expectations for wage bargaining, job creation, worker flows and the efficiency of equilibrium. Second, we analyze the effects of labor market policies on equilibrium outcomes in the presence of biased expectations. As a framework, we adopt the canonical search-and-matching model as described in Pissarides (2000) and used in many of the works described above.[2] In line with the evidence mentioned above, we allow workers

[1] This includes, for example, the study of worker and job flows over the business cycle (Merz 1995; Shimer 2005; Christiano et al. 2016) and across countries (Marimon and Zilibotti 1999; Pries and Rogerson 2005; Ljungqvist and Sargent 2007), as well as the welfare analysis of labor policies and institutions (Acemoglu and Shimer 1999; Flinn 2006; Krusell et al. 2010), and the role of worker and firm heterogeneity for aggregate outcomes (Mortensen and Pissarides 1994). See Rogerson et al. (2005) and Rogerson and Shimer (2011) for surveys of these works.

[2] In a nutshell, in this framework firms need to be matched with a worker in order to produce output. However, there are search frictions in the labor market. Firms post vacancies to attract workers and

in the model to have biased beliefs about future realizations of individual labor market transitions. This includes the transition from unemployment to employment (job finding) and the transition from employment into unemployment (job separation). We define the bias in expectations as the difference between worker's subjective transition probabilities and the statistical probabilities. For example, an optimistic bias in the job finding expectation occurs when the worker over-estimates the probability of transiting from unemployment to employment. Likewise, a pessimistic bias in the job separation expectation occurs when the worker over-estimates the probability of transiting from employment into unemployment.

We derive the equilibrium of the model and characterize analytically the effects of biases on the equilibrium properties of the economy. Several key insights emerge from this analysis. Most importantly, we find that the specific assumption about the frequency of wage bargaining crucially shapes the propagation mechanism through which expectation biases affect equilibrium outcomes. Concretely, when firms and workers renegotiate the wage **every period** – which is the approach predominately used in the literature – then a pessimistic bias in workers' separation expectations leads to a **higher** bargained wage.[3] The intuition is that a pessimistic worker expects the employment relation to end soon, and thus, the firm has to offer a higher wage to make the worker stay in the match. In contrast, when firms and workers negotiate only at the beginning of the employment spell and thereby determine the wage that is paid during the entire **duration of the match**, then pessimistic separation expectations lead to a **lower** bargained wage. In the model, pessimistic workers accept low wages as they heavily discount the effect of future wages on the job value.

In addition to these extreme cases (flexible wages and fixed wages) we also consider a bargaining setting where the wage is renegotiated after a predetermined number of periods. In this context, we isolate two opposing effects of the job separation expectation on the wage, and we characterize a cutoff for the bargaining frequency below which the separation bias positively affects the worker's wage and above which the negative effect dominates. An immediate implication of this result is that, in the presence of biased expectations, the wage is crucially affected by how often workers and firms negotiate.

unemployed workers search for jobs. An aggregate matching function brings together vacant jobs and unemployed workers. Once matched, the worker and the firm negotiate the wage through bilateral Nash bargaining. The employment relation continues until the job is hit by an exogenous separation shock. When the match separates, the worker becomes unemployed and the firm's job becomes vacant.

[3] This result contrasts with recent empirical evidence suggesting a statistically significant and strongly negative relationship between individual wages and the subjective expectations of job separation; see for example, Balleer et al. (2023), Campbell et al. (2007), and Hübler and Hübler (2006).

This finding is in stark contrast to the rational expectations case where the frequency of bargaining is irrelevant for wages.

In our analysis, we also explore the equilibrium effects of workers' subjective job finding expectations. The empirical works mentioned above generally find robust evidence across countries for an optimistic bias – meaning that unemployed workers tend to overestimate the probability of find a job. In our theoretical analysis, we establish analytically that such a bias leads to higher wages and more unemployment in equilibrium. The reason is that optimistic workers overestimate the value of their outside option when bargaining with the firm, and hence, they demand higher wages.[4] Due to higher wages, firms make lower profits and this leads to less vacancy creation and more unemployment in equilibrium. Unlike the job separation bias, the qualitative effect of the job finding bias on the bargained wage is independent of the assumption about the frequency of bargaining.

We consider several extensions to the baseline model. For example, we depart from risk neutrality and allow workers to be risk averse. Risk aversion generally dampens the effect of expectation biases on wages and other equilibrium outcomes. The reason is that due to the declining marginal utility, risk averse workers have a lower (perceived) valuation of the bias than risk neutral workers. In another extension, we study the case where – in addition to workers – also firms have biased beliefs about labor market transitions (vacancy filling and job separation). We establish in this context that only the amount of disagreement between the worker and the firm – but not the size of the biases – matters in the wage negotiations. In other words, if the firm is equally pessimistic (or optimistic) than the worker, then the bargained wage is the same as under rational expectations. We also extend the baseline model to allow for endogenous job separations – along the lines of Mortensen and Pissarides (1994). We show that when workers expect a higher rate of job separation, then this leads to less job separation in equilibrium. The reason is that due to the high expected separation rate the worker has a low valuation of the match. Thus, the firm can extract a higher share of the joint surplus and, as a result, matches with low productivity remain profitable and are not destroyed.

As part of our analysis, we study the efficiency of the decentralized equilibrium. It is well known that the equilibrium of the canonical search-and-matching model is socially efficient if the Hosios condition is satisfied. We show that this condition no longer holds when expectations are biased. This is because the outcome of wage bargaining between

[4]This result is in line with empirical evidence indicating a positive relation between workers' job finding expectations and reservation income; see, for example, Drahs et al. (2018) and Krueger and Mueller (2016).

the firm and the worker is affected by the sign and the magnitude of the biases. We derive a "generalized Hosios condition" which applies when expectations are non-rational.

Lastly, we study labor market policies. Concretely, we consider three widely used policy instruments: unemployment insurance, minimum wage and firing costs. Importantly, we show that in the presence of biased expectations the effects of policies on equilibrium outcomes can be qualitatively different than under rational expectations. For example, higher firing costs can lead to an increase in equilibrium unemployment in an economy with pessimistic workers, whereas unemployment declines in an economy with optimistic workers.

This paper relates and contributes to several strands of literature. First, it relates to the extensive literature mentioned above that uses versions of the Diamond-Mortensen-Pissarides (DMP) search and matching framework to analyze a broad range of labor-related questions. Our paper contributes to this literature in several ways. The first contribution of the paper is methodological. It proposes a framework where the conventional rational expectations assumption is replaced with a flexible belief structure which can accommodate the empirically observed biases in individual labor market expectations. As such, the framework provides a conceptual starting point for further analysis – theoretical and applied – in the context of subjective expectations and labor market outcomes. The paper characterizes the general equilibrium of the model and analytically establishes conditions for the existence and uniqueness of equilibrium. In this context, we show that if workers' beliefs are strongly biased, an equilibrium may not exist. Moreover, we show that the frequency of wage bargaining is key for determining the propagation of expectation biases on wages and equilibrium outcomes.

While the majority of the DMP-literature uses rational expectations, there are a few notable exceptions which are naturally related to our work. Examples are Kennan (2010) and Menzio (2022) who propose non-rational expectations as a mechanism in the DMP-model to endogenously generate wage rigidity and thereby improve the model's ability to explain fluctuations of unemployment and vacancies over the business cycle. Concretely, Kennan (2010) introduces private information into a version of the Mortensen and Pissarides (1994) model. In his framework, aggregate productivity shocks are publicly observed but idiosyncratic shocks are only observed by the firm. Small fluctuations in idiosyncratic productivity gives rise to wage stickiness since after a positive idiosyncratic shock the firm does not adjust the wage. Menzio (2022) assumes that workers have biased beliefs about the aggregate state of the economy. While the model economy is subject to aggregate productivity shocks, some workers believe that aggregate productivity remains constant. As a consequence, workers are optimistic in a downturn and demand higher wages than under rational expectations. This discourages firms to post vacancies in a recession. The opposite effect occurs in a boom. As a result, wages react less to aggregate productivity shocks and unemployment and vacancies are more volatile than under rational expectations.

Several aspects differentiate our paper from their work. First of all, the focus of the papers is different. Kennan (2010) and Menzio (2022) propose non-rational expectations as a mechanism to resolve the Shimer (2005) volatility puzzle. Instead, our paper does not focus on aggregate fluctuations in labor market variables but studies biased expectations a stationary environment. Relatedly, they assume that workers have, on average, correct beliefs about productivity (idiosyncratic or aggregate). Instead, we follow recent empirical studies and allow for systematic and persistent biases in individual expectations. Moreover, while we depart from rational expectations – as they do – we maintain the assumption about all information being public. That is, agents in our framework do not hold private information but truthfully report their beliefs. At the methodological level, the papers differ in terms of the bargaining concept used in the wage negotiation. The existence of private information in their papers precludes the use of the standard Nash bargaining protocol. Therefore, the more general concept of neutral bargaining solution – following Myerson (1984) – is applied in Kennan (2010). Menzio (2022) implements a version of the alternating-offer bargaining protocol of Binmore et al. (1986). Our paper follows the standard approach in the DMP-literature and uses the concept of generalized Nash bargaining. Due to the absence of private information in our setting we do not have to resort to alternative bargaining games.[5] Nevertheless, we show that in our setting the alternating-offer protocol yields the same solution to the bargaining game as Nash bargaining.

The second contribution of the paper is to show that the equilibrium effects of labor market policies are fundamentally shaped by the biases in workers' beliefs. This finding has two implications. First, ignoring these biases can lead to qualitatively different conclusions about the effects of policy. Second, our results can be useful to understand why labor policies may have differential effects across countries. The empirical work mentioned above finds evidence for substantial cross-country variation in the magnitude but also the sign of workers' expectation biases (for example, "German Angst" vs. "American optimism" of employed workers – see Balleer et al. (2023). In light of our findings,

[5] As is well known, the neutral bargaining solution coincides with the Nash bargaining solution under complete information.

one would expect labor markets to react differently to policy changes depending on the underlying belief structure.

A third contribution of the paper is to incorporate also subjective expectations of firms and to show that firms' and workers' biases in beliefs interact in the wage bargain and thereby shape the equilibrium state of the economy in important ways. Clearly, this interaction of biases could not be captured in a partial equilibrium setting. Therefore, this paper underlines the importance of adopting a general equilibrium approach when studying labor-related questions in a non-rational expectations setting.

This paper also contributes to the fast growing literature that studies the implications of workers' expectation biases for individual labor market outcomes. Within this literature, several papers are related to ours. First, Mueller et al. (2021) show in a model of job search how an optimistic bias in workers' job finding beliefs induces individuals to engage less in job search and can thereby help understand the slow exit out of unemployment for certain job seekers. Similarly, Conlon et al. (2018) develop a model of job search and show that learning about future wage offers is key to understand the observed patterns of reservation wages. Spinnewijn (2015) studies the implications of job seekers' optimistic bias for the optimal design of unemployment insurance. Our paper is complementary to these works in that we focus not only biases in the job finding expectations of unemployed workers but also in the separation expectation of employed workers. More importantly, while the aforementioned works adopt a partial equilibrium setting where workers take wages as given, our paper adopts a general equilibrium perspective. This approach has several advantages. First, we can explicitly characterize the propagation mechanism through which expectation biases affect individual wages, as well as firms' vacancy creation and equilibrium unemployment. A central element of this propagation mechanism is the bargaining game between firm and worker. Clearly, this important aspect is absent in a partial equilibrium setting where wages are taken as given. Moreover, by explicitly modeling firms and their choices, our framework can be used to jointly study expectation biases of firms and workers.

Also related are the recent papers by Balleer et al. (2021) and Broer et al. (2021). We complement these works in two ways. First, we study the implication of biased expectations for individual wages and aggregate labor market outcomes, whereas their focus is on asset accumulation and aggregate wealth inequality. Another key difference is the modeling of the labor market. In their model, the transition of workers in an out of unemployment is governed by a stationary Markov process. Thus, the worker flows are invariant to agents' beliefs. Instead in our paper, these flows are fundamentally determined by expectations as they are the result of firms' job creation choices and workers' reservation wage strategy.

The remainder of the paper is structured as follows. In Section 2 we develop the theoretical framework and we use it to derive the main analytical results and study the efficiency of equilibrium. In Section 3 we discuss model extensions. In Section 4 we analyze labor market policies. Section 5 concludes. The Appendix contains supplementary material.

2 Search and matching with biased expectations

In this section, we explore the effects of biased labor market expectations on individual wages and aggregate outcomes within a general equilibrium search-and-matching model of the labor market where wages are determined by generalized Nash bargaining between workers and firms.

2.1 Setup

Time is discrete. The economy is populated by a measure one of workers and a continuum of active firms. Workers are homogeneous, risk-neutral and infinitely lived, and they receive a period-wage ω when employed and income $b \geq 0$ when unemployed. Each active firm has one job that can be vacant or filled with a worker. A vacant job costs $\kappa > 0$ per period and a filled job produces output $z > 0$ per period, with $z > b$. Existing worker-firm pairs separate with exogenous per period probability $0 < \sigma < 1$.

Firms with vacant jobs and unemployed workers are randomly matched together. The matching function $M(u, v)$ determines the number of matches per period, where u is the number of unemployed workers and v is the number of vacant jobs. As is standard, we assume that $M(u, v)$ is homogeneous of degree 1, continuously differentiable, increasing and concave in v and u, and it satisfies $M(0, u) = M(v, 0) = 0$, and $M(v, u) \leq \min(v, u)$.

We refer to the ratio of vacant jobs to unemployed workers as the labor market tightness and we denote it by $\theta \equiv v/u$. Moreover, we define the probability of an unemployment worker to match with a vacant job as $M(u, v)/u = M(1, \theta) \equiv p(\theta)$, and the probability of a vacancy to match with an unemployed worker as $M(u, v)/v = M(1, \theta)/\theta = p(\theta)/\theta$.[6]

[6]We assume that $p(0)$=0, $p(\infty)$=1, p is continuously differentiable and weakly increasing, and $\lim_{\theta \to 0} p(\theta)/\theta = 1$, $\lim_{\theta \to \infty} p(\theta)/\theta = 0$ and that $p(\theta)/\theta$ is continuously differentiable and weakly decreasing.

The common approach in the literature is to assume that firms and workers have rational expectations about the underlying matching and separation probabilities. This includes for the worker the probability to find a job, $p(\theta)$, and the probability to transit from employment to unemployment, σ, whereas for the firm, this includes the probability to hire a worker, $p(\theta)/\theta$, and the probability to separate from the worker, σ. In line with the empirical evidence discussed in Section 1, we depart from this assumption by allowing workers to have biased expectations about the transition probabilities. In our baseline analysis we assume that firms have the correct expectations but later in Section 3.1 we relax this assumption and also allow firms to have biased beliefs.

Concretely, we assume that workers expect to separate from a given job with per period probability $\sigma_w = (1+\Delta_{\sigma w})\sigma$, and to find a job with probability $\lambda_w = (1+\Delta_{\lambda w})p(\theta)$. $\Delta_{\sigma w}$ and $\Delta_{\lambda w}$ denote the bias in workers' expectations about job separation and job finding. Clearly, when $\Delta_{\cdot w} = 0$, there is no bias and workers have rational expectations. When $\Delta_{\sigma w} > 0$, then workers are pessimistic regarding the stability of their job as they expect to separate from their employer with a higher probability than the actual job separation probability. Conversely, when $\Delta_{\lambda w} > 0$, then workers have an optimistic bias in their job finding expectations as they expect to find a new job with a higher probability than the actual job finding probability. We assume that there is no heterogeneity among workers in the magnitude of the bias. That is, all workers are equally pessimistic, optimistic or rational. Moreover, we assume that the expectation biases are constant over time. We leave it to future work to relax these assumptions and study the case where workers are heterogeneous in their bias, and learn over time about the actual transition probabilities.

As in the canonical search and matching model, we assume that the wage ω is determined by generalized Nash bargaining between the firm and the worker.[7] Initially, we consider two versions of the bargaining process that differ in the number of periods for which the wage is determined. First, we follow the majority of the literature and consider the case where the firm and the worker negotiate the wage every period. We refer to this case as **Period-by-period bargaining** (PbP). In the second case, we assume that the firm and the worker set the wage for a fixed number of periods. In the baseline, we consider the extreme case where the firm and the worker never renegotiate. Thus, the firm and the

[7] As an alternative to Nash bargaining we apply in Appendix H the alternating-offer bargaining protocol of Binmore et al. (1986). This bargaining setting can be considered a micro foundation of Nash bargaining as it explicitly specifies the strategic interaction of players in the bargaining game. We show in this context, that in our setting with biased beliefs, the alternating-offer protocol yields the same solution to the bilateral wage negotiation as Nash bargaining. The reason is that the bias in beliefs affects the agents' subjective valuation of payoffs but not their perception of the structure of the bargaining game.

worker negotiate at the beginning of the employment spell and determine the wage that is paid during the entire duration of the match. We refer to this case as **Duration-of-match bargaining** (DM). In Section 3, we relax this assumption and allow renegotiation of the wage after a finite number of periods.

PbP-bargaining is the common approach used in most of the macro-labor literature and appreciated mainly for its tractability and analytical convenience. Versions of DM-bargaining are applied in the context of staggered wages to explain nominal wage rigidity; see Gertler and Trigari (2009) and the subsequent literature. As we show below, these two bargaining settings deliver fundamentally different predictions about how workers' expectation biases affect wages. In what follows, we analyze the stationary equilibrium of the model under each of the two bargaining settings.

2.2 Period-by-period bargaining

Value functions

We assume that firms and workers negotiate the wage in every period of the match. The value of a job for a worker is given by

$$E(\omega) = \omega + \beta(1 - \sigma_w)E(\omega') + \beta\sigma_w U, \tag{1}$$

where $0 < \beta < 1$ is the personal discount factor, $0 < \sigma_w < 1$ is the worker's subjective job separation expectation, ω' is the wage of next period, and U is the value of unemployment. The value of employment depends on the worker's current-period wage, ω, and the discounted continuation value. With (subjective) probability $(1-\sigma_w)$ the match continues and the worker obtains the value of employment also next period. With probability σ_w, the match is dissolved and the worker obtains the value of unemployment next period.

Importantly, E and U are the workers' perceived values of employment and unemployment. With biased expectations, E and U can differ from the actual values.

The value of unemployment for a jobless worker is given by

$$U = b + \beta\lambda_w E(\omega') + \beta(1 - \lambda_w)U, \tag{2}$$

where b is unemployment income and λ_w is the subjective job finding probability. Combining

(1) and (2), we can express the surplus of a match for the worker as

$$E(\omega) - U = \omega - (1-\beta)U + \beta(1-\sigma_w)\big(E(\omega') - U\big).$$

Given this expression, we can define the worker's reservation wage, $\underline{\omega}$, in the standard way as the wage for which the worker is indifferent between working and unemployment. It follows that

$$E(\underline{\omega}) - U = 0 \qquad \Rightarrow \qquad \underline{\omega} = (1-\beta)U - \beta(1-\sigma_w)\big(E(\omega') - U\big).$$

The worker's reservation wage has two terms: the per-period value of unemployment $(1-\beta)U$, and the worker's expected net surplus from continuing the match next period, $\beta(1-\sigma_w)(E(\omega') - U)$. The latter term raises the worker's value of forming a match today, and therefore it reduces the reservation wage. It is straightforward to see that the reservation wage increases in the pessimistic bias of the worker (for given values of E and U). This is intuitive as for higher values of σ_w the worker expects a lower duration of the current job and thus, the expected net surplus from continuing the match is lower.

Next, we define the value of a match for the firm as

$$J(\omega) = z - \omega + \beta(1-\sigma)J(\omega') + \beta\sigma V, \tag{3}$$

where z is match output. The firm expects to separate from the worker with probability σ in which case it obtains the value of a vacant job denoted by V. The latter is defined as

$$V = -\kappa + \beta\lambda J(\omega') + \beta(1-\lambda)V. \tag{4}$$

$\kappa > 0$ is the per-period cost of an open vacancy and $\lambda = p(\theta)/\theta$ is the vacancy filling probability.

Combining (1)-(4) we can express the joint surplus of the match as

$$\begin{aligned} S(\omega) &= J(\omega) - V + E(\omega) - U \\ &= z - (1-\beta)(V+U) + \beta\Big[(1-\sigma)\big(J(\omega') - V\big) + (1-\sigma_w)\big(E(\omega') - U\big)\Big]. \end{aligned}$$

Importantly, as in the case of rational expectations, the total match surplus is independent of the current wage, ω. This holds even if the worker has biased expectations about the duration of the match, $\sigma_w \neq \sigma$. Thus, the role of the wage is to divide the surplus between the firm and the worker. This will be different in the case of DM-bargaining, where the wage also determines the size of the joint surplus.

Wage bargaining

The wage ω is determined through a generalized Nash bargaining process between the firm and the worker. Concretely, the wage is set to maximize the Nash product:

$$\omega = \arg\max \Big[E(\omega) - U\Big]^{\gamma} \Big[J(\omega) - V\Big]^{1-\gamma}, \tag{5}$$

where $0 < \gamma < 1$ is the worker's bargaining weight. The agents' threat points in the negotiation are the respective outside options. These are the value of unemployment for the worker and the value of an open vacancy for the firm.

At this point, it is important to specify the information that is exchanged between the worker and the firm in the bargain. We assume that workers are not aware of their potential expectation bias. That is, they consider their subjective transition probabilities as the actual probabilities. Therefore, a worker with biased beliefs disagrees with the firm about the transition probabilities and, as a consequence, there is disagreement about the implied job values, $J(\omega)$ and $E(\omega)$, and outside options, V and U. In other words, the firm's perceived value of, say, $E(\omega)$ differs from the worker's perceived value. In order to handle this discrepancy, we assume that (i) agents truthfully report their perceived values; hence, there is no private information, (ii) no persuasion takes place as an attempt to inform the counterpart, and (iii) agents agree to disagree; that is, the firm accepts the values of $E(\omega), U$ as reported by the worker, and vice versa.

The optimality condition associated with the maximization problem in (5) is given by

$$\gamma\Big[J(\omega) - V\Big] \underbrace{\frac{\partial E(\omega)}{\partial \omega}}_{1} + (1-\gamma)\Big[E(\omega) - U\Big] \underbrace{\frac{\partial J(\omega)}{\partial \omega}}_{-1} = 0. \tag{6}$$

A marginal change in the wage has the same (absolute) effect on the worker's and the firm's value of the match. That is, $\left|\frac{\partial E(\omega)}{\partial \omega}\right| = \left|\frac{\partial J(\omega)}{\partial \omega}\right|$. Importantly, due to this property, the sharing of the joint match surplus between the worker and the firm is not affected by the worker's subjective separation expectation, σ_w. To see this, we substitute the definition of the joint surplus, $S(\omega)$ into the wage optimality condition to obtain the standard surplus sharing rule

$$\begin{aligned} (1-\gamma)\Big(E(\omega) - U\Big) = \gamma\Big(J(\omega) - V\Big) \quad &\Rightarrow \quad E(\omega) - U &&= \gamma S \\ &\Rightarrow \quad J(\omega) - V &&= (1-\gamma) S. \end{aligned}$$

The wage is negotiated for the current period only, thus, it does not determine the future value of the job. As a consequence, the worker's expected duration of the match plays no role for how the surplus is split.

Partial equilibrium effects of expectation bias on the wage

While the bias has no effect on the sharing of the surplus, it nevertheless affects the bargained wage. In this section we illustrate this property in a partial equilibrium setting, where the value of market tightness, θ, is taken as given. The purpose of the analysis is to build intuition for how the worker's subjective beliefs shape the outcome of the bargaining process. These insights will be useful later to interpret the effects of the bias in general equilibrium.

As a starting point consider the case of no bias where $\Delta_{\sigma w} = \Delta_{\lambda w} = 0$. The solid lines in the upper panel of Figure 1 depict the first and the second part of the optimality condition in Equation (6) as a function of the wage (for the case $\gamma = 1/2$). These terms represent, respectively, the gain for the worker and the loss for the firm of a marginal increase in the wage:

$$\begin{array}{lclclclcl} E(\omega) - U & = & \omega & - & (1-\beta)U & + & \beta(1-\sigma_w)\big(E(\omega') - U\big) \\ J(\omega) - V & = & z - \omega & - & (1-\beta)V & + & \beta(1-\sigma)\big(J(\omega') - V\big). \end{array}$$

The slope of these functions is equal to $+1$ and -1, respectively.[8] The lower panel of the figure shows the Nash product, which reaches its maximum at the wage for which the worker's surplus $E(\omega) - U$ is equal to the firm's surplus $J(\omega) - V$.[9]

Now, consider a worker with a pessimistic bias in the separation expectation. This case is represented by $\sigma_w > \sigma$. A pessimistic worker discounts the future value of the match more strongly. Thus, for any given wage, the worker's surplus is lower than before:[10]

$$E(\omega) - U = \omega - (1-\beta)U + \beta(1-\sigma_w)(E(\omega') - U) \quad \Rightarrow \frac{\partial E(\omega) - U}{\partial \sigma_w} < 0.$$

In Figure 1, this case is represented by a downward-shift in the worker's surplus function (dashed line). As a consequence, the implied reservation wage of the worker increases to $\underline{\omega}'$. In other words, the pessimistic worker would not agree to work for wages that the

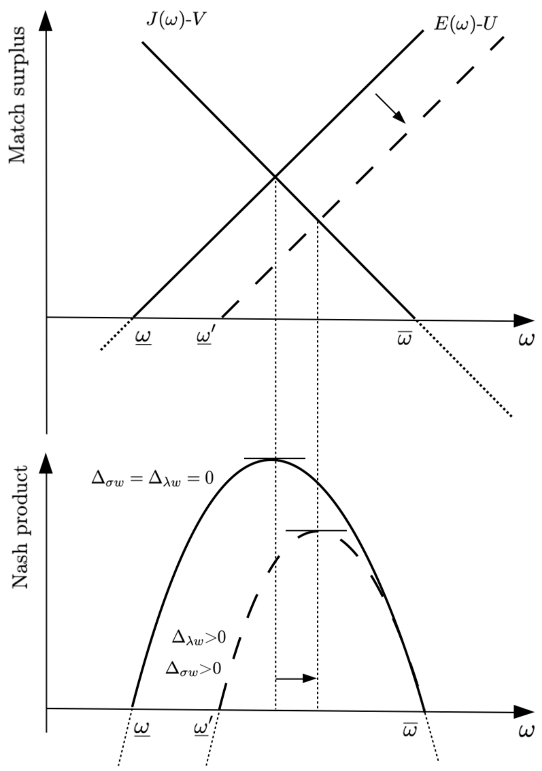
[8] In the figure, the variable $\overline{w}$ represents the maximum wage that the firm is willing to pay. It is given by $J(\overline{\omega}) - V = 0 \Rightarrow \overline{\omega} = z - (1-\beta)V + \beta(1-\sigma)(J(\omega') - V)$.

[9] See optimality condition (6) for $\gamma = 1/2$.

[10] The derivative $\partial(E(\omega) - U)/\partial\sigma_w$ also captures the effect of σ_w on $E(\omega) - U$ via the implied change in U.

rational worker was willing to accept. As a result of the decline in $E(\omega) - U$, the joint match surplus $S = J(\omega) - V + E(\omega) - U$ is reduced for any level of the wage. This is shown in the lower panel of Figure 1 by the lower hump-shaped curve. Due to the downward-shift of the $E(\omega) - U$ line, the new point of intersection between the worker's and the firm's surplus function shifts to the right. Hence, the optimal wage is higher than before. In other words, the pessimistic worker is compensated by the firm for the loss in the surplus with a higher wage.

Figure 1: (PbP) Partial equilibrium wage effects of biased expectations



Pessimistic job separation bias or optimistic job finding bias

In the next step, consider a worker with optimistic job finding expectations. When $\Delta_{\lambda w} > 0$, the worker overestimates the probability of finding employment. Thus, according to Equation (2), the value of unemployment, U, is perceived as higher than without the optimistic bias, and hence, the surplus of employment, $E(\omega) - U$, is lower. This case is also represented in the upper panel of Figure 1 by the dashed line. As before, this shift leads to an increase in the reservation wage. Furthermore, a higher U means that the optimistic worker perceives a more valuable outside option in the bargain which leads to a higher wage.

We formally state these two important results in Proposition 1.[11]

Proposition 1 (Partial equilibrium wage effects under Period-by-period bargaining)**.** *Under Period-by-period bargaining and for any positive market tightness θ, the reservation wage $\underline{\omega}$ and the bargained wage ω are*

i) increasing in the (pessimistic) bias of workers' separation expectation

$$\frac{\partial \underline{\omega}}{\partial \Delta_{\sigma w}} \geq 0, \qquad \frac{\partial \omega}{\partial \Delta_{\sigma w}} \geq 0$$

and

ii) increasing in the (optimistic) bias of worker's job finding expectation

$$\frac{\partial \underline{\omega}}{\partial \Delta_{\lambda w}} \geq 0, \qquad \frac{\partial \omega}{\partial \Delta_{\lambda w}} \geq 0.$$

General equilibrium

In the next step, we derive two conditions that jointly characterize the equilibrium of the model, namely the wage curve and the job creation condition. Combining the optimality condition (6) with the value functions (1)-(4) and the condition that the value of an open vacancy, V, is equal to zero in equilibrium due to free entry of firms, we obtain the following wage schedule:

$$\omega \quad = \quad b + \gamma \left[z - b + \kappa\theta \left(1 + \Delta_{\lambda w} + \sigma \frac{\Delta_{\sigma w}}{p(\theta)} \right) \right]. \tag{7}$$

The structure of the wage schedule is very similar to that in the standard model with rational expectations: The equilibrium wage is given by the sum of unemployment income, b, and the worker's share of period match surplus, γ. The latter is equal to output net of forgone unemployment income, $z - b$, and the worker's compensation for helping the firm to save recruiting costs, $\kappa\theta(\cdot)$. It is straightforward to verify that in the absence of agents' expectation biases, the wage schedule in (7) is identical to the familiar rational expectations solution.[12]

However, with biased beliefs we obtain that $\partial\omega/\partial\Delta_{\sigma w} \geq 0$ and $\partial\omega/\partial\Delta_{\lambda w} \geq 0$ as shown before. Interestingly, while the job finding bias, $\Delta_{\lambda w}$, enters linearly in the wage schedule, the effect of the separation bias, $\Delta_{\sigma w}$, is scaled with the value of the match for the firm. To see this, notice that in Equation (7) the term $\frac{\kappa}{p(\theta)/\theta}$ represents the firm's expected

[11] See Appendix G for the proofs of the propositions.

[12] To see this, set $\Delta_{\lambda w} = \Delta_{\sigma w} = 0$ to obtain $\omega = b + \gamma \left[z - b + \kappa\theta \right]$.

hiring cost.[13] We will show below that the firm's surplus of the match is proportional to these costs. Thus, if the match is very valuable to the firm, then there is more value that can be transferred to the worker via a higher wage.

Combining the firm's value functions (3) and (4) and using free entry ($V = 0$) and the fact that in a stationary equilibrium $J(\omega) = J(\omega')$, we obtain the job creation condition

$$\frac{z - \omega}{1 - \beta(1 - \sigma)} = \frac{\kappa}{\beta p(\theta)/\theta}. \tag{8}$$

Equation (8) is equal to the standard model with rational expectations: The left-hand side of the expression represents the present discounted value of future period-profits, $z - \omega$, whereas the right-hand side represents the expected hiring costs. In equilibrium, the firm's expected profits and expected costs are equalized so that an entering firm expects to make zero profits.

The equilibrium values of the wage and the labor market tightness are jointly determined by the wage curve in (7) and the job creation condition in (8). The job creation condition describes a negative relationship between ω and θ. The standard interpretation of this relationship applies: A high (low) wage implies low (high) period profits for the firm. To break even in expectation, the recruiting costs have to be low (high), on average. A low (high) value of these costs is obtained when vacant jobs are filled quickly which happens when the market tightness, θ, is low (high). We show this more formally by applying the total differential, d, to Equation (8)

$$\frac{d\omega}{d\theta} = -(1 - \epsilon_p(\theta))\frac{\kappa}{p(\theta)}\left(\frac{1}{\beta} - 1 + \sigma\right) < 0,$$

where $\epsilon_p(\theta) = \frac{\partial p(\theta)}{\partial \theta}\frac{\theta}{p(\theta)}$ is the elasticity of the matching probability with respect to labor market tightness. Since the matching function, $M(v, u)$, is homogeneous of degree 1, we get that $\epsilon_m < 1$.

For the wage schedule, the relationship between market tightness and the wage is less clear-cut. In the absence of expectation biases, we obtain the standard result that the wage increases in the tightness, $\frac{d\omega}{d\theta} = \gamma\kappa > 0$. The intuition is as follows. In a tight market (high θ) it takes long for the firm to fill a vacancy. Thus, by staying in the match, the worker helps the firm to save substantial hiring costs. The worker is compensated for these saved costs with a high wage. This can be different in a situation where the worker

[13] κ is the vacancy cost per period and $\frac{1}{p(\theta)/\theta}$ is the expected duration of an open vacancy.

has biased expectations. To analyze this case we rewrite the wage schedule in Equation (7) to obtain

$$\omega = b + \gamma\left[z - b + \kappa\theta(1+\Delta_{\lambda w}) + \sigma_w \frac{\kappa}{p(\theta)/\theta} - \sigma\frac{\kappa}{p(\theta)/\theta}\right].$$

The term $\kappa\theta(1+\Delta_{\lambda w})$ reflects the hiring costs that the firm saves in the current period when the worker agrees to stay in the match. This term enters positively because a higher value of θ implies a higher value of costs that can be saved. An optimistic bias $\Delta_{\lambda w} > 0$ reinforces this effect as it implies that the worker overestimates the value of market tightness. The last two terms in the wage equation can be interpreted as follows. When a match exogenously separates, then the firm is left with an open vacancy and it has to incur recruiting costs to hire a new worker. The expected value of these costs is $\sigma\frac{\kappa}{p(\theta)/\theta}$; where σ is the separation probability and $\frac{\kappa}{p(\theta)/\theta}$ is the average hiring cost. An optimistic worker with $\sigma_w < \sigma$ underestimates the probability of a separation and thus perceives the expected hiring costs to be lower and equal to $\sigma_w\frac{\kappa}{p(\theta)/\theta}$. In the extreme case when $\sigma_w \to 0$, the worker believes the match to last forever, thus the perceived future recruiting costs are zero. A higher market tightness raises the firm's future hiring costs, but since the optimistic worker underestimates this increase, the effect of a higher θ on the wage is negative. When the worker's optimism is sufficiently strong, then this negative effect dominates the positive effect implied by $\kappa\theta(1+\Delta_{\lambda w})$. As a result, the wage can decline with market tightness. More formally, we establish that:

$$\frac{d\omega}{d\theta} = \gamma\kappa\left(1 + \Delta_{\lambda w} + \frac{\sigma\Delta_{\sigma w}}{p(\theta)}(1-\epsilon_m)\right) \gtreqless 0 \qquad \Longleftrightarrow \qquad \sigma - \sigma_w \lesseqgtr \frac{\lambda_w}{1-\epsilon_m}.$$

To reiterate, this condition shows that the wage increases (decreases) with tightness when the worker is sufficiently pessimistic (optimistic) about job separation and sufficiently optimistic (pessimistic) about job finding. An equilibrium of the model is given by the pair (ω, θ) that jointly solves the wage curve in (7) and the job creation condition in (8). The following proposition makes a formal statement about existence and uniqueness of equilibrium.

Proposition 2 (Existence and uniqueness of equilibrium under Period-by-period bargaining)**.** *Under Period-by-period bargaining, an equilibrium with $\theta > 0$ exists if and only if the condition*

$$\gamma(\sigma - \sigma_w) \leq (1-\gamma)\tfrac{z-b}{\kappa} - \left[\tfrac{1}{\beta} - 1 + \sigma\right],$$

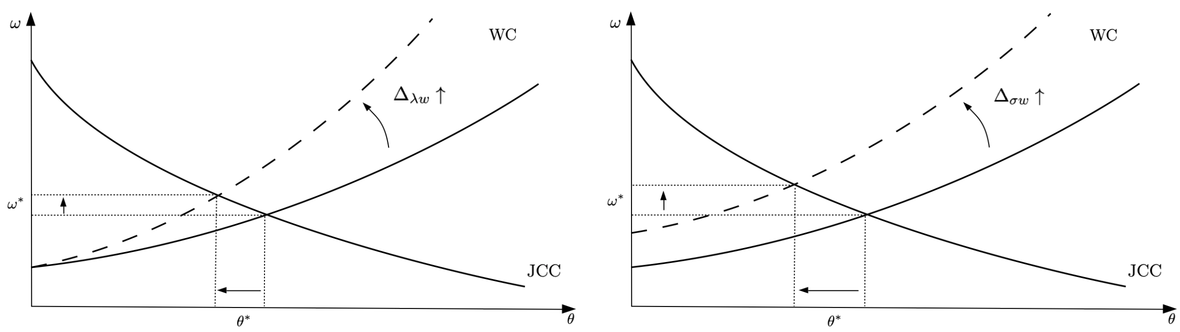
i) holds with strict inequality, or

ii) holds with equality and $\gamma(1+\Delta_{\lambda w}) = 0$ and $\lim_{\theta\to 0}\frac{\partial p(\theta)/\theta}{\partial\theta} = 0$.

If i) holds and $\gamma(1+\Delta_{\lambda w}) > 0$ or $p(\theta)/\theta$ is strictly decreasing, then the equilibrium is unique.

Lastly, we perform a comparative statics exercise to analyze the effects of biases on the equilibrium. For this exercise, we consider the unique equilibrium characterized in Proposition 2. An optimistic bias in the job finding probability $\Delta_{\lambda w} > 0$ leads to an upward-shift of the wage curve (compared to rational expectations) as workers expect to find jobs more easily. This situation is depicted in Panel (a) of Figure 2. As a consequence, the firm pays a higher wage to compensate the worker for staying in the match. The higher wage implies that firms require lower expected recruiting costs to break even. This is achieved by a lower vacancy duration for which the market tightness has to decline. Due to the lower θ the equilibrium unemployment rate $u = \frac{\sigma}{\sigma+p(\theta)}$ and the average unemployment duration $d = 1/p(\theta)$ increase.

Figure 2: General equilibrium effects of bias under PbP-bargaining



(a) Optimistic job finding bias (b) Pessimistic job separation bias

Notes. JCC: Job creation condition, WC: Wage curve.

A pessimistic bias in the worker's separation expectation $\Delta_{\sigma w} > 0$ leads to an upward-shift in the wage curve (compared to rational expectations) - as shown in Panel (b) of Figure 2. The pessimistic worker expects to lose the job soon, thus, the firm has to offer a higher wage to make worker stay in the match. The higher wage leads to same adjustment of tightness as in the previous case. These results are succinctly presented in the following proposition

Proposition 3 (General equilibrium effects under Period-by-period bargaining).
If the conditions for equilibrium uniqueness in Proposition 2 are satisfied, then

i) an optimistic bias in workers' job finding expectation leads to a decrease in equilibrium market tightness θ, and to an increase in the wage ω, unemployment rate u, and *unemployment duration d*

$$\frac{\partial \theta}{\partial \Delta_{\lambda w}} \leq 0, \quad \frac{\partial \omega}{\partial \Delta_{\lambda w}} \geq 0, \quad \frac{\partial u}{\partial \Delta_{\lambda w}} \geq 0, \quad \frac{\partial d}{\partial \Delta_{\lambda w}} \geq 0$$

ii) a pessimistic bias in workers' separation expectation leads to a decrease in equilibrium market tightness θ, and to an increase in the wage ω, unemployment rate u, and unemployment duration d

$$\frac{\partial \theta}{\partial \Delta_{\sigma w}} \leq 0, \quad \frac{\partial \omega}{\partial \Delta_{\sigma w}} \geq 0, \quad \frac{\partial u}{\partial \Delta_{\sigma w}} \geq 0, \quad \frac{\partial d}{\partial \Delta_{\sigma w}} \geq 0.$$

2.3 Duration of match bargaining

In the next step, we analyze the case where the worker and the firm set the wage for a fixed number of periods. First, we consider the extreme case where, upon matching, the worker and the firm negotiate a wage that is paid during the entire duration of the match. Below, in Section 2.5, we relax this assumption and allow for renegotiation. It should be noted that the outcome of this bargaining setting is renegotiation-proof since we consider a stationary environment where z is constant over time and workers do not update their beliefs. Many parts of the model are similar to before and thus, we keep the exposition brief. The value of a match for the worker is given by:

$$E(\omega) = \omega + \beta(1 - \sigma_w)E(\omega) + \beta\sigma_w U. \tag{9}$$

In contrast to the case with PbP-bargaining, the current wage ω now applies also to the next period. The value of unemployment is the same as before and given by

$$U = b + \beta\lambda_w E(\omega) + \beta(1 - \lambda_w)U. \tag{10}$$

We combine (9) and (10) to express the surplus of employment for the worker

$$E(\omega) - U = \frac{\omega - (1 - \beta)U}{1 - \beta(1 - \sigma_w)}. \tag{11}$$

From that we can express the reservation wage, $\underline{\omega}$, as

$$E(\underline{\omega}) - U = 0 \qquad \Rightarrow \qquad \underline{\omega} = (1 - \beta)U.$$

Different to before, the reservation wage depends only on the per-period value of unemployment, $(1-\beta)U$, but not on the continuation value of the job. In other words, whether the worker is indifferent between employment and unemployment does not depend on the expected duration of the match. Therefore, the worker's separation expectation, σ_w, exhibits no direct effect on the reservation wage.

The value of a match for the firm is given by

$$J(\omega) = z - \omega + \beta(1-\sigma)J(\omega) + \beta\sigma V, \tag{12}$$

and the value of a vacancy is

$$V = -\kappa + \beta\lambda J(\omega) + \beta(1-\lambda)V. \tag{13}$$

We combine the value functions in (9)-(13) to express the joint surplus of the match:

$$S(\omega) = J(\omega) - V + E(\omega) - U = \frac{z - \omega - (1-\beta)V}{1-\beta(1-\sigma)} + \frac{\omega - (1-\beta)U}{1-\beta(1-\sigma_w)}. \tag{14}$$

Importantly, under DM-bargaining, the wage not only divides the joint surplus between the firm and the worker but it also determines the level of $S(\omega)$. As we can see from Equation (14), the wage determines the size of $S(\omega)$ whenever the worker has biased separation expectations. For example, when the worker is pessimistic ($\sigma_w > \sigma$) then the joint surplus is negatively related to the wage:

$$\frac{\partial S(\omega)}{\partial \omega} = \frac{\partial J(\omega)}{\partial \omega} + \frac{\partial E(\omega)}{\partial \omega} = -\frac{1}{1-\beta(1-\sigma)} + \frac{1}{1-\beta(1-\sigma_w)} < 0.$$

To understand this relationship, it is important to notice that a marginal change in the wage has a differential impact on the worker's and the firm's surplus. More specifically, when $\sigma_w > \sigma$, then a marginal increase in the wage ω increases the worker's surplus $E(\omega)$ by less (in absolute value) than it decreases the firm's surplus $J(\omega)$. This is because the separation probability determines the expected duration for which the wage is paid. Thus, if the worker expects a shorter duration than the firm, then the perceived gain for the worker from a higher wage is smaller than the loss for the firm.

Wage bargaining

As before, the wage ω is set to maximize the Nash product

$$\omega = \arg\max \Big[E(\omega) - U\Big]^{\gamma} \Big[J(\omega) - V\Big]^{1-\gamma}.$$

The optimality condition associated with this problem is given by

$$\gamma\Big[J(\omega)-V\Big]\underbrace{\frac{\partial E(\omega)}{\partial\omega}}_{\frac{1}{1-\beta(1-\sigma_w)}}+(1-\gamma)\Big[E(\omega)-U\Big]\underbrace{\frac{\partial J(\omega)}{\partial\omega}}_{-\frac{1}{1-\beta(1-\sigma)}}=0.$$

This condition differs in two aspects from that obtained under PbP-bargaining. First, the derivative of the worker's and the firm's value function with respect to the wage are larger than unity (in absolute value). This is because a marginal change in ω affects not only the instantaneous value of the match (as under PbP-bargaining) but also the future values. Second, as mentioned before, when $\sigma_w \neq \sigma$, then a marginal change in the wage affects the worker's value differently than the firm's value. Thus, we have that

$$\left|\frac{\partial E(\omega)}{\partial\omega}\right| \neq \left|\frac{\partial J(\omega)}{\partial\omega}\right|.$$

We can write the optimality condition as

$$(1-\gamma)\Big(E(\omega)-U\Big)=\frac{1-\beta(1-\sigma)}{1-\beta(1-\sigma_w)}\gamma\Big(J(\omega)-V\Big). \tag{15}$$

from which we derive the worker's share in the total surplus

$$\frac{E(\omega)-U}{S(\omega)}=\gamma\left[\frac{1}{\gamma+(1-\gamma)\frac{1-\beta(1-\sigma_w)}{1-\beta(1-\sigma)}}\right].$$

As already alluded to, the sharing of the total surplus between the firm and the worker depends on the worker's subjective expectation. As we can see from the previous expression, the worker's share in the surplus is equal to γ when $\sigma_w = \sigma$. However, it is less than γ when the worker is pessimistic about the match duration, and it is larger than γ when the worker is optimistic. The intuition for this relationship will be discussed in the next section.

Partial equilibrium effect of expectation bias on the wage

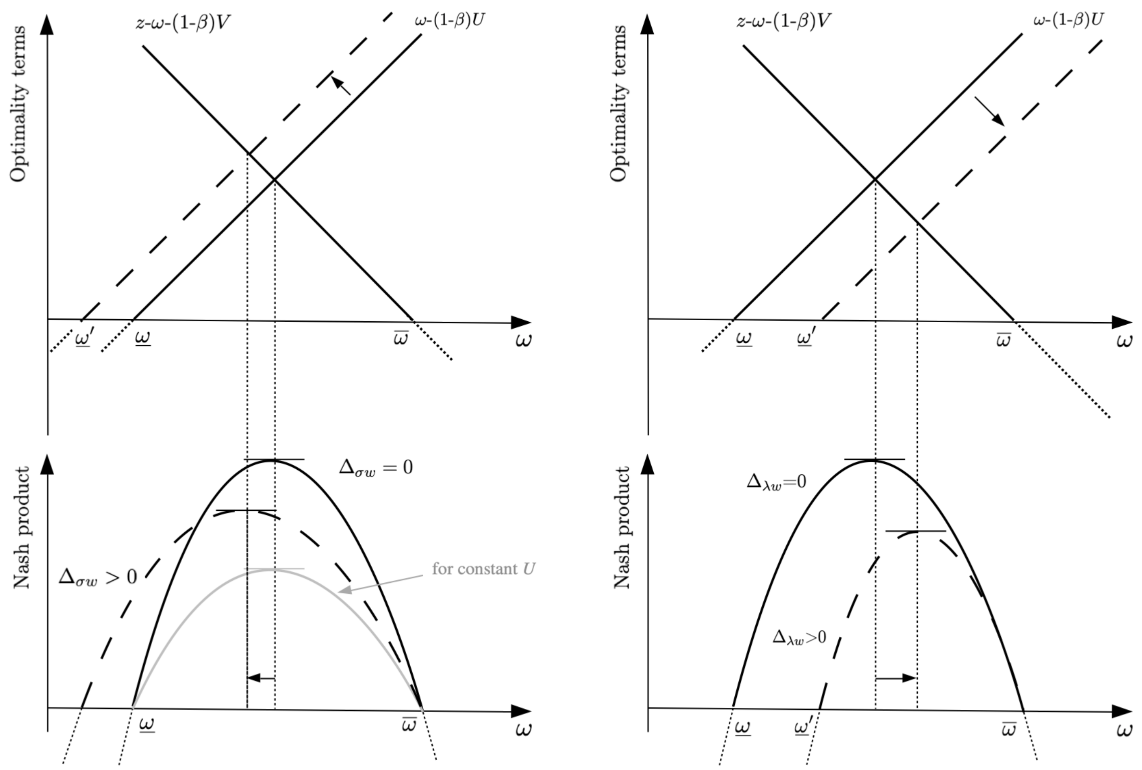
In the next step, we study as before – in partial equilibrium – how workers' expectation bias affects the bargained wage. We consider the case of no bias as a reference point. For the interpretation, it will be useful to rewrite the wage optimality condition (15) by using the value functions in (9)-(13). This yields the following expression

$$
\begin{aligned}
(1-\gamma)\Big(E(\omega)-U\Big)\Big(1-\beta(1-\sigma_w)\Big) &= \gamma\Big(J(\omega)-V\Big)\Big(1-\beta(1-\sigma_f)\Big) \\
\Rightarrow \qquad (1-\gamma)\Big(\omega-(1-\beta)U\Big) &= \gamma\Big(z-\omega-(1-\beta)V\Big).
\end{aligned}
$$

The solid lines in the upper panels of Figure 3 depict both sides of the wage optimality condition (for $\gamma = 1/2$). As we can see from the previous expression, both sides of the optimality condition have a slope of one (in absolute value). As a result, the Nash product – depicted in the lower panels of the figure – reaches its maximum at the wage ω for which the two lines intersect.

Now, consider a worker with pessimistic separation expectations, $\sigma_w > \sigma$. As we can see from Equation (11), a higher value of σ_w implies that the worker discounts future wages more heavily which leads to a decline in the worker's match surplus $E(\omega)-U$. However, as mentioned before, the reservation wage is unaffected by changes in the worker's separation expectation. Hence, there is no direct effect of σ_w on the solid line in the upper part of Panel (a).

Figure 3: (DM) Partial equilibrium wage effects of biased expectations



(a.) Pessimistic job separation bias (b.) Optimistic job finding bias

The decline in the worker's surplus $E(\omega) - U$ reduces the value of the Nash product for any wage within the bargaining set $(\underline{\omega}, \overline{w})$. This is represented by the grey line in the lower part of Panel (a) in Figure 3. While the Nash product is lower than before, its maximum is attained for the same wage. To see this, consider the wage optimality condition from above. It implies that for a given value of U, the optimal wage is unaffected by the worker's separation expectation.

However, clearly, the value of unemployment, U, is different for a pessimistic worker. In particular, a higher σ_w implies a lower value of U since the pessimistic worker considers future employment less attractive. As a result, the surplus of the current match rises. This situation is represented in the upper part of Panel (a) by an upward shift in the line representing the term $(\omega - (1-\beta)U)$. Due to the reduction in the worker's outside option, U, the reservation wage declines to $\underline{\omega}'$. As a result, the new maximum of the Nash product is attained for a lower wage ω. Intuitively, the lower wage is optimal because the pessimistic worker discounts the future more than the firm and thus, the lower wage decreases the worker's value of employment by less than it increases the value of the match for the firm. We state this result more formally below in Proposition 4. Notice that this finding is the exact opposite of what we obtained under PbP-bargaining. In Section 2.4 we discuss in detail why the two bargaining settings deliver these opposing results regarding the wage effect of the separation bias.

As before under PbP-bargaining, an optimistic bias in the worker's job finding expectation increases the value of unemployment and thus, leads to a higher reservation wage. This situation is depicted in Panel (b) of Figure 3. With a more valuable outside option, the worker ends up with a better bargain and obtains a higher wage.

Proposition 4 (Partial equilibrium wage effects under duration-of-match bargaining)**.** *Under duration-of-match bargaining and for any positive market tightness θ, the reservation wage $\underline{\omega}$ and the bargained wage ω are*

i) increasing in the (optimistic) bias of worker's job finding expectation

$$\frac{\partial \underline{\omega}}{\partial \Delta_{\lambda w}} \geq 0, \qquad \frac{\partial \omega}{\partial \Delta_{\lambda w}} \geq 0$$

and

ii) decreasing in the (pessimistic) bias of worker's separation expectation

$$\frac{\partial \underline{\omega}}{\partial \Delta_{\sigma w}} \leq 0, \qquad \frac{\partial \omega}{\partial \Delta_{\sigma w}} \leq 0.$$

General equilibrium

In the next step, we analyze the equilibrium of the model. For this purpose, we derive the wage curve and the job creation condition. We obtain the wage curve by combining the optimality condition (15) with the agents' value functions (9)-(13) and we use the fact that $V = 0$ in equilibrium. As a result, we obtain:

$$\omega = b + \gamma \Big[z - b + \frac{1 - \beta(1-\sigma)}{1 - \beta(1-\sigma_w)} \kappa\theta \left(1 + \Delta_{\lambda w}\right) \Big]. \tag{16}$$

As before under PbP-bargaining, the wage is equal to the sum of unemployment income, b, and the worker's share γ in the per-period match surplus. Also, it is straightforward to verify that in the absence of expectation biases, the wage curve is identical to the rational expectations solution and the PbP-solution.

With biased beliefs, the effects of the bias on the wage are as described before with $\partial\omega/\partial\Delta_{\sigma w} < 0$ and $\partial\omega/\partial\Delta_{\lambda w} > 0$. Moreover, the specific bargaining setting does not affect the value functions of the firm and the free entry condition. Thus, the job creation condition under DM-bargaining is identical to that under PbP-bargaining and given by Equation (8).

As before, the equilibrium of the model is described by the pair (ω, θ) that jointly solves the job creation condition and the wage curve. As we can see from Equation (16), the wage curve describes a positive relationship between the wage and labor market tightness. This relationship holds for any value of σ_w. Thus, unlike in the case of PbP-bargaining, the existence and uniqueness of equilibrium under DM-bargaining is independent of the workers' subjective separation probabilities. Concretely, we can state the following proposition.

Proposition 5 (Existence and uniqueness of equilibrium under duration-of-match bargaining). *Under duration-of-match bargaining, an equilibrium with $\theta > 0$ exists if and only if the condition*

$$0 \leq (1-\gamma)\tfrac{z-b}{\kappa} - \left[\tfrac{1}{\beta} - 1 + \sigma\right]$$

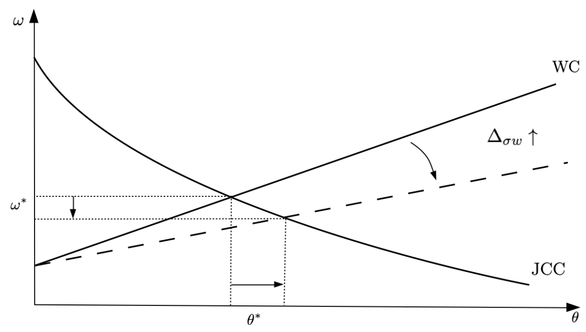
i) holds with strict inequality, or

ii) holds with equality and $\gamma(1+\Delta_{\lambda w}) = 0$ *and* $\lim_{\theta\to 0} \frac{\partial p(\theta)/\theta}{\partial \theta} = 0$.

If i) holds and $\gamma(1+\Delta_{\lambda w}) > 0$ *or* $p(\theta)/\theta$ *is strictly decreasing, then the equilibrium is unique.*

In the last step, we perform comparative statics to investigate how workers' expectation biases affect equilibrium outcomes. We focus only on the separation expectations, because the bias in job finding expectations have the same effects as in the case of PbP-bargaining. As before, we use rational expectations as the reference point. A pessimistic bias in the worker's separation expectation $\Delta_{\sigma w} > 0$ leads to a downward-shift in the wage curve. This situation is depicted in Figure 4. The pessimistic worker expects a shorter duration of the match and, thus, perceives a lower value of saved recruiting costs. The firm compensates the worker for these costs by paying a lower wage. As the job creation condition is unaffected by the worker's separation expectation, the shift in the wage curve leads to an increase in the equilibrium labor market tightness. Lower wages imply higher firms' profits which encourages vacancy creation and leads to a higher value of θ and lower unemployment rate and duration.

Figure 4: General equilibrium effects of separation bias under DM-bargaining



Notes. JCC: Job creation condition, WC: Wage curve

Proposition 6 (General equilibrium effects under duration-of-match bargaining).
If the conditions for equilibrium uniqueness in Proposition 5 are satisfied, then

i) an optimistic bias in workers' job finding expectation leads to a decrease in equilibrium market tightness θ*, and an increase in the wage* ω*, unemployment rate* u*, and unemployment duration* d

$$\frac{\partial \theta}{\partial \Delta_{\lambda w}} \leq 0, \quad \frac{\partial \omega}{\partial \Delta_{\lambda w}} \geq 0, \quad \frac{\partial u}{\partial \Delta_{\lambda w}} \geq 0, \quad \frac{\partial d}{\partial \Delta_{\lambda w}} \geq 0$$

ii) a pessimistic bias in workers' separation expectation leads to an increase in equilibrium market tightness θ, and a decrease in the wage ω, unemployment rate u, and unemployment duration d

$$\frac{\partial \theta}{\partial \Delta_{\sigma w}} \geq 0, \quad \frac{\partial \omega}{\partial \Delta_{\sigma w}} \leq 0, \quad \frac{\partial u}{\partial \Delta_{\sigma w}} \leq 0, \quad \frac{\partial d}{\partial \Delta_{\sigma w}} \leq 0.$$

2.4 Discussion

The purpose of this section is to briefly discuss why the two bargaining settings considered in Sections 2.2 and 2.3 imply such different effects of job separation expectations on the wage.

Start with a worker with unbiased expectations who earns a given wage ω. Suppose that this worker becomes pessimistic with $\Delta_{\sigma w} > 0$. For the same wage ω, the perceived match surplus, $E(\omega) - U$, is now lower than before because the pessimistic worker expects a lower duration of the match. This logic holds irrespective of the bargaining setting. A first important difference between the two bargaining settings emerges since with PbP-bargaining, the pessimistic worker's reservation wage is higher than before while it is lower with DM-bargaining. The underlying intuition is as follows. In both settings, the value of unemployment U declines with the bias – because the pessimistic worker considers future employment less valuable – which negatively affects the reservation wage in both cases. However, under PbP-bargaining this negative effect is dominated by the additional positive effect which is due to the decline in the (subjective) discounted value of future employment $\beta(1 - \sigma_w)(E(\omega') - U)$. The firm compensates the worker for the lower value of future employment by paying a higher wage. The higher wage raises the Nash product because, under PbP-bargaining, the wage is set for just one period and hence it does not affect the future surplus of the match. Consequently, the worker's biased discounting of the future (represented by $\beta(1 - \sigma_w)$) does not matter for the effect of the current-period wage ω on the worker' perceived match surplus $E(\omega) - U$. As a result, the increase in the wage raises the workers surplus by the same amount (in absolute value) than it decreases the firm's surplus. In the end, the worker obtains the same share, γ, of the total match surplus $S(\omega)$, as before with unbiased expectations.

The situation is very different with DM-bargaining. In this setting, the wage is set for all periods and, thus, it affects the future surplus of the match. As a result, and in contrast to PbP-bargaining, the worker's subjective discount factor now plays a key role. Since the pessimistic worker strongly discounts the future effects of the wage on the match surplus, it is optimal for the firm to set a lower wage. The reason is that due to the biased discounting, the reduction in worker's perceived match surplus due to the lower wage is less (in absolute value) than the gain in the firm's surplus. As a result, the pessimistic worker can only extract a share of the total match surplus of less than γ.

2.5 Generalization: Period-T bargaining

Clearly, the bargaining settings considered in Sections 2.2 and 2.3 are two extreme cases. In reality, wages are neither fixed for the entire employment spell nor are they renegotiated every period. A more realistic scenario is an intermediate setting where the worker and the firm renegotiate the wage after a certain number of periods. We analyze this case in this section. The analysis of such a setting is of interest per se and, moreover, it will be useful to bring the findings of the previous sections into perspective. Of particular interest is the question to what extent the bargaining horizon, i.e. the number of periods for which the wage is fixed, matters for how the worker's separation expectation affects the wage.

Specifically, consider the framework from before but suppose that the worker and the firm bargain over the wage every T periods. Once the wage is set, it stays fixed until the next bargaining round. The worker's match surplus can be derived from the value function in (1) and it is given by

$$E_T(\omega) - U = \underbrace{[\omega - (1-\beta)U] \sum_{t=1}^{T} (\beta(1-\sigma_w))^{t-1}}_{\text{Period surplus}} + \underbrace{[\beta(1-\sigma_w)]^T \Big(E_T(\omega') - U\Big)}_{\text{Continuation value}}. \quad (17)$$

The worker obtains a given wage ω for a total of T periods. Thus, the first term in the expression represents the discounted sum of period surplus, $\omega - (1-\beta)U$, that accrues from these wage payments. After T periods, the worker and the firm negotiate again. Thus, the second term reflects the discounted continuation value of the match that the worker obtains for the new wage ω'.

It is straightforward to see that this specification nests the two bargaining settings from above. When $T = 1$ the worker obtains the period surplus only once. Instead when $T \to \infty$, then the continuation value vanishes since $\lim_{T\to\infty} [\beta(1-\sigma_w)]^T = 0$.

$$\text{PbP} \qquad T = 1 \qquad E_1(\omega) - U = \omega - (1-\beta)U + \beta(1-\sigma_w)\Big(E(\omega') - U\Big)$$

$$\text{DM} \qquad T \to \infty \qquad E_\infty(\omega) - U = \frac{\omega - (1-\beta)U}{1-\beta(1-\sigma_w)}$$

Using the worker's surplus in Equation (17) and the firm's surplus we solve the Nash bargaining problem and obtain the following wage schedule.[14]

$$\omega_T = b + \gamma \left[z - b + \kappa\theta \left(\overbrace{\frac{\sum_{t=1}^{T} \Big(\beta(1-\sigma_w)\Big)^{t-1}}{\sum_{t=1}^{T} \Big(\beta(1-\sigma)\Big)^{t-1}}}^{\frac{\partial \cdot}{\partial \sigma_w}<0} (1+\Delta_{\lambda w}) + \overbrace{\beta^{T-1} \frac{(1-\sigma)^T - (1-\sigma_w)^T}{p(\theta) \sum_{t=1}^{T} \Big(\beta(1-\sigma)\Big)^{t-1}}}^{\frac{\partial \cdot}{\partial \sigma_w}>0} \right) \right]. \tag{18}$$

This expression is a convex combination of the wage schedules obtained for the PbP- and the DM-bargaining setting. It is straightforward to verify that for $T = 1$ ($T \to \infty$) the wage schedule boils down to the PbP- (DM-)solution. Next, we use the wage schedule to show how the bargaining frequency T matters for the effect of the worker's separation expectation on the wage. In this context, it is useful to recall the findings from above: With PbP-bargaining we obtain a positive effect of the separation bias on the wage, whereas with DM-bargaining the effect was negative.

As we can see from Equation (18), the total effect of the separation bias on the wage depends on the two opposing effects represented by the two terms inside the round brackets. The first term depends negatively on the bias. The underlying intuition is as discussed in the context of the DM-bargaining. To reiterate, a pessimistic worker discounts future wages more strongly than the firm and therefore agrees to a lower wage. In contrast, the second term in brackets depends positively on the bias. The intuition is as in the case of PbP-bargaining: A pessimistic worker strongly discounts the (future) surplus of the match that is obtained in the next bargaining round. The firm compensates the worker for the lower perceived job value by paying a higher wage.

As we can see from Equation (18), the bargaining horizon affects the strength of these two effects. When $T = 1$, then the wage is fixed for just one period. Therefore, the difference in the effective discount factor between the worker $\beta(1-\sigma_w)$ and the firm $\beta(1-\sigma)$ is irrelevant. More specifically, when $T = 1$, then a higher wage raises the worker's surplus by the same absolute value that it lowers the firm's surplus. This relationship is illustrated in Panel (a) of Figure 5 where $\frac{\partial E(\omega)-U}{\partial \omega} = \left|\frac{\partial J(\omega)-V}{\partial \omega}\right| = 1$ for $T = 1$. As a result, the first (negative) effect is not present and we obtain that $\frac{\partial \omega_T}{\partial \sigma_w} > 0$. However, for $T > 1$ the first effect comes into play and it becomes more important as the bargaining horizon increases. The reason is that, for $T > 1$, the differential discounting between the firm

[14] The derivations are in Appendix A.

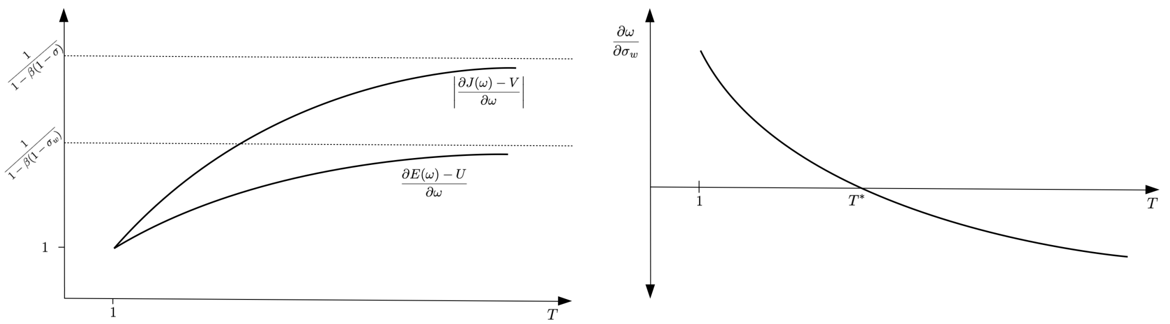
and the worker starts to matter. This implies that in the case of a pessimistic worker, a higher wage raises the worker's surplus by less than it lowers the firm's surplus. More formally:

$$\frac{\partial E(\omega) - U}{\partial \omega} = \sum_{t=1}^{T} \Big(\beta(1-\sigma_w)\Big)^{t-1} < \sum_{t=1}^{T} \Big(\beta(1-\sigma)\Big)^{t-1} = \left|\frac{\partial J(\omega) - V}{\partial \omega}\right| \qquad \text{for } \; T > 1. \tag{19}$$

This relationship is illustrated in Panel (a) of Figure 5 where $\frac{\partial E(\omega)-U}{\partial \omega} < \left|\frac{\partial J(\omega)-V}{\partial \omega}\right|$ for $T > 1$. The maximum difference in the discounting is obtained for the longest possible bargaining horizon, i.e. for $T \to \infty$. In this case – which is the DM-bargaining setting – the separation bias has the largest negative effect on the wage.

At the same time, as T increases, the next bargaining round moves further and further into the distant future. Therefore, the worker discounts more strongly the surplus which is associated with the next round, $E(\omega') - U$. As a result, a pessimistic worker requires less and less compensation for the decline in future surplus. Thus, the second (positive) effect of the separation bias on the wage becomes less important as the bargaining horizon increases.

Figure 5: Marginal effects under period-T bargaining



(a) Marginal effect of wage on match surplus (b) Marginal effect of separation bias on wage

To summarize, as the bargaining horizon increases, the negative effect of the separation bias on the wage becomes larger and the positive effect becomes smaller. This implies that the derivative $\frac{\partial \omega}{\partial \sigma_w}$ is monotonically decreasing in T, as shown in Panel (b) of Figure 5. Together with a positive derivative for $T = 1$ and a negative derivative for $T \to \infty$, there exists a unique cutoff for the bargaining frequency T^* such that the wage increases with the separation bias for all $T < T^*$ and the wage decreases with the bias for all $T \geq T^*$. One can show that this unique cutoff is given by the smallest integer T^* which satisfies the following condition:

$$\frac{\partial \omega_T}{\partial \sigma_w} < 0 \qquad \Longleftrightarrow \qquad \frac{T^*}{(1+\Delta_{\lambda w})p(\theta)} < \beta \sum_{t=1}^{T^*-1} (T^* - t)\Big(\beta(1-\sigma_w)\Big)^{-t}.$$

Lastly, it is important to notice that the job creation condition is not affected by the length of the bargaining horizon. Thus, the expression in Equation (8) also applies to the case of period-T bargaining. Using the job creation condition together with the wage schedule in Equation (18) we can derive the equilibrium of the economy. The following proposition makes a formal statement about existence and uniqueness of equilibrium.

Proposition 7 (Existence and uniqueness of equilibrium under period-T-bargaining)**.** *Under period-T-bargaining, an equilibrium with $\theta > 0$ exists if and only if the condition*

$$\gamma\left((1-\sigma)^T - (1-\sigma_w)^T\right)\beta^{T-1} \leq (1-\gamma)\tfrac{z-b}{\kappa}\textstyle\sum_{t=1}^{T}\Big(\beta(1-\sigma)\Big)^{t-1} - \left[\tfrac{1}{\beta} + (1-\sigma)^T\beta^{T-1}\right]$$

i) holds with strict inequality, or

ii) holds with equality and $\gamma(1+\Delta_{\lambda w}) = 0$ and $\lim_{\theta\to 0}\frac{\partial p(\theta)/\theta}{\partial\theta} = 0$.

If i) holds and $\gamma(1+\Delta_{\lambda w}) > 0$ or $p(\theta)/\theta$ is strictly decreasing, then the equilibrium is unique.

3 Extensions

3.1 Bias in workers' and firms' expectations

As an extension to the baseline model, we also allow firms to have biased expectations about job separation and vacancy filling. Concretely, we assume that firms expect to separate from a worker with probability $\sigma_f = (1+\Delta_{\sigma f})\sigma$, and to meet an unemployed worker with probability $\lambda_f = (1+\Delta_{\lambda f})p(\theta)/\theta$. Thus, $\Delta_{\sigma f}$ and $\Delta_{\lambda f}$ denote the bias in firms' expectations about match separation and vacancy filling.

In order to analyze the equilibrium of this extended model, we derive the job creation condition and the wage curve – see Appendix B for the details. As in Section 2.5, we assume that wages are renegotiated after T periods. The job creation condition is given by the following expression:

$$\frac{z-\omega}{1-\beta(1-\sigma_f)} = \frac{\kappa}{\beta(1+\Delta_{\lambda f})p(\theta)/\theta}. \tag{20}$$

As before, the left-hand side of the expression represents the present discounted value of future period-profits, whereas the right-hand side represents the expected hiring costs. Biases in firm's subjective matching and separation probabilities affect the perceived costs and profits, and thereby shape firm entry and vacancy creation. Concretely, a pessimistic bias in the separation probability, $\Delta_{\sigma f} > 0$ lowers the left-hand side of Equation (20) because a pessimistic firm expects to reap the period-profits $z - \omega$ for a shorter duration. For a given wage, ω, this leads to fewer vacancy creation and a drop in the labor market tightness, θ. Similarly, a pessimistic bias in the matching expectation, $\Delta_{\lambda f} < 0$ increases the right-hand side of Equation (20) as the firm expects higher recruiting costs to find a worker. Again, for a given wage, this leads to fewer vacancies in equilibrium.

Following the same steps as in the previous section, we obtain the following expression for the wage schedule

$$\omega_T = b + \gamma \left[z - b + \kappa\theta \left(\frac{\sum_{t=1}^{T} \Big(\beta(1-\sigma_w)\Big)^{t-1}}{\sum_{t=1}^{T} \Big(\beta(1-\sigma_f)\Big)^{t-1}} \left(\frac{1+\Delta_{\lambda w}}{1+\Delta_{\lambda f}} \right) + \beta^{T-1} \frac{(1-\sigma_f)^T - (1-\sigma_w)^T}{(1+\Delta_{\lambda f})p(\theta) \sum_{t=1}^{T} \Big(\beta(1-\sigma_f)\Big)^{t-1}} \right) \right]. \tag{21}$$

The wage schedule reveals the interesting implication that only the amount of disagreement between the worker and the firm – but not the size of expectation biases – matters for the wage. In other words, if the firm and the worker are equally pessimistic (or optimistic), ($\Delta_{\sigma w} = \Delta_{\sigma f}$ and $\Delta_{\lambda w} = \Delta_{\lambda f}$), then the wage curve is the same as under rational expectations.[15] The reason for this "equivalence result" is twofold. First, the perceived value of (saved) recruiting costs depends on the ratio of worker's subjective job finding expectation to the firm's subjective vacancy filling expectation. This ratio is equivalent to the agents' perceived labor market tightness, $\theta\left(\frac{1+\Delta_{\lambda w}}{1+\Delta_{\lambda f}}\right)$. If agents' agree on the value of tightness, then they perceive the same value of recruiting costs. Second, when $\Delta_{\sigma w} = \Delta_{\sigma f}$, then the worker and the firm agree on the expected duration of the match. In this case, agents share the same discount factor ($\beta(1-\sigma_f) = \beta(1-\sigma_w)$), and thus, they identically evaluate the future contributions of the wage on the match surplus.

As before, the equilibrium of the model is given by the pair (ω, θ) that jointly solves the wage curve and the job creation condition. The existence and uniqueness of equilibrium depends on – among other things – on the amount of disagreement in the agents' separation expectation. More formally, we establish the following proposition.

[15]While the wage curve is the same as under rational expectations, the equilibrium wage may not be the same, because the value of θ can be different in equilibrium.

Proposition 8 (Existence and uniqueness of equilibrium under period-T-bargaining with biased worker and firm expectations)**.**
Under period-T-bargaining and biased worker and firm expectations, an equilibrium with $\theta > 0$ *exists if and only if* $\Delta_{\lambda f} > -1$ *and following condition*

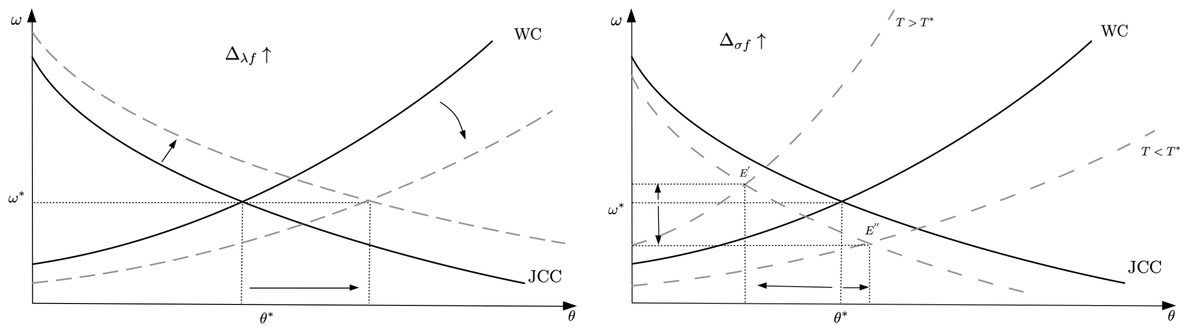
$$\gamma \left((1-\sigma_f)^T - (1-\sigma_w)^T\right) \beta^{T-1} \leq (1-\gamma)\tfrac{z-b}{\kappa}(1+\Delta_{\lambda f}) \textstyle\sum_{t=1}^{T} \left(\beta(1-\sigma_f)\right)^{t-1} - \left[\tfrac{1}{\beta} + (1-\sigma_f)^T \beta^{T-1}\right]$$

i) holds with strict inequality, or

ii) holds with equality and $\gamma(1+\Delta_{\lambda w}) = 0$ *and* $\lim_{\theta \to 0} \frac{\partial p(\theta)/\theta}{\partial \theta} = 0$.

If i) holds and $\gamma(1+\Delta_{\lambda w}) > 0$ *or* $p(\theta)/\theta$ *is strictly decreasing, then the equilibrium is unique.*

Figure 6: General equilibrium effects of firm bias under period-T-bargaining



(a) Optimistic job finding bias (b) Pessimistic job separation bias

Notes. JCC: Job creation condition, WC: Wage curve, $\Delta_{\sigma f}$: Firm's job separation bias, $\Delta_{\lambda f}$: Firm's vacancy filling bias.

Lastly, we perform a comparative statics exercise to analyze the effects of firm's expectation biases on the equilibrium. An optimistic bias in firms' vacancy expectation, $\Delta_{\lambda f} > 0$ leads to a downward shift in the wage curve. This situation is represented in Panel (a) of Figure 6. As a firm expects to find a new worker more easily, it pays a lower compensation to the worker for staying in the match. At the same time, the job creation condition shifts up because, due to the lower perceived recruiting costs, a higher wage is required for the firm to break even. As can be seen in the figure, the total effect on the wage is can be positive or negative but the labor market tightness unambiguously increases which implies more vacancy creation, lower unemployment rate and a shorter unemployment duration.

A pessimistic bias in firm's separation expectation $\Delta_{\sigma f} > 0$ leads to a downward shift of the job creation condition. See Panel (b) in Figure 6. Pessimistic firms expect a lower job duration and therefore require a lower wage to break even. The reaction of the wage curve depends on the bargaining horizon T. For sufficiently low values of T, the increase in $\Delta_{\sigma f} > 0$ leads to a downward shift of the wage curve. The firm expects the worker to leave soon and, thus, it pays a lower compensation for saved hiring cost. As a result, a pessimistic bias in the firm's separation expectation leads to a lower wage, whereas the effect on vacancy creation and labor market tightness is ambiguous. In contrast, when bargaining occurs only infrequently (large T), then the wage curve shifts upward. The pessimistic firm strongly discounts the future, and thus, the worker is able to extract a larger share of the surplus via a higher wage. Consequently, the total effect on the equilibrium wage is ambiguous, but vacancy creation drops which leads to higher unemployment and longer unemployment duration. The results of the comparative statics exercise are presented in Table 1.

Table 1: General equilibrium effects of expectation bias

			$T < T^*$			$T > T^*$		
Worker								
Job finding	$\Delta_{\lambda w} \uparrow$	$\Rightarrow$	$\omega \uparrow$	$\theta \downarrow$	$u \uparrow$	$\omega \uparrow$	$\theta \downarrow$	$u \uparrow$
Job separation	$\Delta_{\sigma w} \uparrow$	$\Rightarrow$	$\omega \uparrow$	$\theta \downarrow$	$u \uparrow$	$\omega \downarrow$	$\theta \uparrow$	$u \downarrow$
Firm								
Vacancy filling	$\Delta_{\lambda f} \uparrow$	$\Rightarrow$	$\omega ?$	$\theta \uparrow$	$u \downarrow$	$\omega ?$	$\theta \uparrow$	$u \downarrow$
Job separation	$\Delta_{\sigma f} \uparrow$	$\Rightarrow$	$\omega \downarrow$	$\theta ?$	$u ?$	$\omega ?$	$\theta \downarrow$	$u \uparrow$

3.2 Endogenous separations

As another extension to the baseline model, we allow for endogenous match separation. Concretely, we follow Mortensen and Pissarides (1994) and assume that match-specific productivity, z, is a random variable. Each period, with probability σ a new productivity z' is drawn from the distribution $G(z')$. The support of G is assumed to be $[0, 1]$. All newly created matches start with highest productivity $z = 1$. If the match specific productivity is sufficiently low, then the match is dissolved. Hence, there is a cutoff productivity z^* below which a match is destroyed. The cutoff productivity is such that $J(z^*) = E(z^*) - U = 0$. Thus, the true match separation probability is $\sigma G(z^*)$. As in the baseline model, we assume that workers' subjective probability is $\sigma_w = (1 + \Delta_{\sigma w})\sigma$. Hence, a pessimistic worker with $\Delta_{\sigma w} > 0$ overestimates the true separation probability and $\sigma_w G(z^*) > \sigma G(z^*)$.

The equilibrium of the model is given by the values of the cutoff productivity z^* and labor market tightness θ that jointly solve the job creation condition (JC) and the job destruction condition (JD).[16] See Appendix C for the value functions and the derivations.

$$\textbf{JC} \qquad \frac{(1-\gamma)(1-z^*)}{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma)} = \frac{\kappa}{\beta\lambda(\theta)}$$

$$\textbf{JD} \qquad z^* - b + \frac{\beta(\gamma\sigma_w+(1-\gamma)\sigma)}{1-\beta+\beta(\gamma\sigma_w+(1-\gamma)\sigma)}\int_{z^*}^{1}(z'-z^*)dG(z') = \frac{\gamma}{1-\gamma}\kappa(1+\Delta_{\lambda w})\theta$$

The job creation condition represents a negative relationship between labor market tightness and the productivity cutoff. The intuition is straightforward. A higher cutoff z^* implies a higher probability of separation; hence, the firm expects to make lower profits and requires better labor market conditions, i.e. a lower market tightness to break even in expectations. The job destruction condition describes a positive relation between θ and z^*. When the market is tight (high θ) workers easily find new jobs and, thus, there is more job destruction in equilibrium, as implied by a high z^*. Graphically, the equilibrium is given by the intersection of the job creation condition and the job destruction condition. The following proposition formalizes the conditions for existence and uniqueness of equilibrium.

Proposition 9 (Existence and uniqueness of equilibrium with endogenous separations)**.** *In the model with endogenous separations, an equilibrium with $z^* \in (0,1)$ and $\theta > 0$ exists and is unique if and only if: $\gamma(1+\Delta_{\lambda w}) > 0$, and $p(\theta)/\theta$ is strictly decreasing, and the following condition is satisfied:*

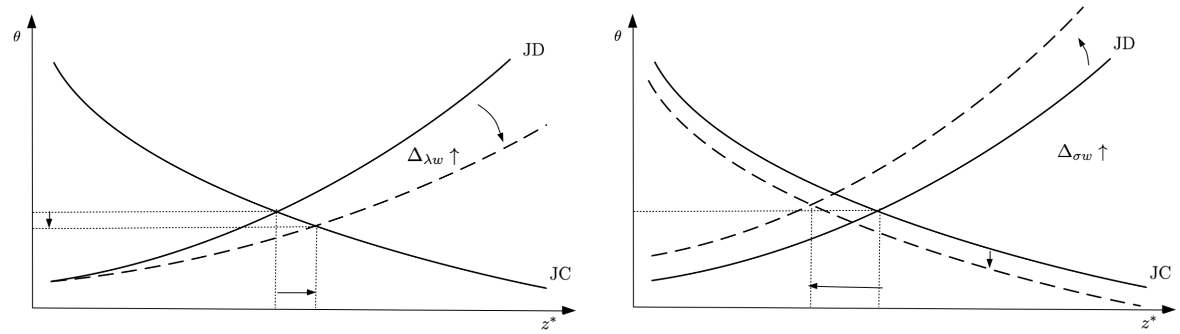
$$\frac{\gamma}{1-\gamma}\kappa(1+\Delta_{\lambda w})\lambda^{-1}\left(\frac{\kappa}{\beta}\frac{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma)}{(1-\gamma)}\right) > -b + \frac{\beta(\gamma\sigma_w+(1-\gamma)\sigma)}{1-\beta+\beta(\gamma\sigma_w+(1-\gamma)\sigma)}\int_0^1 z'dG(z')$$

Using the equilibrium conditions, it is straightforward to show the equilibrium effects of biased expectations. An optimistic job finding bias leads to a downward-shift of the job destruction condition. For $\Delta_{\lambda w} > 0$ the worker expects to move out of unemployment faster. This implies a higher opportunity cost of employment for the worker and thus leads to more job destruction. The job creation condition is unaffected and, hence, an optimistic job finding bias leads to more job separations in equilibrium and, hence, lower labor market tightness. This case is depicted in Panel (a) of Figure 7. Moreover, the equilibrium effect of an optimistic job finding bias on the wage is positive – as in the baseline case.

[16] Here we consider the case of PbP-bargaining.

A pessimistic job separation bias leads to a downward-shift of the job creation condition. For $\Delta_{\sigma w} > 0$, the worker expects a lower job duration and values the match less. Hence, on average, the firm can extract a higher share of the surplus. As a result, also matches with a lower productivity remain active, which is represented by a decrease in the cutoff z^*. At the same time, the job destruction condition shifts upward since the bias lowers the worker's perceived opportunity cost of employment and thereby leads to less frequent job destruction. Together, these effects give rise to the interesting result that when workers expect a higher rate of job separations, there are, in fact, fewer job separations in equilibrium – as can be seen in Panel (b) of Figure 7.

Figure 7: General equilibrium effects of bias with endogenous separations



(a) Optimistic job finding bias (b) Pessimistic job separation bias

Notes. JC: Job creation condition, JD: Job destruction condition.

The equilibrium wage effect of a pessimistic separation bias is different from that in the baseline model of Section 2. There we established that the pessimistic worker obtained a higher wage than the unbiased worker. In contrast, in the model without endogenous separation one can show that the total effect of the separation bias on the wage is composed of the direct effect $\frac{\partial \omega(z)}{\partial \sigma_w}$ and the general equilibrium effects through changes in z^* and θ (see the wage equation in Appendix C). More formally:

$$\frac{d\omega(z)}{d\sigma_w} = \frac{\partial \omega(z)}{\partial \sigma_w} + \underbrace{\frac{dz^*}{d\sigma_w}\frac{\partial \omega(z)}{\partial z^*}}_{\geq 0} + \underbrace{\frac{d\theta}{d\sigma_w}\frac{\partial \omega(z)}{\partial \theta}}_{\geq 0}$$

One can establish that in the presence of a pessimistic bias ($\sigma_w > \sigma$), the two GE effects are always non-negative. The sign of the direct effect $\frac{\partial \omega(z)}{\partial \sigma_w}$, depends on the level of match productivity. For matches with productivity z higher (lower) than $z^* + \int_{z^*}^{1}(z' - z^*)dG(z')$ the direct effect is positive (negative). The direction of the total effect cannot be unambiguously determined analytically but depends on the shape of $G(z')$ as well as the values of the model parameters.

3.3 Risk aversion

In another extension of the model, we relax the assumption of risk neutrality and allow workers to be risk averse. The purpose of this modification is to assess the extent to which risk aversion shapes the effects of expectation biases on wages. Concretely, we assume that worker's instantaneous utility function is given by $u(y)$ with $u'(y) > 0$, $u''(y) < 0$, $\lim_{y\to 0} u'(y) = \infty$, $\lim_{y\to\infty} u'(y) = 0$, where $y = \omega$, when the worker is employed, and $y = b$, when the worker is unemployed. Most importantly, we find that risk aversion tends to dampen the effect of worker's biases on the bargained wage. We refer the reader to Appendix D for the details of the calculations.

4 Labor market policy

In the last part of our analysis we explore whether in the presence of expectation biases labor market policies affect equilibrium outcomes differently than under rational expectations. We consider three widely used policy instruments: unemployment insurance, minimum wage and firing costs. We study these instruments within the general setting of Section 2.5 where wages are renegotiated after T periods. To keep the analysis tractable, we abstract from the firms' bias.

4.1 Unemployment insurance

In our framework, unemployment income b can be considered as capturing a variety of factors, including the value of leisure, home production, transfers but also unemployment benefits. Thus, an increase in b can be considered as representing a more generous unemployment insurance system. As we can see from Equation (18), the level of b enters linearly in the wage schedule and it does not interact with the expectation biases. Thus, a change in b shifts the wage curve but the magnitude of the shift does not depend on workers' expectations. However, as we have shown in Section 2.5, the slope of the wage curve depends on expectation biases. As a result, a change in b affects the equilibrium market tightness θ and wage ω differently in an economy with pessimistic workers than in an economy with optimistic workers.

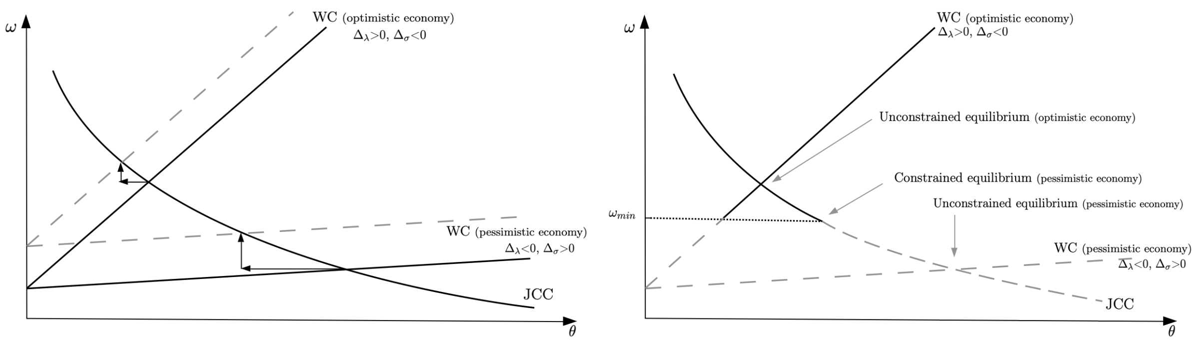
To illustrate this point, consider Panel (a) in Figure 8. Together with the job creation curve we plot in this figure the wage schedules implied by different expectation biases. Moreover, we consider the case where $T > T^*$.[17] According to Equation (18), the wage curve is flatter in an economy where workers are pessimistic – about job finding or job

[17] The wage schedule in Figure 8 is drawn for $T \to \infty$.

separation – than in an economy with optimistic workers. At the same time, the job creation curve is unaffected by workers' biases. An increase in the level of unemployment benefits b leads to an upward-shift in the wage curve (dashed lines) but it does not affect the job creation condition. Since the latter is a convex function, we can establish that more generous UI implies a larger (smaller) reduction of equilibrium market tightness θ and wage ω in a labor market with pessimistic (optimistic) workers.

This result can be useful to understand why labor policies may have differential effects across countries. The empirical evidence discussed in Section 1 shows that workers' expectation biases differ across countries, not only in terms of the magnitude but also in the terms of the sign. For example, German workers tend to be pessimistic about job stability, whereas U.S. workers are optimistic. Our finding suggests that – everything else equal – the same increase in unemployment benefits should lead to a larger adjustment of (θ, ω) in the country with pessimistic workers.

Figure 8: UI and minimum wage (for $T > T^*$)



(a) Increase in unemployment benefits b (b) Minimum wage

Notes. JCC: Job creation condition, WC: Wage curve, Δ_σ: Job separation bias, Δ_λ: Job finding bias, ω_{min}: Wage floor.

Importantly, when $T < T^*$, we obtain the opposite results that the adjustment of (θ, ω) is larger in a labor market where workers are optimistic about job separation. This is because the wage curve is flatter in the economy with optimistic workers. In other words, in the presence of biased expectations the bargaining frequency T is key for determining how changes in UI affect labor market outcomes.

4.2 Minimum wage

In the next step, we study the minimum wage. As before in the case of UI, the equilibrium effect of a minimum wage depends on the sign of the expectation bias and the bargaining frequency. First, we consider the case where $T > T^*$. In this case, a minimum wage is more likely to be binding in a labor market with pessimistic workers. To see this, consider Panel (b) in Figure 8 where the minimum wage is represented by the line at ω_{min}. The pessimistic bias implies a wage curve that is less steep and thus the equilibrium is at the point where ω_{min} intersects with the job creation condition. Importantly, a small increase in the minimum wage leads to a higher equilibrium wage and unemployment rate (through the decline in θ) in the pessimistic labor market whereas it has no effect in the optimistic labor market.

As before in the case of UI, these findings crucially depend on the bargaining frequency T. If $T < T^*$, then we obtain the opposite result that a minimum wage more likely binds when workers have an optimistic bias.

4.3 Firing costs

Lastly, we study firing costs. We follow Pries and Rogerson (2005) and consider costs that a firm has to pay when it separates from its worker. This happens when the match is hit by a separation shock, or the bargaining breaks down. Importantly, these costs apply only to existing matches and not to newly formed firm-worker pairs that bargain for the first time. Hence, the bargaining – and therefore the wage – is different in existing and in newly formed matches. The value function of the firm is

$$J_i(\omega_i) = z - \omega_i + \beta(1-\sigma)J_c(\omega_c') + \beta\sigma V - \beta\sigma F.$$

where $F \geq 0$ is the firing cost and i indicates the type of the match, with $i = n$ for new matches and $i = c$ for continuing matches. We refer the reader to Appendix F for the remaining value functions. The wages in new matches and in continuing matches are given by

$$\begin{aligned}
\omega_n &= \arg\max \quad \Big(E_n(\omega_n) - U\Big)^{\gamma}\Big(J_n(\omega_n) - V\Big)^{1-\gamma}, \\
\omega_c &= \arg\max \quad \Big(E_c(\omega_c) - U\Big)^{\gamma}\Big(J_c(\omega_c) + F - V\Big)^{1-\gamma}.
\end{aligned}$$

In Appendix F we solve the Nash bargaining problem and derive the equilibrium conditions. The equilibrium is given by the pair (ω_n, θ) that jointly solves the job creation condition and the wage schedule for new matches:

$$\omega_n = \frac{1}{1-\gamma\left(1-\left(\frac{1-\sigma_w}{1-\sigma}\right)^T\right)}\left[b+\gamma\left(z\left(\frac{1-\sigma_w}{1-\sigma}\right)^T - b + \kappa\theta\frac{\beta p(\theta)(1+\Delta_{\lambda w})\sum_{t=1}^T(\beta(1-\sigma_w))^{t-1}+1-\left(\frac{1-\sigma_w}{1-\sigma}\right)^T}{\beta p(\theta)\sum_{t=1}^T(\beta(1-\sigma))^{t-1}} - F\left[\beta\sigma\left(\frac{1-\sigma_w}{1-\sigma}\right)^T + \frac{(\beta(1-\sigma_w))^T}{\sum_{t=1}^T(\beta(1-\sigma))^{t-1}}\right]\right)\right],$$

$$\frac{\kappa\theta}{\beta p(\theta)} = \frac{z-\omega_n-\beta\sigma F}{1-\beta(1-\sigma)} - \gamma F\frac{(\beta(1-\sigma))^T}{1-(\beta(1-\sigma))^T}.$$

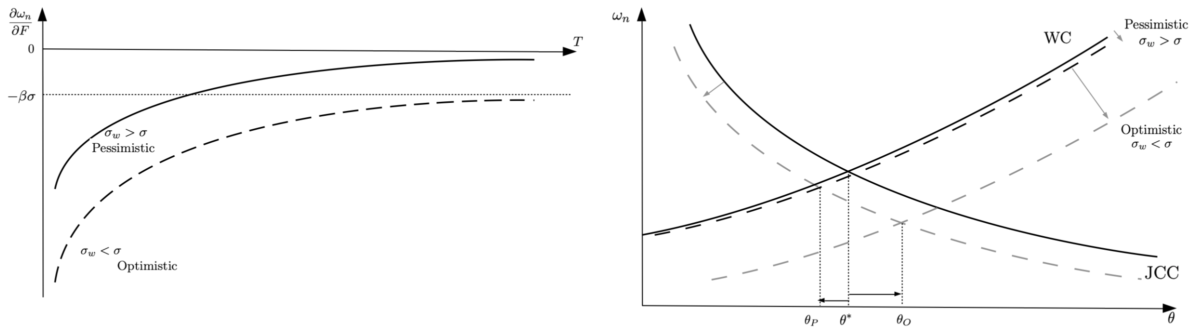
The firing cost enters negatively in both expressions. This is intuitive, as F is a resource cost (and not a transfer from the firm to the worker) and thus it reduces the joint surplus of the match. For this reason, an increase in F leads to a downward-shift of both, the job creation condition and the wage curve. This situation is depicted in Panel (b) of Figure 9. However importantly, the magnitude of the shift in the wage curve depends on the workers' job separation expectations and the bargaining frequency. Formally, we obtain from the wage schedule the following relationship:

$$\frac{\partial\omega_n}{\partial F} = -\gamma\frac{1}{1-\gamma\left(1-\left(\frac{1-\sigma_w}{1-\sigma}\right)^T\right)}\left(\beta\sigma\left(\frac{1-\sigma_w}{1-\sigma}\right)^T + \frac{(\beta(1-\sigma_w))^T}{\sum_{t=1}^T(\beta(1-\sigma))^{t-1}}\right).$$

In the case of worker pessimism ($\sigma_w > \sigma$), the reaction of the wage is largest for $T = 1$, and equal to $\partial\omega_n/\partial F = -\gamma\beta\frac{1-\sigma_w}{1-[(1-\gamma)\sigma+\gamma\sigma_w]}$, but is becoming smaller as T increases. See Panel (a) in Figure 9 for an illustration of this pattern. In the limit, as $T \to \infty$, the firing cost has no effect on the wage curve and we obtain that $\partial\omega_n/\partial F = 0$. This is very different when workers are optimistic ($\sigma_w < \sigma$). For $T = 1$, we obtain as before that $\partial\omega_n/\partial F = -\gamma\beta\frac{1-\sigma_w}{1-[(1-\gamma)\sigma+\gamma\sigma_w]}$. However, as T increases, the reaction of the wage does not diminish to zero but converges in the limit to $\lim_{T\to\infty}\partial\omega_n/\partial F = -\beta\sigma$. It is straightforward to show that for any given T, the response of the bargained wage to a change in firing costs is always larger (more negative) in the optimistic economy than in the pessimistic economy. That is, $\frac{\partial\omega_n}{\partial F}\Big|_{\sigma_w<\sigma} < \frac{\partial\omega_n}{\partial F}\Big|_{\sigma_w>\sigma}$.

This differential reaction of the wage curve implies that higher firing costs can lead to a new equilibrium with lower market tightness – and a higher unemployment – in the economic with pessimistic workers, whereas in the economy with optimistic workers the new equilibrium can have higher tightness and lower unemployment. See Panel (b) in Figure 9 for an illustration of these cases.

Figure 9: Firing costs



(a) $\partial\omega_n/\partial F$ as a function of T (b) Increase in F (large T)

Notes. JCC: Job creation condition, WC: Wage curve, ω_n: Wage in new matches.

5 Conclusion

This paper incorporates subjective expectations of workers into a general equilibrium search-and-matching model of the labor market and analytically studies the implications of biased expectations for wage bargaining, firms' job creation, as well as equilibrium outcomes and efficiency. As extensions, the paper also considers a framework with endogenous job separations, and it studies the equilibrium implications of biases in firms' beliefs. Moreover, the paper studies labor market policies and it shows that the presence of biased beliefs can qualitatively alter the equilibrium effects of labor market policies.

The framework developed in this paper can serve as a starting for further analysis of labor market outcomes in the presence of biased beliefs. An example of such an analysis is Balleer et al. (2023) where we establish that workers in East-Germany are significantly pessimistic compared to West-German workers about job finding and job separation. We use the theoretical insights of this paper to study quantitatively the implications of cross-region differences in workers' expectations for the non-convergence of East-German labor market outcomes.

Based on the analysis in this paper, we see several avenues for further research. First, we consider it worthwhile to study worker heterogeneity in biases in a general equilibrium setting. The empirical studies discussed above generally find evidence for significant and persistent heterogeneity among workers in terms of expectation biases – even after controlling for observables. It is conceivable that these biases create externalities whereby, say, the pessimistic workers in an economy affect the labor market conditions of the optimists, and vice versa. A different but related direction of research is to explore the implications of learning for wage bargaining and aggregate labor market outcomes.

Lastly, while we consider the standard random search-and-matching model in our analysis, it would be worthwhile to study subjective expectations in the context of an equilibrium directed search model of the labor market.

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Appendix

A Period-T Bargaining

The surplus of a match for the firm is:

$$J_T(\omega) - V = [z - \omega - (1-\beta)V] \sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1} + (\beta(1-\sigma))^T (J_T(\omega') - V) \quad \text{(A.1)}$$

The surplus of a match for the worker is:

$$E_T(\omega) - U = [\omega - (1-\beta)U] \sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1} + (\beta(1-\sigma_w))^T (E_T(\omega') - U) \quad \text{(A.2)}$$

The value functions for V and U are as in the baseline case. Every T periods, the firm and the worker set the wage to maximize the Nash product:

$$\omega = \arg\max \left(E_T(\omega) - U\right)^{\gamma} \left(J_T(\omega) - V\right)^{1-\gamma}$$

The first-order condition to this optimization problem is given by

$$\gamma(J_T(\omega) - V) \frac{1}{\sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1}} = (1-\gamma)(E_T(\omega) - U) \frac{1}{\sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1}} \quad \text{(A.3)}$$

which can be rearranged to obtain

$$E_T(\omega) - U = \frac{\gamma}{1-\gamma} \frac{\sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1}}{\sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1}} (J_T(\omega) - V) \quad \text{(A.4)}$$

Insert into the first-order condition (A.3) the firm's surplus (A.1) and the worker's surplus (A.2) to obtain.

$$\gamma \left[z - \omega + \frac{(\beta(1-\sigma))^T}{\sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1}} J_T(\omega') \right] = (1-\gamma) \left[\omega - (1-\beta)U + \frac{(\beta(1-\sigma_w))^T}{\sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1}} (E_T(\omega') - U) \right]$$

Replace the term $(1-\beta)U$ by the expression $(1-\beta)U = b + \beta\lambda_w(E_T(\omega) - U)$ which is obtained by rearranging the worker's value function for U. Lastly, use (A.4) to replace $E_T(\omega) - U$ and use $J_T(\omega) = \frac{\kappa}{\beta p(\theta)/\theta}$ to obtain the wage schedule in Equation (18) in the main text.

B Bias in workers' and firms' expectations

The value functions of the firm are given by

$$J_T(\omega) - V = [z - \omega - (1-\beta)V] \sum_{t=1}^{T} [\beta(1-\sigma_f)]^{t-1} + (\beta(1-\sigma_f))^T (J_T(\omega') - V)$$

$$V = -\kappa + \beta\lambda_f J_T(\omega) + \beta(1-\lambda_f) J_T(\omega)$$

where $\sigma_f = (1+\Delta_{\sigma f})\sigma$ and $\lambda_f = (1+\Delta_{\lambda f})p(\theta)/\theta$. The value functions of the worker and the equilibrium calculations are as in Section A.

C Endogenous separations

The value functions are given by

$$J(z) = z - \omega(z) + \beta(1-\sigma)J(z) + \beta\sigma \int_{z^*}^{1} J(z')dG(z') \qquad V = 0 \text{ is already incorporated}$$

$$V = -\kappa + \lambda\beta J(1) + \beta(1-\lambda)V \qquad \Rightarrow \quad J(1) = \frac{\kappa}{\beta\lambda}$$

$$E(z) = \omega(z) + \beta(1-\sigma_w)E(z) + \beta\sigma_w \int_{z^*}^{1} E(z')dG(z') + \beta\sigma_w G(z^*)U$$

$$U = b + \beta\lambda_w E(1) + \beta(1-\lambda_w)U$$

Period-by-period bargaining leads to the familiar optimality condition

$$\Big(E(z) - U\Big)^{\gamma}\Big(J(z) - V\Big)^{1-\gamma} \qquad \Rightarrow \quad \gamma J(z) = (1-\gamma)(E(z) - U)$$

In order to derive the wage equation, we insert value functions into the optimality condition to obtain

$$(1-\gamma)\Big[\omega(z) + \beta(1-\sigma_w)E(z) + \beta\sigma_w \int_{z^*}^{1} E(z')dG(z') + \beta\sigma_w G(z^*)U - b - \beta\lambda_w E(1) - \beta(1-\lambda_w)U\Big]$$
$$= \gamma\Big[z - \omega(z) + \beta(1-\sigma)J(z) + \beta\sigma \int_{z^*}^{1} J(z')dG(z')\Big]$$

Adding and subtracting the term $(1-\gamma)\sigma_w \int_{z^*}^{1} UdG(z^*)$ and using the conditions $\gamma J(z) =$

$(1-\gamma)(E(z)-U)$ and $J(1)=\frac{\kappa}{\beta\lambda}$ leads to the wage equation

$$\omega(z)=b+\gamma\Big[z-b+(\sigma_w-\sigma)\beta\Big(J(z)-\int_{z^*}^{1}J(z')dG(z')\Big)+\kappa(1+\Delta_{\lambda w})\theta\Big]$$

Next, insert the expression for $\omega(z)$ into the value function $J(z)$ to obtain:

$$J(z)(1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma))=(1-\gamma)(z-b)+\beta(\gamma\sigma_w+\sigma(1-\gamma))\int_{z^*}^{1}J(z')dG(z')-\gamma\kappa(1+\Delta_{\lambda w})\theta$$

Evaluate this expression for $z=z^*$ (which leads to $J(z^*)=0$) and insert the resulting expression into the previous one:

$$J(z)(1+\beta\gamma(\sigma_w-\sigma)+\beta(1-\sigma))=(1-\gamma)(z-z^*)$$

Evaluate this expression at $z=1$ to obtain:

$$J(1)(1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma))=(1-\gamma)(1-z^*)$$

Combine it with the expression for $J(1)$ from above to obtain the job creation condition

$$\frac{(1-\gamma)(1-z^*)}{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma)}=\frac{\kappa}{\beta\lambda}$$

Use the value function $J(z)$ from above and insert the expression $J(z)=\frac{(1-\gamma)(z-z^*)}{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma)}$ into the integral

$$\begin{aligned}&J(z)(1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma))=(1-\gamma)(z-b)-\gamma\kappa(1+\Delta_{\lambda w})\theta\\&+\beta(\gamma\sigma_w+(1-\gamma)\sigma)\textstyle\int_{z^*}^{1}\frac{(1-\gamma)(z'-z^*)}{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma)}dG(z')\end{aligned}$$

Evaluate this expression at $z=z^*$ to obtain the job destruction condition:

$$z^*-b+\frac{\beta(\gamma\sigma_w+(1-\gamma)\sigma)}{1-\beta+\beta(\gamma\sigma_w+(1-\gamma)\sigma)}\int_{z^*}^{1}(z'-z^*)dG(z')=\frac{\gamma}{1-\gamma}\kappa(1+\Delta_{\lambda w})\theta$$

The wage equation is given by:

$$\omega(z)=b+\gamma\left[z-b+\frac{(1-\gamma)\beta(\sigma_w-\sigma)}{1-\beta+\beta(\gamma\sigma_w+(1-\gamma)\sigma)}\left(z-z^*-\int_{z^*}^{1}(z'-z^*)dG(z')\right)+\kappa\theta(1+\Delta_{\lambda w})\right]$$

D Risk aversion

The value functions of the firm are as in Section A. The value functions of the worker are given by

$$E_T(\omega) - U = [u(\omega) - (1-\beta)U] \sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1} + (\beta(1-\sigma_w))^T (E_T(\omega') - U)$$

$$U = u(b) + \beta\lambda_w E_T(\omega) + \beta(1-\lambda_w)U$$

The first-order condition to the Nash bargaining problem is given by

$$\gamma(J_T(\omega) - V)u'(\omega) \sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1} = (1-\gamma)(E_T(\omega) - U) \sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1}$$

By following the same steps as in Section A one obtains the following wage equation:

$$(1-\gamma)\left(\frac{u(\omega)-u(b)}{u'(\omega)}\right) + \gamma\omega = \gamma\left[z + \kappa\theta\left[\frac{\sum_{t=0}^{T}(\beta(1-\sigma_w))^{t-1}}{\sum_{t=0}^{T}(\beta(1-\sigma))^{t-1}}(1+\Delta_{\lambda w}) + \beta^{T+1}\frac{(1-\sigma)^T - (1-\sigma_w)^T}{p(\theta)\sum_{t=0}^{T}(\beta(1-\sigma))^{t-1}}\right]\right]$$

Using the wage equation we compute the total differential to establish that

$$\frac{d\omega_{RA}}{d\sigma_w} = \frac{d\omega/d\sigma_w}{\gamma + (1-\gamma)\left(1 - \epsilon_u \frac{u(\omega)-u(b)}{u'(\omega)\omega}\right)}, \qquad \frac{d\omega_{RA}}{d\lambda_w} = \frac{d\omega/d\lambda_w}{\gamma + (1-\gamma)\left(1 - \epsilon_u \frac{u(\omega)-u(b)}{u'(\omega)\omega}\right)}.$$

where $\left(\frac{d\omega_{RA}}{d\sigma_w}, \frac{d\omega_{RA}}{d\lambda_w}\right)$ and $\left(\frac{d\omega}{d\sigma_w}, \frac{d\omega}{d\lambda_w}\right)$ represent the reaction of wages to a change in the worker's subjective separation and job finding probabilities – for risk aversion and risk neutrality, respectively. Moreover, $\epsilon_u \equiv \frac{\partial u'(\omega)}{\partial \omega}\frac{\omega}{u'(\omega)}$ is the elasticity of the marginal utility. Risk aversion implies that $\epsilon_u < 0$. Since $\omega > b$ and $u' > 0$, the denominator in the expressions above is larger than unity. As a result, we obtain that $\left|\frac{d\omega_{RA}}{d\sigma_w}\right| < \left|\frac{d\omega}{d\sigma_w}\right|$ and $\frac{d\omega_{RA}}{d\lambda_w} < \frac{d\omega}{d\lambda_w}$. For an increasing degree of risk aversion, the denominator grows larger and leads to a smaller effect of a change in subjective expectations on the wage.

The intuition for this result is as follows. Risk aversion implies that workers associate a positive but declining value with each incremental increase in the wage. Formally, this can be observed from the partial derivative of the worker's surplus function (for period-T

bargaining)

$$\frac{\partial E(\omega) - U}{\partial \omega} = u'(\omega) \sum_{t=1}^{T} \Big(\beta(1-\sigma_w)\Big)^{t-1} > 0 \qquad \frac{\partial^2 E(\omega) - U}{\partial \omega^2} < 0.$$

The higher is the degree of risk aversion, the stronger is the decline in the extra value. Instead, under risk neutrality we have that $\frac{\partial^2 E(\omega) - U}{\partial \omega^2} = 0$. An increase in, say, the subjective job finding probability, λ_w, leads to an improvement of the worker's outside option and thus raises the threat point in the bargain. The firm reacts by paying a higher wage. However, it is optimal to raise the wage for risk averse workers by less than the wage for risk neutral workers. The reason is that risk averse workers perceive a smaller gain in the surplus for each marginal increase in the wage.

E Efficiency

We explore the efficiency of equilibrium in the economy with biased worker beliefs. As is well known, in the standard version of the model (with unbiased beliefs), the equilibrium is efficient, if the Hosios condition holds: $\gamma = 1 - \epsilon_p(\theta^*)$. This condition is satisfied when the worker's bargaining power γ is equal to 1 minus the elasticity of the matching function, evaluated at the socially optimal level of labor market tightness, θ^*.

The objective of the social planner is to maximize social welfare which is given by the discounted stream of aggregate income. The planner is subject to the same matching frictions as firms and workers. Therefore, it cannot freely allocate workers to jobs but it chooses the sequence of $\{u_{t+1}, \theta_t\}_t$ subject to the reallocation constraint. The optimization problem of the social planner is given by

$$\max_{\{\theta_t, u_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left\{ z(1-u_t) + bu_t - \kappa\theta_t u_t - \lambda_t[u_{t+1} - u_t - \sigma(1-u_t) + p(\theta_t)u_t] \right\}$$

The first-order conditions are

$$\begin{aligned} \theta_t: &\quad -\beta^t \kappa u_t - \beta^t \lambda_t \tfrac{\partial p(\theta_t)}{\partial \theta_t} u_t && = 0 \\ u_t: &\quad -\beta^t \lambda_t + \beta^{t+1}\Big(-z + b - \kappa\theta_{t+1} + \lambda_{t+1} - \lambda_{t+1}\sigma - \lambda_{t+1} p(\theta_{t+1})\Big) && = 0 \end{aligned}$$

Collapsing these two expressions and imposing stationarity $\theta_t = \theta_{t+1} = \theta$, $\lambda_t = \lambda_{t+1} = \lambda$ yields the following condition for the socially optimal level of θ

$$\kappa \left(\frac{1}{\beta} - (1-\sigma) \right) = \epsilon(\theta) p(\theta) \theta^{-1} (z-b) - \kappa p(\theta)(1-\epsilon(\theta)) \tag{A.5}$$

In Appendix E, we state the social planner problem and derive the condition which implicitly defines the socially efficient level of labor market tightness, θ^*:

$$\epsilon_m(\theta^*)\frac{p(\theta^*)}{\theta^*}(z-b) - \kappa p(\theta^*)(1-\epsilon_p(\theta^*)) = \kappa\left(\frac{1}{\beta} - 1 + \sigma\right). \tag{A.6}$$

Next, we turn to the market equilibrium. First, we substitute the wage ω – from the job creation condition in Equation (8) – into the wage schedule in Equation ((18) to obtain the following condition

$$\begin{aligned}&\kappa\left(\tfrac{1}{\beta} - (1-\sigma)\right)\\ &= \left(z - b - \gamma\left[z - b + \kappa\theta\left[\tfrac{\sum_{t=0}^{T}(\beta(1-\sigma_w))^{t-1}}{\sum_{t=0}^{T}(\beta(1-\sigma))^{t-1}}(1+\Delta_{\lambda w}) + \beta^{T-1}\tfrac{(1-\sigma)^T-(1-\sigma_w)^T}{p(\theta)\sum_{t=0}^{T}(\beta(1-\sigma))^{t-1}}\right]\right]\right)\tfrac{p(\theta)}{\theta}\end{aligned} \tag{A.7}$$

that implicitly defines the equilibrium market tightness. In the next step, combine the Equations (A.6) and (A.7) to obtain the Generalized Hosios condition for the worker's bargaining power

$$\gamma = \frac{(1-\epsilon_p(\theta^*))\left(\frac{z-b}{\theta^*}+\kappa\right)}{\frac{z-b}{\theta^*} + \kappa\left(\frac{\sum_{t=1}^{T}(\beta(1-\sigma_w))^{t-1}}{\sum_{t=1}^{T}(\beta(1-\sigma))^{t-1}}(1+\Delta_{\lambda w}) + \beta^{T-1}\frac{(1-\sigma)^T-(1-\sigma_w)^T}{p(\theta^*)\sum_{t=1}^{T}(\beta(1-\sigma))^{t-1}}\right)}. \tag{A.8}$$

It is straightforward to verify that in the absence of any bias – when $\sigma_w = \sigma$ and $\Delta_{\lambda w} = 0$ – this condition collapses to the standard Hosios condition $\gamma = 1 - \epsilon_p(\theta^*)$. The interpretation of the condition in (A.8) follows directly from the partial effects of the expectation biases on the wage schedule. For example, an optimistic job finding bias, $\Delta_{\lambda u} > 0$, leads to a higher bargained wage for each level of θ. To offset this effect, the worker's bargaining power must be lower than the level implied by the standard Hosios condition without biased beliefs. Likewise, a pessimistic job separation bias implies a lower (higher) wage for a sufficiently long (short) bargaining horizon, $T > T^*$ ($T < T^*$). A higher (lower) value of the worker's bargaining power is required to reach the socially optimal level of market tightness.

F Firing cost

The surplus of a match to the firm and to the worker are given by:

$$J_i(\omega_i) - V = [z - \omega_i - (1-\beta)V - \beta\sigma F] \sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1} + (\beta(1-\sigma))^T (J_c(\omega_c') - V) \tag{A.9}$$

$$E_i(\omega_i) - U = [\omega_i - (1-\beta)U] \sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1} + (\beta(1-\sigma_w))^T (E_c(\omega_c') - U) \tag{A.10}$$

where $i = n$ indicates newly formed matches and $i = c$ indicates continuing matches. The value functions for a vacancy and for unemployment are:

$$V = -\kappa + \beta\lambda J_n(\omega_n) + \beta(1-\lambda)V$$

$$U = b + \beta\lambda_w E_n(\omega_n) + \beta(1-\lambda_w)U$$

In a newly formed match, the worker and the firm set the wage to solve

$$\omega_n = \arg\max \Big(E_n(\omega_n) - U\Big)^{\gamma} \Big(J_n(\omega_n) - V\Big)^{1-\gamma}$$

This problem yields the following first-order condition

$$\gamma(J_n(\omega_n) - V) \sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1} = (1-\gamma)(E_n(\omega_n) - U) \sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1} \tag{A.11}$$

which can be rearranged to obtain

$$E_n(\omega_n) - U = \frac{\gamma}{1-\gamma} \frac{\sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1}}{\sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1}} (J_n(\omega_n) - V)$$

In a continuing match, the worker and the firm set the wage to solve

$$\Big(E_c(\omega_c) - U\Big)^{\gamma} \Big(J_c(\omega_c) + F - V\Big)^{1-\gamma}$$

This problem yields the following first-order condition

$$\gamma(J_c(\omega_c) + F - V) \sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1} = (1-\gamma)(E_c(\omega_c) - U) \sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1}$$

which can be rearranged to obtain

$$E_c(\omega_c) - U = \frac{\gamma}{1-\gamma} \frac{\sum_{t=1}^{T} [\beta(1-\sigma_w)]^{t-1}}{\sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1}} (J_c(\omega_c) + F - V) \tag{A.12}$$

Next, solve for the wage in newly formed matches. For this purpose, substitute into the optimality condition (A.11) the expressions for the firm's and the worker's match surplus from Equations (A.9) and (A.10):

$$\begin{aligned} \gamma\Big(z - \omega_n - \beta\sigma F \quad + \quad & \tfrac{(\beta(1-\sigma))^T}{\sum_{t=1}^{T}[\beta(1-\sigma)]^{t-1}} J_c(\omega')\Big) \\ & = (1-\gamma)\Big(\omega_n - (1-\beta)U + \tfrac{(\beta(1-\sigma_w))^T}{\sum_{t=1}^{T}[\beta(1-\sigma_w)]^{t-1}} \left(E_c(\omega_c') - U\right)\Big) \end{aligned} \tag{A.13}$$

Rearrange the value function for U to obtain $(1-\beta)U = b + \beta\lambda_w(E_n(\omega_n) - U)$. Use this expression to replace $(1-\beta)U$ in Equation (A.13) and Equation (A.12) to replace $E_c(\omega_c) - U$. These transformations yield the following expression:

$$\begin{aligned} \gamma\Big(z - \omega_n - \beta\sigma F \quad + \quad & \tfrac{(\beta(1-\sigma))^T}{\sum_{t=1}^{T}[\beta(1-\sigma)]^{t-1}} J_c(\omega')\Big) \\ & = (1-\gamma)\Big(\omega_n - (1-\beta)U + \tfrac{(\beta(1-\sigma_w))^T}{\sum_{t=1}^{T}[\beta(1-\sigma)]^{t-1}} \tfrac{\gamma}{1-\gamma} \left(J_c(\omega_c') + F\right)\Big) \end{aligned}$$

which can be rearranged to express the wage ω_n:

$$\omega_n = b + \gamma\Big[z - b + \beta\lambda_w \tfrac{\sum_{t=1}^{T}[\beta(1-\sigma_w)]^{t-1}}{\sum_{t=1}^{T}[\beta(1-\sigma)]^{t-1}} J_n(\omega_n) - \tfrac{(\beta(1-\sigma_w))^T}{\sum_{t=1}^{T}[\beta(1-\sigma)]^{t-1}} F - \beta\sigma F + \beta^T \tfrac{(1-\sigma)^T - (1-\sigma_w)^T}{\sum_{t=1}^{T}[\beta(1-\sigma)]^{t-1}} J_c(\omega_c')\Big]$$

Replace $J_n(\omega_n)$ in this expression by $J_n(\omega_n) = \frac{\kappa}{\beta p(\theta)/\theta}$ obtained from the value function for V. To replace $J_c(\omega_c')$ use the value function $J_i(\omega_i)$ for $i = n$ and rearrange it to obtain:

$$\frac{J_c(\omega_c')}{\sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1}} = \Big[\frac{J_n(\omega_n)}{\sum_{t=1}^{T} [\beta(1-\sigma)]^{t-1}} - (z - \omega_n - \beta\sigma F)\Big] \frac{1}{(\beta(1-\sigma))^T}$$

After substituting for $J_c(\omega_c'$ one can rearrange the expression to obtain the wage schedule for ω_n reported in Section 4.3. The wage schedule for continuing matches ω_c can be derived analogously. It is given by:

$$\omega_c = \omega_n + \frac{\gamma}{\sum_{t=1}^{T}(\beta(1-\sigma))^{t-1}} F$$

To derive the job creation condition, use the firm's value function for $i = n$:

$$J_n(\omega_n) = (z - \omega_n - \beta\sigma F) \sum_{t=1}^{T} (\beta(1-\sigma))^{t-1} + (\beta(1-\sigma))^T J_c(\omega_c')$$

Use the expression $J_n(\omega_n) = \frac{\kappa\theta}{\beta p(\theta)}$ to replace $J_n(\omega_n)$. Next, take the firm's value function for continuing matches:

$$J_c(\omega_c) = (z - \omega_c - \beta\sigma F)\sum_{t=1}^{T}(\beta(1-\sigma))^{t-1} + (\beta(1-\sigma))^T J_c(\omega_c')$$

and use the property that wages are symmetric in equilibrium, $\omega = \omega'$, to obtain

$$J_c(\omega_c) = \frac{z - \omega_c - \beta\sigma F}{1 - \beta(1-\sigma)}$$

Use this expression to replace $J_c(\omega_c)$ in the equation above. These steps yield the following condition:

$$\frac{\kappa\theta}{\beta p(\theta)} = \frac{z - \omega_n - (\omega_c - \omega_n)(\beta(1-\sigma))^T - \beta\sigma F}{1 - \beta(1-\sigma)}$$

Lastly, use the expression from above to replace ω_c. After rearranging, one can obtain the job creation condition reported in Section 4.3.

G Proofs

Proof of Proposition 1

Consider any partial equilibrium wage. This wage determines U and in combination with the current wage, ω, $E(\omega)$, which is given recursively by

$$E(\omega) = \omega + \beta(1-\sigma_w)E(\omega') + \beta\sigma_w U$$

Subtracting U results in a recursive definition of the worker surplus $E(\omega) - U$

$$E(\omega) - U = \omega - (1-\beta)U + \beta(1-\sigma_w)(E(\omega') - U)$$

i)

Taking the partial derivative with respect to λ_w yields

$$\frac{\partial E(\omega) - U}{\partial \lambda_w} = -(1-\beta)\frac{\partial U}{\partial \lambda_w} + \beta(1-\sigma_w)\frac{\partial E(\omega') - U}{\partial \lambda_w}$$

As $E(\omega) - U$ is linear in ω, the derivative does not depend on ω, implying that $\frac{\partial E(\omega)-U}{\partial \lambda_w} = \frac{\partial E(\omega')-U}{\partial \lambda_w}$ and thus

$$\frac{\partial E(\omega) - U}{\partial \lambda_w}\Big[1 - \beta(1 - \sigma_w)\Big] = -(1-\beta)\frac{\partial U}{\partial \lambda_w}$$

To obtain the derivative of U with respect to σ_w, rearrange the the value function of U

$$\begin{aligned}
U &= b + \beta\lambda_w E(\omega') + \beta(1-\lambda_w)U \\
U &= \frac{1}{1-\beta}\Big(b + \beta\lambda_w(E(\omega') - U)\Big) \\
\frac{\partial U}{\partial \lambda_w} &= \frac{1}{1-\beta}\beta\left[\lambda_w \frac{\partial E(\omega') - U}{\partial \lambda_w} + E(\omega') - U\right]
\end{aligned}$$

Use this expression in the derivative of $E(\omega) - U$, to get

$$\begin{aligned}
\frac{\partial E(\omega) - U}{\partial \lambda_w}\Big[1 - \beta(1-\sigma_w)\Big] &= -\beta\lambda_w \frac{\partial E(\omega') - U}{\partial \lambda_w} - \beta(E(\omega') - U) \\
&= -\beta\lambda_w \frac{\partial E(\omega) - U}{\partial \lambda_w} - \beta(E(\omega') - U) \\
\implies \frac{\partial E(\omega) - U}{\partial \lambda_w}\Big[1 - \beta + \beta\sigma_w + \beta\lambda_w\Big] &= -\beta(E(\omega') - U) \\
\implies \frac{\partial E(\omega) - U}{\partial \lambda_w} &= -(E(\omega') - U)\frac{1}{\frac{1}{\beta} - 1 + \sigma_w + \lambda_w}
\end{aligned}$$

Since $\beta \in (0, 1)$ and $\sigma_w, \lambda_w \geq 0$ the last term is strictly positive. In a partial equilibrium with a (strictly) positive market tightness, $E(\omega') \geq U$. Hence,

$$\frac{\partial E(\omega) - U}{\partial \lambda_w} \leq 0.$$

First, for the reservation wage

$$\underline{\omega} = (1-\beta)U - \beta(1-\sigma_w)(E(\omega') - U)$$

the positive derivative is directly obtained as

$$E(\omega) - U = \omega - \underline{\omega}$$

implies

$$\frac{\partial \underline{\omega}}{\partial \lambda_w} = -\frac{\partial E(\omega) - U}{\partial \lambda_w} \geq 0$$

Second, the partial equilibrium wage has to satisfy the Nash bargaining optimality condition

$$(1-\gamma)(E(\omega) - U) = \gamma(J(\omega) - V)$$

Hence, the wage is implicitly defined as the root of

$$f(\omega) = (1-\gamma)(E(\omega) - U) - \gamma(J(\omega) - V).$$

Since $J(\omega) - V = z - \omega - (1-\beta)V + \beta(1-\sigma)(J(\omega') - V)$

$$\frac{\partial J(\omega) - V}{\partial \lambda_w} = 0$$

for any partial equilibrium wage. Together, as γ might be one, this implies that

$$\frac{\partial f(\omega)}{\partial \lambda_w} \leq 0.$$

Moreover, as

$$f(\omega) = \omega + (1-\gamma)(-(1-\beta)U + \beta(1-\sigma_w)(E(\omega') - U)) - \gamma(z - (1-\beta)V + \beta(1-\sigma)(J(\omega') - V))$$

the derivative with respect to the wage is equal to one, $\frac{\partial f(\omega)}{\partial \omega} = 1$. Finally, as $f(\omega)$ is continuously differentiable with $\frac{\partial f(\omega)}{\partial \omega} \neq 0$, by the implicit function theorem

$$\frac{\partial \omega}{\partial \lambda_w} = -\frac{\frac{\partial f(\omega)}{\partial \lambda_w}}{\frac{\partial f(\omega)}{\partial \omega}} = -\frac{\partial f(\omega)}{\partial \lambda_w} \geq 0$$

Since $\lambda_w = (1 + \Delta_{\lambda w})p(\theta)$, the claim from i) follows.

ii)
Taking the partial derivative of $E(\omega) - U$ with respect to σ_w yields

$$\frac{\partial E(\omega) - U}{\partial \sigma_w} = -(1-\beta)\frac{\partial U}{\partial \sigma_w} + \beta\Big((1-\sigma_w)\frac{\partial E(\omega') - U}{\partial \sigma_w} - (E(\omega') - U)\Big)$$

As $E(\omega) - U$ is linear in ω, the derivative does not depend on ω, implying that $\frac{\partial E(\omega) - U}{\partial \sigma_w} = \frac{\partial E(\omega') - U}{\partial \sigma_w}$ and thus

$$\frac{\partial E(\omega) - U}{\partial \sigma_w}\Big[1 - \beta(1-\sigma_w)\Big] = -(1-\beta)\frac{\partial U}{\partial \sigma_w} - \beta(E(\omega') - U)$$

To obtain the derivative of U with respect to σ_w, rearrange the the value function of U

$$\begin{aligned} U &= b + \beta\lambda_w E(\omega') + \beta(1-\lambda_w)U \\ U &= \frac{1}{1-\beta}\Big(b + \beta\lambda_w(E(\omega') - U)\Big) \\ \frac{\partial U}{\partial \sigma_w} &= \frac{1}{1-\beta}\beta\lambda_w\frac{\partial E(\omega') - U}{\partial \sigma_w} \end{aligned}$$

Use this expression in the derivative of $E(\omega) - U$, to get

$$\begin{aligned} \frac{\partial E(\omega) - U}{\partial \sigma_w}\Big[1 - \beta(1-\sigma_w)\Big] &= -\beta\lambda_w\frac{\partial E(\omega') - U}{\partial \sigma_w} - \beta(E(\omega') - U) \\ &= -\beta\lambda_w\frac{\partial E(\omega) - U}{\partial \sigma_w} - \beta(E(\omega') - U) \\ \implies \frac{\partial E(\omega) - U}{\partial \sigma_w}\Big[1 - \beta + \beta\sigma_w + \beta\lambda_w\Big] &= -\beta(E(\omega') - U) \\ \implies \frac{\partial E(\omega) - U}{\partial \sigma_w} &= -(E(\omega') - U)\frac{1}{\frac{1}{\beta} - 1 + \sigma_w + \lambda_w} \end{aligned}$$

Since $\beta \in (0,1)$ and $\sigma_w, \lambda_w \geq 0$ the last term is strictly positive. In a partial equilibrium with a (strictly) positive market tightness, $E(\omega') \geq U$. Hence,

$$\frac{\partial E(\omega) - U}{\partial \sigma_w} \leq 0.$$

First, for the reservation wage

$$\underline{\omega} = (1-\beta)U - \beta(1-\sigma_w)(E(\omega') - U)$$

the positive derivative is directly obtained as

$$E(\omega) - U = \omega - \underline{\omega}$$

implies

$$\frac{\partial \underline{\omega}}{\partial \sigma_w} = -\frac{\partial E(\omega) - U}{\partial \sigma_w} \geq 0$$

Second, the partial equilibrium wage has to satisfy the Nash bargaining optimality condition

$$(1-\gamma)(E(\omega) - U) = \gamma(J(\omega) - V)$$

Hence, the wage is implicitly defined as the root of

$$f(\omega) = (1-\gamma)(E(\omega) - U) - \gamma(J(\omega) - V).$$

Since $J(\omega) - V = z - \omega - (1-\beta)V + \beta(1-\sigma)(J(\omega') - V)$

$$\frac{\partial J(\omega) - V}{\partial \sigma_w} = 0$$

for any future sequence of wages. Together, as γ might be one, this implies that

$$\frac{\partial f(\omega)}{\partial \sigma_w} \leq 0.$$

Moreover, as

$$f(\omega) = \omega + (1-\gamma)(-(1-\beta)U + \beta(1-\sigma_w)(E(\omega')-U)) - \gamma(z - (1-\beta)V + \beta(1-\sigma)(J(\omega')-V))$$

the derivative with respect to the wage is equal to one, $\frac{\partial f(\omega)}{\partial \omega} = 1$. Finally, as $f(\omega)$ is continuously differentiable with $\frac{\partial f(\omega)}{\partial \omega} \neq 0$, by the implicit function theorem

$$\frac{\partial \omega}{\partial \sigma_w} = -\frac{\frac{\partial f(\omega)}{\partial \sigma_w}}{\frac{\partial f(\omega)}{\partial \omega}} = -\frac{\partial f(\omega)}{\partial \sigma_w} \geq 0$$

Since $\sigma_w = (1 + \Delta_{\sigma w})\sigma$, the claim from ii) follows.

■

Proof of Proposition 2

Set $\Delta_{\lambda f} = 0$, $\Delta_{\sigma f} = 0$, and $T = 1$ in the proof of Proposition 8.

■

Proof of Proposition 3

First, the job-creation condition, the unemployment rate, as well as the average unemployment duration are given by

$$\omega(\theta) = z - \frac{\kappa}{\beta p(\theta)/\theta}(1 - \beta(1-\sigma))$$
$$u = \frac{\sigma}{\sigma + p(\theta)}$$
$$d = \frac{1}{p(\theta)}$$

As $p(\theta)$ is increasing and $p(\theta)/\theta$ decreasing, clearly,

$$\frac{\partial \omega}{\partial \theta} \leq 0, \quad \frac{\partial u}{\partial \theta} \leq 0, \quad \frac{\partial d}{\partial \theta} \leq 0$$

Hence, we only need to show that (i) $\frac{\partial\theta}{\partial\Delta_{\lambda w}} \leq 0$ and (ii) $\frac{\partial\theta}{\partial\Delta_{\sigma w}} \leq 0$.

Using the notation and results from the proof of Proposition 8, we know that for any $\Delta_{\lambda w}$ a unique root of $h(\theta)$ exists. Furthermore, we know that $h(\theta)$ is continuously differentiable with $h'(\theta) < 0$. Finally, from

$$h(\theta) = z - \frac{\kappa}{\beta p(\theta)/\theta}(1-\beta(1-\sigma)) - b - \gamma\left[z - b + \kappa\theta\left(1 + \Delta_{\lambda w} + \sigma\frac{\Delta_{\sigma w}}{p(\theta)}\right)\right]$$

we get that

$$\frac{\partial h(\theta)}{\partial\Delta_{\lambda w}} = -\gamma\kappa\theta \leq 0 \quad \text{for } \theta > 0.$$

Together, by the implicit function theorem,

$$\frac{\partial\theta}{\partial\Delta_{\lambda w}} = -\frac{\frac{\partial h(\theta)}{\partial\Delta_{\lambda w}}}{\frac{\partial h(\theta)}{\partial\theta}} \leq 0.$$

Similarly, from the definition of $h(\theta)$, it follows directly that

$$\frac{\partial h(\theta)}{\partial\Delta_{\sigma w}} = -\gamma\kappa\sigma\frac{\theta}{p(\theta)} \leq 0$$

Therefore, again by the implicit function theorem,

$$\frac{\partial\theta}{\partial\Delta_{\sigma w}} = -\frac{\frac{\partial h(\theta)}{\partial\Delta_{\sigma w}}}{\frac{\partial h(\theta)}{\partial\theta}} \leq 0.$$

■

Proof of Proposition 4

Consider any partial equilibrium wage. This wage determines U and in combination with the current wage, ω, $E(\omega)$, which is given recursively by

$$E(\omega) = \omega + \beta(1-\sigma_w)E(\omega) + \beta\sigma_w U$$

Subtracting U results in the following definition of the worker surplus $E(\omega) - U$

$$E(\omega) - U = \frac{\omega - (1-\beta)U}{1-\beta(1-\sigma_w)}$$

Moreover, the reservation wage $\underline{\omega}$ can be expressed as

$$E(\underline{\omega}) - U = 0 \qquad \Longrightarrow \qquad \underline{\omega} = (1-\beta)U$$

i)

Taking the partial derivative of $E(\omega) - U$ with respect to λ_w yields

$$\frac{\partial E(\omega) - U}{\partial \lambda_w} = -\frac{1-\beta}{1-\beta(1-\sigma_w)} \frac{\partial U}{\partial \lambda_w}$$

which is independent of ω. Hence, implying that $\frac{\partial E(\omega)-U}{\partial \lambda_w} = \frac{\partial E(\omega')-U}{\partial \lambda_w}$.
To obtain the derivative of U with respect to σ_w, rearrange the the value function of U

$$\begin{aligned} U &= b + \beta\lambda_w E(\omega') + \beta(1-\lambda_w)U \\ U &= \frac{1}{1-\beta}\Big(b + \beta\lambda_w(E(\omega') - U)\Big) \\ \frac{\partial U}{\partial \lambda_w} &= \frac{1}{1-\beta}\beta\left[\lambda_w \frac{\partial E(\omega') - U}{\partial \lambda_w} + E(\omega') - U\right] \end{aligned}$$

Use this expression in the derivative of $E(\omega) - U$, to get

$$\begin{aligned} \frac{\partial E(\omega) - U}{\partial \lambda_w}\Big[1-\beta(1-\sigma_w)\Big] &= -\beta\lambda_w \frac{\partial E(\omega') - U}{\partial \lambda_w} - \beta(E(\omega') - U) \\ &= -\beta\lambda_w \frac{\partial E(\omega) - U}{\partial \lambda_w} - \beta(E(\omega') - U) \\ \Longrightarrow \frac{\partial E(\omega) - U}{\partial \lambda_w}\Big[1-\beta+\beta\sigma_w+\beta\lambda_w\Big] &= -\beta(E(\omega') - U) \\ \Longrightarrow \frac{\partial E(\omega) - U}{\partial \lambda_w} &= -(E(\omega') - U)\frac{1}{\frac{1}{\beta} - 1 + \sigma_w + \lambda_w} \end{aligned}$$

Since $\beta \in (0,1)$ and $\sigma_w, \lambda_w \geq 0$ the last term is strictly positive. In a partial equilibrium with a (strictly) positive market tightness, $E(\omega') \geq U$. Hence,

$$\frac{\partial E(\omega) - U}{\partial \lambda_w} \leq 0.$$

First, for the reservation wage

$$\underline{\omega} = (1-\beta)U$$

the positive derivative is directly obtained as

$$E(\omega) - U = \omega - \underline{\omega}$$

implies

$$\frac{\partial \underline{\omega}}{\partial \lambda_w} = -\frac{\partial E(\omega) - U}{\partial \lambda_w} \geq 0$$

Second, the partial equilibrium wage has to satisfy the Nash bargaining optimality condition

$$(1-\gamma)(E(\omega) - U)(1 - \beta(1-\sigma_w)) = \gamma(J(\omega) - V)(1 - \beta(1-\sigma))$$

From $J(\omega) = z - \omega + \beta(1-\sigma)J(\omega) + \beta\sigma V$, the surplus of a firm is given by

$$J(\omega) - V = \frac{z - \omega - (1-\beta)V}{1 - \beta(1-\sigma)}$$

simplifying the optimality condition to

$$\begin{aligned}(1-\gamma)(\omega - (1-\beta)U) &= \gamma(z - \omega - (1-\beta)V) \\ \omega &= \gamma(z - (1-\beta)V) + (1-\gamma)(1-\beta)U\end{aligned}$$

Hence, the wage is implicitly defined as the root of

$$f(\omega) = \omega - \gamma(z - (1-\beta)V) - (1-\gamma)(1-\beta)U.$$

Since $\frac{\partial J(\omega) - V}{\partial \lambda_w} = 0$, $\frac{\partial V}{\partial \lambda_w} = 0$ for any partial equilibrium wage. Together, as γ might be one, this implies that

$$\frac{\partial f(\omega)}{\partial \lambda_w} \leq 0.$$

Moreover, the derivative with respect to the current wage is equal to one, $\frac{\partial f(\omega)}{\partial \omega} = 1$. Finally, as $f(\omega)$ is continuously differentiable with $\frac{\partial f(\omega)}{\partial \omega} \neq 0$, by the implicit function theorem

$$\frac{\partial \omega}{\partial \lambda_w} = -\frac{\frac{\partial f(\omega)}{\partial \lambda_w}}{\frac{\partial f(\omega)}{\partial \omega}} = -\frac{\partial f(\omega)}{\partial \lambda_w} \geq 0$$

Since $\lambda_w = (1 + \Delta_{\lambda w})p(\theta)$, the claim from i) follows.

ii)

Taking the partial derivative of $E(\omega) - U$ with respect to σ_w yields

$$\begin{aligned}\frac{\partial E(\omega) - U}{\partial \sigma_w} &= -\frac{(1-\beta)}{1-\beta(1-\sigma_w)}\frac{\partial U}{\partial \sigma_w} - \beta\frac{\omega - (1-\beta)U}{(1-\beta(1-\sigma_w))^2} \\ &= -\frac{(1-\beta)}{1-\beta(1-\sigma_w)}\frac{\partial U}{\partial \sigma_w} - \beta\frac{E(\omega) - U}{1-\beta(1-\sigma_w)}\end{aligned}$$

for any wage ω. Hence, for the partial equilibrium wage ω'

$$\frac{\partial E(\omega') - U}{\partial \sigma_w} = -\frac{(1-\beta)}{1-\beta(1-\sigma_w)}\frac{\partial U}{\partial \sigma_w} - \beta\frac{E(\omega') - U}{1-\beta(1-\sigma_w)}$$

To obtain the derivative of U with respect to σ_w, rearrange the the value function of U

$$\begin{aligned}
U &= b + \beta\lambda_w E(\omega') + \beta(1-\lambda_w)U \\
U &= \frac{1}{1-\beta}\Big(b + \beta\lambda_w(E(\omega') - U)\Big) \\
\frac{\partial U}{\partial \sigma_w} &= \frac{1}{1-\beta}\beta\lambda_w\frac{\partial E(\omega') - U}{\partial \sigma_w}
\end{aligned}$$

Use this expression in the derivative of $E(\omega') - U$, to get

$$\begin{aligned}
\frac{\partial E(\omega') - U}{\partial \sigma_w}\Big[1 - \beta(1-\sigma_w)\Big] &= -\beta\lambda_w\frac{\partial E(\omega') - U}{\partial \sigma_w} - \beta(E(\omega') - U) \\
\implies \frac{\partial E(\omega') - U}{\partial \sigma_w}\Big[1 - \beta + \beta\sigma_w + \beta\lambda_w\Big] &= -\beta(E(\omega') - U) \\
\implies \frac{\partial E(\omega') - U}{\partial \sigma_w} &= -(E(\omega') - U)\frac{1}{\frac{1}{\beta} - 1 + \sigma_w + \lambda_w}
\end{aligned}$$

Since $\beta \in (0,1)$ and $\sigma_w, \lambda_w \geq 0$ the last term is strictly positive. In a partial equilibrium with a (strictly) positive market tightness, $E(\omega') \geq U$. Hence,

$$\frac{\partial E(\omega') - U}{\partial \sigma_w} \leq 0.$$

Furthermore, the value of unemployment decreases if σ_w increases

$$\frac{\partial U}{\partial \sigma_w} = \frac{1}{1-\beta}\beta\lambda_w\frac{\partial E(\omega') - U}{\partial \sigma_w} \leq 0$$

First, by the definition of the reservation wage

$$\underline{\omega} = (1-\beta)U$$

the positive derivative is directly obtained

$$\frac{\partial \underline{\omega}}{\partial \sigma_w} = (1-\beta)\frac{\partial U}{\partial \sigma_w} \leq 0$$

Second, the partial equilibrium wage has to satisfy the Nash bargaining optimality condition

$$(1-\gamma)(E(\omega)-U)(1-\beta(1-\sigma_w)) = \gamma(J(\omega)-V)(1-\beta(1-\sigma))$$

From $J(\omega) = z - \omega + \beta(1-\sigma)J(\omega) + \beta\sigma V$, the surplus of a firm is given by

$$J(\omega) - V = \frac{z-\omega-(1-\beta)V}{1-\beta(1-\sigma)}$$

simplifying the optimality condition to

$$\begin{aligned}(1-\gamma)(\omega-(1-\beta)U) &= \gamma(z-\omega-(1-\beta)V)\\ \omega &= \gamma(z-(1-\beta)V) + (1-\gamma)(1-\beta)U\end{aligned}$$

Hence, the wage is implicitly defined as the root of

$$f(\omega) = \omega - \gamma(z-(1-\beta)V) - (1-\gamma)(1-\beta)U.$$

Since $\frac{\partial J(\omega)-V}{\partial \sigma_w} = 0$, $\frac{\partial V}{\partial \sigma_w} = 0$ for any partial equilibrium wage. Together, as γ might be one, this implies that

$$\frac{\partial f(\omega)}{\partial \sigma_w} \geq 0.$$

Moreover, the derivative with respect to the current wage is equal to one, $\frac{\partial f(\omega)}{\partial \omega} = 1$. Finally, as $f(\omega)$ is continuously differentiable with $\frac{\partial f(\omega)}{\partial \omega} \neq 0$, by the implicit function theorem

$$\frac{\partial \omega}{\partial \sigma_w} = -\frac{\frac{\partial f(\omega)}{\partial \sigma_w}}{\frac{\partial f(\omega)}{\partial \omega}} = -\frac{\partial f(\omega)}{\partial \sigma_w} \leq 0$$

Since $\sigma_w = (1+\Delta_{\sigma w})\sigma$, the claim from ii) follows.

■

Proof of Proposition 5

Set $\Delta_{\lambda f} = 0$, $\Delta_{\sigma f} = 0$, and $T \to \infty$ in the proof of Proposition 8.

■

Proof of Proposition 6

First, the job-creation condition, the unemployment rate, as well as the average unemployment duration are given by

$$\begin{aligned}\omega(\theta) &= z - \frac{\kappa}{\beta p(\theta)/\theta}(1-\beta(1-\sigma)) \\ u &= \frac{\sigma}{\sigma + p(\theta)} \\ d &= \frac{1}{p(\theta)}\end{aligned}$$

As $p(\theta)$ is increasing and $p(\theta)/\theta$ decreasing, clearly,

$$\frac{\partial \omega}{\partial \theta} \leq 0, \quad \frac{\partial u}{\partial \theta} \leq 0, \quad \frac{\partial d}{\partial \theta} \leq 0$$

Hence, we only need to show that (i) $\frac{\partial \theta}{\partial \Delta_{\lambda w}} \leq 0$ and (ii) $\frac{\partial \theta}{\partial \Delta_{\sigma w}} \geq 0$.

Using the notation and results from the proof of Proposition 8, we know that for any $\Delta_{\lambda w}$ a unique root of $h(\theta)$ exists. Furthermore, we know that $h(\theta)$ is continuously differentiable with $h'(\theta) < 0$. Finally, from

$$h(\theta) = z - \frac{\kappa}{\beta p(\theta)/\theta}(1-\beta(1-\sigma)) - b - \gamma \left[z - b + \frac{1-\beta(1-\sigma)}{1-\beta(1-(1+\Delta_{\sigma w})\sigma)} \kappa\theta \left(1+\Delta_{\lambda w}\right) \right]$$

we get that

$$\frac{\partial h(\theta)}{\partial \Delta_{\lambda w}} = -\gamma \frac{1-\beta(1-\sigma)}{1-\beta(1-(1+\Delta_{\sigma w})\sigma)} \kappa\theta \leq 0 \quad \text{for } \theta > 0.$$

Together, by the implicit function theorem,

$$\frac{\partial \theta}{\partial \Delta_{\lambda w}} = -\frac{\frac{\partial h(\theta)}{\partial \Delta_{\lambda w}}}{\frac{\partial h(\theta)}{\partial \theta}} \leq 0.$$

Similarly, from the definition of $h(\theta)$, it follows directly that

$$\frac{\partial h(\theta)}{\partial \Delta_{\sigma w}} = -\gamma \frac{(1-\beta+\beta\sigma)\beta\sigma}{(1-\beta+\beta\sigma+\beta\Delta_{\sigma w}\sigma)^2} \kappa\theta \left(1+\Delta_{\lambda w}\right) \leq 0$$

Therefore, again by the implicit function theorem,

$$\frac{\partial\theta}{\partial\Delta_{\sigma w}} = -\frac{\frac{\partial h(\theta)}{\partial\Delta_{\sigma w}}}{\frac{\partial h(\theta)}{\partial\theta}} \geq 0.$$

■

Proof of Proposition 7

Set $\Delta_{\lambda f} = 0$, $\Delta_{\sigma f} = 0$ in the proof of Proposition 8.

■

Proof of Proposition 8

The wage curve under Period-T renegotiation with biased firm expectations is given (after rewriting the finite sums) by

$$\begin{aligned} f(\theta) = \gamma z + (1-\gamma)b + \gamma\frac{1-\beta(1-\sigma_f)}{1-\beta(1-\sigma_w)}\frac{1-(\beta(1-\sigma_w))^T}{1-(\beta(1-\sigma_f))^T}\frac{1+\Delta_{\lambda w}}{1+\Delta_{\lambda f}}\theta\kappa \\ + \gamma\beta^{T-1}\left[(1-\sigma_f)^T - (1-\sigma_w)^T\right]\frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T}\frac{1}{(1+\Delta_{\lambda f})p(\theta)/\theta}\kappa \end{aligned}$$

and the job-creation condition by

$$g(\theta) = z - \frac{\kappa}{\beta(1+\Delta_{\lambda f})p(\theta)/\theta}(1-\beta(1-\sigma_f))$$

Notice that from the job-creation condition requires that $\Delta_{\lambda f} > -1$, otherwise, no firm would enter the market.

Define the difference of these two functions as $h(\theta) = g(\theta) - f(\theta)$. An equilibrium of this model solves $g(\theta) = f(\theta)$ or, equivalently, $h(\theta) = 0$.

First, we establish that $\frac{\partial h(\theta)}{\partial \theta} \leq 0 \;\; \forall \theta > 0$ independent of the proposition's condition:

$$
\begin{aligned}
\frac{\partial h(\theta)}{\partial \theta} =& -\frac{\kappa}{\beta}\frac{p(\theta)-\theta m'(\theta)}{p(\theta)^2}\frac{1-\beta(1-\sigma_f)}{1+\Delta_{\lambda f}} \\
& -\gamma\frac{1-\beta(1-\sigma_f)}{1-\beta(1-\sigma_w)}\frac{1-(\beta(1-\sigma_w))^T}{1-(\beta(1-\sigma_f))^T}\frac{1+\Delta_{\lambda w}}{1+\Delta_{\lambda f}}\kappa \\
& -\kappa\gamma\beta^{T-1}\left[(1-\sigma_f)^T-(1-\sigma_w)^T\right]\frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T}\frac{1}{1+\Delta_{\lambda f}}\frac{p(\theta)-\theta m'(\theta)}{p(\theta)^2} \\
=& -\frac{\kappa}{1+\Delta_{\lambda f}}\frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T}\left[\frac{1-\epsilon_p(\theta)}{p(\theta)}\left(\frac{1}{\beta}(1-(\beta(1-\sigma_f))^T)+\gamma\beta^{T-1}((1-\sigma_f)^T-(1-\sigma_w)^T)\right)\right. \\
& \left.+\gamma\frac{1-(\beta(1-\sigma_w))^T}{1-\beta(1-\sigma_w)}(1+\Delta_{\lambda w})\right] \\
=& -\frac{\kappa}{1+\Delta_{\lambda f}}\frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T}\left[\frac{1-\epsilon_p(\theta)}{p(\theta)}\beta^{T-1}\left(\frac{1}{\beta^T}-[(1-\gamma)(1-\sigma_f)^T+\gamma(1-\sigma_w)^T]\right)\right. \\
& \left.+\gamma\frac{1-(\beta(1-\sigma_w))^T}{1-\beta(1-\sigma_w)}(1+\Delta_{\lambda w})\right] \\
\leq& \, 0
\end{aligned}
$$

where the last inequality follows, as all parts are weakly larger than zero. In particular, $(1-\gamma)(1-\sigma_f)^T+\gamma(1-\sigma_w)^T$ represents a convex combination of two elements which are each bounded by zero and one, and therefore cannot exceed $1/\beta^T$ which is larger than one.
$\frac{\partial h(\theta)}{\partial \theta} \leq 0$ implies that the wage curve always exhibits a larger slope than the job-creation curve. Before we proceed, we simplify $h(\theta)$ analogously to the derivative before:

$$
\begin{aligned}
h(\theta) =& (1-\gamma)(z-b)-\frac{\kappa}{1+\Delta_{\lambda f}}\frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T} \\
& \times\left[\gamma\frac{1-(\beta(1-\sigma_w))^T}{1-\beta(1-\sigma_w)}(1+\Delta_{\lambda w})\theta+\frac{1}{p(\theta)/\theta}\beta^{T-1}\left(\frac{1}{\beta^T}-[(1-\gamma)(1-\sigma_f)^T+\gamma(1-\sigma_w)^T]\right)\right]
\end{aligned}
$$

Next, we show that for large θ, the job-creation curve is below the wage curve, i.e., $h(\theta) < 0$ for $\theta >> 0$.

$$
\begin{aligned}
\lim_{\theta\to\infty} h(\theta) =& (1-\gamma)(z-b)-\frac{\kappa}{1+\Delta_{\lambda f}}\frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T} \\
& \times\lim_{\theta\to\infty}\left[\gamma\frac{1-(\beta(1-\sigma_w))^T}{1-\beta(1-\sigma_w)}(1+\Delta_{\lambda w})\theta+\frac{1}{p(\theta)/\theta}\beta^{T-1}\left(\frac{1}{\beta^T}-[(1-\gamma)(1-\sigma_f)^T+\gamma(1-\sigma_w)^T]\right)\right] \\
=& (1-\gamma)(z-b)-\infty \\
=& -\infty
\end{aligned}
$$

where the limit diverges to $+\infty$ as $p(\theta)$ converges by assumption to unity and all the components within the limit are weakly positive (and the last part is strictly positive) as argued previously. Hence, by continuity of $h(\theta)$, there are finite θ for which $h(\theta)$ is negative.

Since $h(\theta)$ is continuous, weakly decreasing and negative for large values of θ, by the intermediate value theorem, there exists $\theta > 0$ solving $h(\theta) = 0$ if and only if $\lim_{\theta \to 0} h(\theta) > 0$ or $\lim_{\theta \to 0} h(\theta) = 0$ with $\lim_{\theta \to 0} h'(\theta) = 0$. This condition can be re-written to yield the condition from the proposition:

$$
\begin{aligned}
&\lim_{\theta \to 0} h(\theta) \geq 0 \\
\Longleftrightarrow &(1-\gamma)(z-b) \geq \frac{\kappa}{1+\Delta_{\lambda f}} \frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T} \\
&\times \lim_{\theta \to 0} \left[\gamma \frac{1-(\beta(1-\sigma_w))^T}{1-\beta(1-\sigma_w)} (1+\Delta_{\lambda w})\theta + \frac{1}{p(\theta)/\theta} \beta^{T-1} \left(\frac{1}{\beta^T} - [(1-\gamma)(1-\sigma_f)^T + \gamma(1-\sigma_w)^T] \right) \right] \\
\Longleftrightarrow &(1-\gamma)(z-b) \geq \frac{\kappa}{1+\Delta_{\lambda f}} \frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T} \beta^{T-1} \left(\frac{1}{\beta^T} - [(1-\gamma)(1-\sigma_f)^T + \gamma(1-\sigma_w)^T] \right) \\
\Longleftrightarrow & -[(1-\gamma)(1-\sigma_f)^T + \gamma(1-\sigma_w)^T] \frac{1}{1+\Delta_{\lambda f}} \frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T} \beta^{T-1} \leq (1-\gamma)\frac{z-b}{\kappa} - \frac{1}{1+\Delta_{\lambda f}} \frac{1-\beta(1-\sigma}{1-(\beta(1-\sigma_f} \\
\Longleftrightarrow &\gamma \left((1-\sigma_f)^T - (1-\sigma_w)^T \right) \frac{1}{1+\Delta_{\lambda f}} \frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T} \beta^{T-1} \\
&\leq (1-\gamma)\frac{z-b}{\kappa} - \frac{1}{1+\Delta_{\lambda f}} \frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T} \left[\frac{1}{\beta} + (1-\sigma_f)^T \beta^{T-1} \right] \\
\Longleftrightarrow &\gamma \left((1-\sigma_f)^T - (1-\sigma_w)^T \right) \beta^{T-1} \leq (1-\gamma)\frac{z-b}{\kappa}(1+\Delta_{\lambda f}) \frac{1-(\beta(1-\sigma_f))^T}{1-\beta(1-\sigma_f)} - \left[\frac{1}{\beta} + (1-\sigma_f)^T \beta^{T-1} \right] \\
\Longleftrightarrow &\gamma \left((1-\sigma_f)^T - (1-\sigma_w)^T \right) \beta^{T-1} \leq (1-\gamma)\frac{z-b}{\kappa}(1+\Delta_{\lambda f}) \sum_{t=1}^{T} \Big(\beta(1-\sigma_f) \Big)^{t-1} - \left[\frac{1}{\beta} + (1-\sigma_f)^T \beta^{T-1} \right]
\end{aligned}
$$

Hence, if this condition hold strictly, there exists at least one $\theta > 0$ solving $h(\theta) = 0$. If the condition holds with equality, $\lim_{\theta \to 0} h'(\theta) = 0$ is additionally required which is equivalent to

$$
\begin{aligned}
&\lim_{\theta \to 0} h'(\theta) = 0 \\
\Longleftrightarrow &\lim_{\theta \to 0} \left[\frac{1-\epsilon_p(\theta)}{p(\theta)} \beta^{T-1} \left(\frac{1}{\beta^T} - [(1-\gamma)(1-\sigma_f)^T + \gamma(1-\sigma_w)^T] \right) + \gamma \frac{1-(\beta(1-\sigma_w))^T}{1-\beta(1-\sigma_w)} (1+\Delta_{\lambda w}) \right] = 0 \\
\Longleftrightarrow &\lim_{\theta \to 0} \left[\frac{1-\epsilon_p(\theta)}{p(\theta)} \right] \beta^{T-1} \left(\frac{1}{\beta^T} - [(1-\gamma)(1-\sigma_f)^T + \gamma(1-\sigma_w)^T] \right) + \gamma \frac{1-(\beta(1-\sigma_w))^T}{1-\beta(1-\sigma_w)} (1+\Delta_{\lambda w}) = 0 \\
\Longleftrightarrow &\lim_{\theta \to 0} \left[\frac{1-\epsilon_p(\theta)}{p(\theta)} \right] = 0 \qquad \text{and} \qquad \gamma(1+\Delta_{\lambda w}) = 0
\end{aligned}
$$

where the first condition is satisfied if and only if the job-filling rate is constant in the limit, i.e. $\lim_{\theta \to 0} \frac{\partial \theta}{\partial} p(\theta)/\theta = 0$.

Next, uniqueness of the equilibrium follows from the fact that under the additional assumptions of the proposition, $h'(\theta) < 0$ and $\lim_{\theta \to 0} h(\theta) > 0$, implying a unique root for $\theta > 0$.

Finally, we show that the previous equilibria candidates are indeed admissible, as they imply a wage above b, which is not guaranteed for the case of a decreasing wage function. To show this, we start by assuming that $\Delta_{\lambda w} = -1$. Then, rearranging $h(\theta) = 0$ yields

$$\frac{\kappa}{p(\theta)/\theta} = (1-\gamma)(z-b)(1+\Delta_{\lambda f})\frac{1-(\beta(1-\sigma_f))^T}{1-\beta(1-\sigma_f)}\frac{1}{\beta^{T-1}}\left(\frac{1}{\beta^T} - [(1-\gamma)(1-\sigma_f)^T + \gamma(1-\sigma_w)^T]\right)^{-1}$$

Plugging this expression into the wage curve, we get

$$\begin{aligned}
\omega &= b + \gamma(z-b) + \gamma\beta^{T-1}\left[(1-\sigma_f)^T - (1-\sigma_w)^T\right]\frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T}\frac{1}{(1+\Delta_{\lambda f})}\frac{\kappa}{p(\theta)/\theta} \\
&= b + \gamma(z-b) + \gamma\left[(1-\sigma_f)^T - (1-\sigma_w)^T\right](1-\gamma)(z-b)\left(\frac{1}{\beta^T} - [(1-\gamma)(1-\sigma_f)^T + \gamma(1-\sigma_w)^T]\right)^{-1} \\
&= b + \gamma(z-b)\left[1 + (1-\gamma)\frac{(1-\sigma_f)^T - (1-\sigma_w)^T}{\frac{1}{\beta^T} - [(1-\gamma)(1-\sigma_f)^T + \gamma(1-\sigma_w)^T]}\right] \\
&= b + \gamma(z-b)\frac{\frac{1}{\beta^T} - (1-\sigma_w)^T}{\frac{1}{\beta^T} - [(1-\gamma)(1-\sigma_f)^T + \gamma(1-\sigma_w)^T]} \\
&\geq b
\end{aligned}$$

using $z > b$ and the fact that $\frac{1}{\beta^T} > 1$ while $(1-\sigma_i)^T \in [0, 1]$.
Thus, the unique root of $h(\theta)$ implies a wage above b. To extend this relation to all $\Delta_{\lambda w} \in [-1, 1]$, notice that

$$\begin{aligned}
\frac{\partial^2 h(\theta)}{\partial\theta\partial\Delta_{\lambda w}} &= -\frac{\kappa}{1+\Delta_{\lambda f}}\frac{1-\beta(1-\sigma_f)}{1-(\beta(1-\sigma_f))^T}\gamma\frac{1-(\beta(1-\sigma_w))^T}{1-\beta(1-\sigma_w)} < 0 \\
\frac{\partial \lim_{\theta\to 0} h(\theta)}{\partial\Delta_{\lambda w}} &= 0
\end{aligned}$$

Hence, for larger values of $\Delta_{\lambda w}$, $h(\theta)$ starts at the same level for $\theta = 0$ but approaches zero faster, thus, the root must be smaller (by applying the implicit function theorem; see the proof for Proposition 6). Since

$$\begin{aligned}
\frac{\partial g(\theta)}{\partial\theta} &< 0 \\
\frac{\partial^2 g(\theta)}{\partial\theta\partial\Delta_{\lambda w}} &= 0
\end{aligned}$$

this implies that $\frac{\partial w}{\partial\Delta_{\lambda w}} > 0$. As $\omega \geq b$ for $\Delta_{\lambda w} = -1$, this relation finally leads to $\omega \geq b \ \forall\Delta_{\lambda w} \in [-1, 1]$.

■

Proof of Proposition 9

Equilibrium conditions:

$$\frac{(1-\gamma)(1-z^*)}{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma)}=\frac{\kappa}{\beta\lambda(\theta)}$$

$$z^*-b+\frac{\beta(\gamma\sigma_w+(1-\gamma)\sigma)}{1-\beta+\beta(\gamma\sigma_w+(1-\gamma)\sigma)}\int_{z^*}^{1}(z'-z^*)dG(z')=\frac{\gamma}{1-\gamma}\kappa(1+\Delta_{\lambda w})\theta$$

Express θ from first expression:

$$\theta=\lambda^{-1}\left(\frac{\kappa}{\beta}\frac{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma)}{(1-\gamma)(1-z^*)}\right)$$

and plug it into second expression:

$$z^*-b+\frac{\beta(\gamma\sigma_w+(1-\gamma)\sigma)}{1-\beta+\beta(\gamma\sigma_w+(1-\gamma)\sigma)}\int_{z^*}^{1}(z'-z^*)dG(z')=\frac{\gamma}{1-\gamma}\kappa(1+\Delta_{\lambda w})\lambda^{-1}\left(\frac{\kappa}{\beta}\frac{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-}{(1-\gamma)(1-z^*)}\right.$$

The left-hand side is strictly increasing in z^*

$$\frac{\partial L}{\partial z^*}=1+\frac{\beta(\gamma\sigma_w+(1-\gamma)\sigma)}{1-\beta+\beta(\gamma\sigma_w+(1-\gamma)\sigma)}G(z^*)>0$$

For $z^*=1$, the left-hand side is equal to $1-b>0$. For $z^*=0$, the left-hand side is equal to $-b+\frac{\beta(\gamma\sigma_w+(1-\gamma)\sigma)}{1-\beta+\beta(\gamma\sigma_w+(1-\gamma)\sigma)}\int_0^1 z'dG(z')$.

Since $\frac{\partial\lambda}{\partial z^*}=\frac{\kappa}{\beta}\frac{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma)}{(1-\gamma)(1-z^*)}>0$, we get that $\frac{\partial\lambda^{-1}}{\partial z^*}<0$ (by continuity of λ).

Hence (for $(1+\Delta_{\lambda w})>0$), the right-hand side is strictly decreasing in z^*. For $z^*=0$ the right-hand side is strictly positive and equal to $\frac{\gamma}{1-\gamma}\kappa(1+\Delta_{\lambda w})\lambda^{-1}\left(\frac{\kappa}{\beta}\frac{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma)}{(1-\gamma)}\right)>0$. For $z^*\to 1$ the right-hand side goes to zero.

For $z^*=1$, the left-hand side of the equilibrium condition is strictly larger than the right-hand side. For $z^*=0$, the left-hand side is strictly smaller than the right-hand side if and only iff

$$\frac{\gamma}{1-\gamma}\kappa(1+\Delta_{\lambda w})\lambda^{-1}\left(\frac{\kappa}{\beta}\frac{1+\beta\gamma(\sigma_w-\sigma)-\beta(1-\sigma)}{(1-\gamma)}\right)>-b+\frac{\beta(\gamma\sigma_w+(1-\gamma)\sigma)}{1-\beta+\beta(\gamma\sigma_w+(1-\gamma)\sigma)}\int_0^1 z'dG(z')$$

A sufficient condition for this expression to hold is that $\int_0^1 z dG(z) < b$; that is, the mean of the shock-distribution is smaller than unemployment income.

If this condition is satisfied, then there exists a z^* such that the left-hand side is equal to the right-hand side. The right-hand side is always positive, hence the equilibrium z^*, implies a strictly positive value of θ. Moreover, since the left-hand side and the right-hand side are both monotonous, the equilibrium is unique.

■

H Alternating-offer bargaining

The alternating-offer bargaining game follows Binmore et al. (1986). One of the players, say the worker (without loss of generality), starts the game by making a wage demand to the firm. If the firm accepts the demand, the game ends and production starts. If the firm rejects it, the bargain either breaks down or continues. It breaks down with probability $1 - e^{-\phi\tau}$, where $\tau > 0$ measures time and $\phi > 0$. In this case, the match separates and the worker returns to unemployment and the firm's vacancy remains unfilled. The bargain continues with probability $e^{-\phi\tau}$ and the firm makes a wage offer. If the worker accepts the firm's offer, the game ends and production starts. If the worker rejects it, the bargain breaks down with probability $1 - e^{-\mu\tau}$, where $\mu > 0$ and continues with probability $e^{-\mu\tau}$. If it continues, the worker makes another wage demand. The game continues until an agreement is reached or the bargain breaks down.

The optimal strategy of the firm is to offer the wage $\underline{\omega}_o$ and to accept any wage $\omega \leq \overline{\omega}_d$ such that $\underline{\omega}_o$ and $\overline{\omega}_d$ satisfy the following conditions

$$\begin{aligned} J(\overline{\omega}_d) &= (1 - e^{-\phi\tau})V + e^{-\phi\tau} J(\underline{\omega}_o) \\ E(\underline{\omega}_o) &= (1 - e^{-\mu\tau})U + e^{-\mu\tau} E(\overline{\omega}_d) \end{aligned} \tag{A.14}$$

The first condition states that, $\overline{\omega}_d$ is the highest possible wage that the firm is willing to accept, given its offer $\underline{\omega}_o$. For this wage, the firm is indifferent between accepting $\overline{\omega}_d$, and rejecting it. Any demand $\omega > \overline{\omega}_d$ would be rejected by the firm. Likewise, the second condition states that $\underline{\omega}_o$ is the lowest possible wage the firm can offer. Any offer $\omega < \underline{\omega}_o$ would be rejected by the worker.

Given the firm's bargaining strategy, the worker's best response is to demand the wage $\overline{\omega}_d$ and to accept any offer $\omega \geq \underline{\omega}_o$. As a result of these strategies, the bargained wage will be equal to $\overline{w_d}$ satisfying the conditions in Equation (A.14).[18] To shorten notation, these two conditions can be written in terms of the surplus:

$$\begin{aligned} J(\overline{w}_d) - V &= e^{-\phi\tau}\left[J(\underline{w}_o) - V\right] \\ E(\underline{w}_o) - U &= e^{-\mu\tau}\left[E(\overline{w}_d) - U\right] \end{aligned} \tag{A.15}$$

In the next step, we solve for the bargained wage. We consider the case of period-T bargaining. First, we substitute the firm's and the worker's match surplus in Equations (A.1) and (A.2) into the first and second condition in Equation (A.15). This gives, after some rearranging, the following expressions

$$\overline{w}_d = e^{-\phi\tau}\underline{w}_o + (1 - e^{-\phi\tau})\left[z - (1-\beta)V + \frac{(\beta(1-\sigma))^T}{\sum_{t=1}^T(\beta(1-\sigma))^{t-1}}\big(J(\omega') - V\big)\right]$$

$$\underline{w}_o = e^{-\mu\tau}\overline{w}_d + (1 - e^{-\mu\tau})\left[(1-\beta)U - \frac{(\beta(1-\sigma_w))^T}{\sum_{t=1}^T(\beta(1-\sigma_w))^{t-1}}\big(E(\omega') - U\big)\right]$$

Substitute the second into the first expression to obtain:

$$\begin{aligned} \overline{w}_d = \quad & \frac{e^{-\phi\tau} - e^{-(\phi+\mu)\tau}}{1 - e^{-(\phi+\mu)\tau}}\left[(1-\beta)U - \frac{(\beta(1-\sigma_w))^T}{\sum_{t=1}^T(\beta(1-\sigma_w))^{t-1}}\big(E(\omega') - U\big)\right] \\ + \quad & \frac{1 - e^{-\phi\tau}}{1 - e^{-(\phi+\mu)\tau}}\left[z - (1-\beta)V + \frac{(\beta(1-\sigma))^T}{\sum_{t=1}^T(\beta(1-\sigma))^{t-1}}\big(J(\omega') - V\big)\right] \end{aligned}$$

As standard in the literature, we consider the limiting case of the bargaining game for $\tau \to 0$. Applying L'Hôpital's rule to this expression and defining $\gamma \equiv \frac{\phi}{\phi+\mu}$ as the worker's bargaining power, we obtain the following expression for the bargained wage:

$$\begin{aligned} w = \quad & (1-\gamma)\left[(1-\beta)U - \frac{(\beta(1-\sigma_w))^T}{\sum_{t=1}^T(\beta(1-\sigma_w))^{t-1}}\big(E(\omega') - U\big)\right] \\ + \quad & \gamma\left[z - (1-\beta)V + \frac{(\beta(1-\sigma))^T}{\sum_{t=1}^T(\beta(1-\sigma))^{t-1}}\big(J(\omega') - V\big)\right] \end{aligned}$$

Using again the agents' surplus functions to substitute for the terms in square brackets, we obtain:

$$(1-\gamma)\frac{(\beta(1-\sigma_w))^T}{\sum_{t=1}^T(\beta(1-\sigma_w))^{t-1}}\big(E(\omega') - U\big) = \gamma\frac{(\beta(1-\sigma))^T}{\sum_{t=1}^T(\beta(1-\sigma))^{t-1}}\big(J(\omega') - V\big)$$

This condition is identical to the optimality condition of the Nash bargaining game shown in Equation (A.3). Therefore, in our setting the alternative-offer bargaining protocol yields the same wage equation as the Nash bargaining game that we assume throughout the paper.

[18] If the firm was to move first in the bargaining game, the resulting wage would be equal to $\underline{\omega}_o$.