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Internal Meta-Analysis for Monte Carlo Simulations

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Internal Meta-Analysis for Monte Carlo Simulations

Abstract

Monte Carlo (MC) simulations are one of the dominant approaches to compare statistical methods. To date, there is no standard procedure for MC simulations. Although internally valid, they exhibit a certain degree of arbitrariness through the various choices that researchers make. In this paper, we propose the use of an internal meta-analysis for MC simulations to allow a standardized analysis, synthesis and presentation of MC simulation results in a transparent manner. The use of an internal meta-analysis allows (i) a much more standardized procedure and (ii) comprehensive analysis of a large variety and number of simulations. To exemplify the procedure, we conduct an extensive set of simulations to compare the empirical performance of three different estimators of the generalized stochastic frontier panel data model. Besides contributing to the literature on efficiency analysis by improving the understanding of the merits of the three different estimators, we demonstrate the applicability and usefulness of internal meta-analysis for MC simulations in general.

JEL-Codes: C1, C15

Keywords: Monte Carlo simulation; meta-analysis; stochastic frontier analysis; production function; panel data

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1 Mark A. Andor, RWI; David H. Bernstein, University of Miami, USA; Christopher F. Parmeter, University of Miami, USA; Stephan Sommer, Bochum University of Applied Sciences and RWI. *– This paper was presented at the North American Productivity Workshop (NAPW) on June 11, 2021. We thank Oleg Badunenko, and other NAPW participants for helpful feedback. We gratefully acknowledge that this work has been partly supported by the Collaborative Research Center "Statistical Modeling of Nonlinear Dynamic Processes" (SFB 823) of the German Research Foundation (DFG), within the framework of Project A3, "Dynamic Technology Modeling". The views expressed here are those of the authors and not necessarily those of the Federal Energy Regulatory Commission or the U.S. Government. The authors are responsible for all errors and omissions.* *– All correspondence to: Mark A. Andor, RWI, Hohenzollernstr. 1-3, 45128 Essen, Germany, e-mail: mark.andor@rwi-essen.de*

1 Introduction

Monte Carlo (MC) studies are widely recognized as a "statistical referee" (Perelman & Santin 2009, p. 303), used to verify the potential strengths and weaknesses of competing statistical methods. Owing to the fact that the performance evaluation of various statistical approaches is not possible using empirical data, researchers use MC simulations to make formal comparison in known settings. MC studies enable researchers to generate their own data set and compare "true" effects with estimated ones. A fundamental problem with such an approach is that the results depend on the underlying data generating process (DGP). Therefore, it is important to vary the assumptions in the DGP to derive a reasonable set of scenarios over which to make comparisons. Furthermore, it is useful to replicate the DGP with the same assumptions several times in order to obtain reliability. However, there is no standard procedure for MC simulations so far; simulation setups vary across disciplines, journals and authors alike, making cross-study comparisons difficult. Although internally valid, MC simulations exhibit a certain degree of arbitrariness, allowing researchers to choose the DGP. Moreover, the set of simulations considered is often limited so that the presentation of the results can be undertaken in a reasonable fashion.

In this paper, we posit that the use of an internal meta-analysis for MC simulations can provide additional value to the overall interpretation of the final results. We argue that this helps "standardize" the statistical referee as it allows (i) a transparent procedure by which to aggregate results and (ii) comprehensive analysis of a large variety and number of simulations.

Meta-analysis has traditionally provided a simple and informative way to summarize results across a large number of distinct studies that investigate a common problem. While meta-analysis is commonly viewed as the analysis of analyses Glass (1976), it can also be used in the context of a single study to summarize how subjective modeling decisions influence various outcomes of interest (Banzhaf & Smith 2007). As no Monte Carlo study is beyond reproach, internal meta-analysis can provide objective insights into how the various aspects of the MC simulations impact the assessed performance of each of the methods (or of a singular method). The proposed internal approach begins by conducting the MC simulations as normal. From here each specification is defined by a set of indicator variables describing its features (e.g. functional form, sample size, relative size of the variances, etc.) and by a set of variables describing the outcome of interest, say root mean square error or bias. Regressing these outcomes on indicators for features of the statistical specification can then help to summarize the ways in which the researcher's modeling decisions influence their findings. This approach is new in the operational research and efficiency modeling arena (see Section 2 for a review of the literature). However, internal meta-analyses seem to be new to MC simulations in general, as we could not find any application in other contexts. We are convinced that internal meta-analyses have great value for any future large-scale Monte Carlo study involving many different "choices" made by the analyst.

To exemplify the procedure, we conduct an extensive set of simulations to compare the performance of three of the most popular estimators of the generalized stochastic frontier panel data model. In our Monte Carlo simulation study, we mimic the simulation structure that appeared in Badunenko & Kumbhakar (2016) but also include the full maximum likelihood estimator of Colombi et al. (2014) and the plug-in likelihood estimator of Kumbhakar et al. (2014). While it is well known that, given a correct distributional specification, full maximum likelihood is the theoretically dominant approach, it is unknown whether this fact reveals itself in practice. This is particularly questionable in the four component setting where the likelihood function itself requires evaluation of a multi-dimensional integral, which grows as the time dimension of the panel increases. Thus, any large sample gains may not materialize with the sample sizes likely encountered by practitioners. The computationally simpler approaches of Kumbhakar et al. (2014) and Filippini & Greene (2016) are thus likely to be appealing to applied efficiency researchers. However, no rigorous assessment across all of these methods exists.[1]

Besides demonstrating the applicability and usefulness of internal meta-analysis for MC simulations in general, our internal meta-analysis reveals several interesting features that are difficult to see directly from the results. Namely, the mean square error of each of the three estimators of technical efficiency shrinks as the signal-to-noise ratio grows, at roughly the same rate. Further, we see that increasing the panel dimension has a more pronounced effect on the mean square error for transient technical efficiency for the two maximum likelihood estimators beyond that for the OLS based estimator. Further, for the logit based internal meta-analysis we see that there exist scenarios where any one of the three estimators is the best in terms of mean square error for transient, persistent and overall technical efficiency. However, when looking at relative and upward bias, the OLS based estimator never wins in the MC simulations for overall technical efficiency and very rarely wins for the estimation of transient technical efficiency.

The paper proceeds as follows. In the next section, we provide a review of the literature on MC simulations that evaluate efficiency analysis methods to demonstrate the merits of the application of an internal meta-analysis in MC studies. In Section 3, we introduce the methods, both the three estimators of the generalized stochastic frontier panel data model as well as the internal meta-analysis approach. In Section 4, we present the MC simulation design, the results, the internal meta-analysis and the limitations of this simulation study. Section 5 concludes.

[1] Colombi et al. (2014) provide a brief comparison between their proposed full maximum likelihood approach and the plug-in likelihood approach of Kumbhakar et al. (2014) while Badunenko & Kumbhakar (2016) assessed the ability of Filippini & Greene's (2016) simulated maximum likelihood proposal to recover the parameters of interest. A detailed comparison is important for several reasons. First, in the cross-sectional setting, a variety of simulation studies have revealed that multistep method of moment or plug-in likelihood approaches perform almost equally to maximum likelihood (Olson et al. 1980, Coelli 1995, Andor & Parmeter 2017) but do not require potentially cumbersome nonlinear optimization and the corresponding convergence issues. Moreover, these approaches can typically be implemented with far less computational panache than the more sophisticated methods. This can mitigate the impact that starting values, multi-starts and the number of simulations that are common choices for users with the more advanced methods.

2 Review of the Literature on MC Simulations for Efficiency Analysis Methods

In this section, we provide a review of the literature on Monte Carlo (MC) simulations that examine the merits and limitations of efficiency analysis methods. This is a non-systematic review and its purpose is not to provide a complete overview of every single study comparing efficiency analysis methods using MC simulations. Nevertheless, the review includes 39 studies published in the most relevant and prestigious journals in operations research and efficiency analysis, such as *European Journal of Operations Research* (12 studies), *Journal of Econometrics* (4), *Journal of Productivity Analysis* (7), and *Operations Research* (6). The purpose of this review is twofold. First, to give readers an idea that MC simulations are widely used to compare efficiency analysis methods and to present the main features of existing MC studies. Second, to emphasize that while MC studies are generally an objective, insightful, and systematic means of identifying the merits and limitations of existing and newly developed methods, the set of assumptions in MC simulations and the approaches used to aggregate results are to some extent arbitrary. Moreover, the number of scenarios considered in any MC study is limited, in the past primarily due to computer and time constraints, and more recently by the challenge of providing readers with a comprehensive, structured, and meaningful synthesis of the results. In particular, the approaches researchers have used to aggregate results vary, but no best practice has yet been achieved. We propose to use internal meta-analysis in MC simulations as a new standard.

In Table 1, we report the summary statistics from our review of the 39 considered studies. To get an idea of how MC studies have changed over time, in addition to the overall numbers, we split the sample into pre- and post-2010 studies. Of the 39 studies, 20 MC studies have been published after the year 2009, indicating an increase in studies per year. Furthermore, the number of replications and scenarios in each study has considerably increased on average. Overall, MC studies have an average number of

Table 1: Summary Statistics from Literature Review

	Overall	Pre 2010	Post 2010
# of considered studies	39	19	20
Characteristics of the MC studies			
# of replications, mean (median)	1106 (200)	339 (100)	1782 (500)
# of scenarios, mean (median)	104 (38)	58 (36)	150 (56)
sample sizes (%)	50 (72), 100 (69), 200 (54)	50 (76), 100 (57), 200 (33)	50 (67), 100 (83), 200 (78)
# of covariates, mean (median)	2.78 (2)	2.73 (2)	2.83 (2)
# of estimators, mean (median)	3.18 (3)	3.05 (2)	3.33 (3)
# of sample sizes, mean (median)	4.18 (4)	4.1 (4)	4.28 (4)
Approach to aggregate the results			
Each single scenario	34	18	16
Mixed approach	3	0	3
Graphical approach	2	1	1
Meta-Analysis	0	0	0

Notes: MSE = mean squared error; Mixed approach = aggregate some/all scenarios and look at individual scenarios. The considered studies are: Adler & Yazhemsky (2010), Aigner et al. (1977), Andor & Hesse (2014), Andor & Parmeter (2017), Andor et al. (2019), Badunenko et al. (2012), Badunenko & Kumbhakar (2016), Banker & Natarajan (2008), Banker et al. (1993), Behr & Tente (2008), Bojani et al. (1998), Chen & Delmas (2012), Coelli (1995), Cordero et al. (2009), Cordero et al. (2020), Cordero et al. (2016), Giorgio et al. (2016), Fan et al. (1996), Gong & Sickles (1992), Henningsen et al. (2015), Holland & Lee (2002), Jensen (2005), Krüger (2012), Kuosmanen & Kortelainen (2012), Nieswand & Seifert (2018), Olson et al. (1980), Parmeter & Zelenyuk (2019), Pedraja-Chaparro et al. (1999), Parmeter & Zelenyuk (2019), Podinovski et al. (2018), Resti (2000), Ruggiero (2007), Ruggiero (1999), Schaefer & Clermont (2018), Sickles (2005), Simar & Zelenyuk (2011), Simar & Zelenyuk (2018), Yu (1998).

1,106 replications, and 104 scenarios, yet for studies published after the year 2009 the numbers increase to 1,782 replications and 150 scenarios. In addition, the considered sample sizes increased over time, for example a sample size of 200 is considered in 33% of the MC studies published before 2010, versus 78% of the studies published after the year 2009. By contrast, the number of considered covariates, estimators and number of sample sizes seems to be relatively constant with around 2.8 covariates, 3.2 estimators and 4.2 sample sizes.

For the purpose of our study, the approach to aggregate the results from the various scenarios in each MC study is of major relevance. Table 1 demonstrates that most MC studies show and discuss each single considered scenario, 34 of the 39 studies follow this approach. This approach implies, of course, that the number of scenarios considered must be limited so that a discussion of all scenarios is possible within the framework of a scientific article. Others apply graphical approaches (2) or a mixed approach (3), meaning that they aggregate some or all scenarios but also look at individual scenarios to discuss specific topics. None of the MC studies apply an internal meta-analysis, what we suggest.

3 Methods

The introduction of the cross-section stochastic frontier model by Aigner et al. (1977) and Meeusen & van den Broeck (1977) has seen extensive and widespread application across a range of economic and operational research milieus. One area where this model has witnessed great interest recently has been in the construction of models with panel data designed to separate time varying and time constant firm level inefficiency (for a recent review, see Kumbhakar & Parmeter 2019). The essence of accounting for the panel structure of the data is that one has the potential ability to model both firm heterogeneity and time constant (persistent) inefficiency along with the usual time varying inefficiency. The initial work of Colombi (2010) and Colombi et al. (2011) leveraged results on the Closed Skew Normal distribution to produce a tractable likelihood form, demonstrating the potential to separately identify the parameters of the assumed distributions for each of the four components of the model.

Since this early work on the four component model there has been a large literature that has developed paying specific attention to this style of panel data stochastic frontier model. Colombi et al. (2014) more rigorously assessed the estimation of the model, while Kumbhakar et al. (2014) presented a plug-in likelihood approach, Kumbhakar & Tsionas (2014) studied the model in a Bayesian context and Filippini & Greene (2016) presented a computationally appealing simulated maximum likelihood approach. Kumbhakar & Lien (2018) provided a multistep approach based on method of moments, which was applied to study electricity regulation in Norway.

3.1 Estimation of the Generalized Stochastic Frontier Panel Data Model

The generalized stochastic production frontier panel data model is

$$y_{it} = \alpha_0 + m(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + \mu_i + v_{it} - \eta_i - u_{it}, \tag{1}$$

where μ_i are unobserved firm effects which capture time invariant heterogeneity, η_i represents time constant firm level inefficiency, u_{it} captures inefficiency which changes across both firms and time, and v_{it} is the standard idiosyncratic shock. The elegance of the four component model is that it can separately distinguish firm heterogeneity and factors which may have permanent effects on inefficiency (time-invariant). This model is a generalization of Greene (2005), Kumbhakar & Wang (2005), Wang & Ho (2010), and Chen et al. (2014).

The four component panel data model in equation (1) can be estimated in a single stage based on distributional assumptions for the four components Colombi et al. (2014). However, the likelihood function is somewhat complex and a simple algorithm has been proposed by Kumbhakar et al. (2014) based on plug-in likelihood. Alternatively, maximum simulated likelihood estimation can also be deployed Filippini & Greene (2016).

3.1.1 Plug-In Likelihood Estimation

To describe the algorithm of Kumbhakar et al. (2014) first rewrite the model in (1) as

$$y_{it} = \alpha_0^* + m(\boldsymbol{x}_{it}; \boldsymbol{\beta}) + \alpha_i + \varepsilon_{it}, \tag{2}$$

where $\alpha_i = \mu_i - \eta_i + E(\eta_i)$, $\varepsilon_{it} = v_{it} - u_{it} + E(u_{it})$ and $\alpha_0^* = \alpha_0 - E(\eta_i) - E(u_{it})$. This specification leads to both α_i and ε_{it} having mean zero. It is further assumed that both α_i and ε_{it} have constant variance – implying that there are no determinants of inefficiency present.

With the respecified model, the three step algorithm of Kumbhakar et al. (2014) is as follows:

Step 1: Estimate the model in equation (2) using the standard random effects panel estimator, which produces a consistent estimator of β. This will also produce estimates of α_i and ε_{it}.

Step 2: Using the predicted values of ε_{it} from Step 1, coupled with the assumptions that $v_{it} \sim N(0, \sigma_v^2)$ and $u_{it} \sim N_+(0, \sigma_u^2)$, estimate the unknown parameters using pseudolikelihood. The residuals need to be corrected by $E(u_{it}) = \sqrt{2/\pi}\sigma_u)$. This produces time-varying technical inefficiency using the Jondrow et al. (1982) estimator.

Step 3: Using the predicted values of α_i from Step 1, coupled with the assumptions that $\mu_i \sim N(0, \sigma_\mu^2)$ and $\eta_i \sim N_+(0, \sigma_\eta^2)$, estimate the unknown parameters using pseudolikelihood. The residuals need to be corrected by $E(\eta_i) = \sqrt{2/\pi}\sigma_\eta)$. This produces estimates of persistent (time-constant) technical inefficiency using the Jondrow et al. (1982) estimator.

Once time-varying and persistent technical efficiency have been calculated, RTE and PTE, respectively, overall technical efficiency, OTE, can be calculated as $OTE = RTE \times PTE$. This procedure could be modified with alternative distributional assumptions, such as Truncated Normal. This would simply involve changing the means by which the residuals need to be augmented by in the likelihood estimation. See Andor & Parmeter (2017) for a detailed account of plug-in likelihood estimation of the stochastic frontier panel data model.

3.1.2 Full Likelihood Estimation

Full likelihood based estimation of the four component model is derived in Colombi (2010). The full log likelihood is:

$$\ln L_F(\alpha_0, \beta, \Sigma, A, V, \Lambda) = \sum_{i=1}^{N} \Big[\ln \phi_T \Big(y_i - X_i\beta, 1_T\alpha_0, \Sigma + AVA' \Big) + \ln \bar{\Phi}_{T+1} \Big(R(y_i - X_i\beta - 1_T\alpha_0), \Lambda \Big) \Big], \quad (3)$$

where ϕ_T is the density for a $1 \times T$ normal random vector with mean $1_T\alpha_0$ and variance $\Sigma + AVA'$ and $\bar{\Phi}_{T+1}$ is the probability that a $1 \times (T+1)$ normal random vector with mean $R(y_i - X_i\beta - 1_T\alpha_0)$ and variance Λ is part of the positive orthant. Furthermore, $A = -[1_T I_T]$, $V = \begin{pmatrix} \sigma_\eta^2 & 0_T' \\ 0_T & \sigma_u^2 I_T \end{pmatrix}$, $\Sigma = \sigma_v^2 + \sigma_\mu^2 1_T 1_T'$, $\Lambda = (V^{-1} + A'\Sigma^{-1}A)^{-1}$, and $R = \Lambda A \Sigma^{-1}$. Estimation of $\ln L_F(\alpha_0, \beta, \Sigma, A, V, \Lambda)$ is complicated by the linear growth in integrals with T.

Technical efficiency estimation for the FML four-component model follows Colombi (2010):

$$\mathbb{E}(\exp(t \cdot (\underbrace{\eta_i}_{\text{persistent TE}}, \underbrace{u_{i1}, \ldots, u_{iT}}_{\text{transient TE}})')|\epsilon_i) = \left[\frac{\bar{\Phi}_{T+1}(R\hat{\epsilon}_i + \Lambda t', \Lambda)}{\bar{\Phi}_{T+1}(R\hat{\epsilon}_i, \Lambda)}\right] \exp(tR\hat{\epsilon}_i + \frac{1}{2}t\Lambda t'), \quad (4)$$

for $i = 1, \ldots, N$, $t = (-1, 0, \ldots, 0), (0, -1, 0, \ldots, 0), \cdots, (0, \ldots, 0, -1)$, and R, Λ, A, Σ and V, as defined in Equation (3). Again, $\bar{\Phi}_{T+1}(x, \Sigma_x)$ is the joint probability that a $T+1$-order normal vector is in the non-negative orthant with mean x and variance Σ_x. Further, η_i represents persistent technical efficiency (TE), u_{it} represents transient TE, and their product is overall TE $(\eta_1 \cdot u_{1,1} \ldots \eta_1 \cdot u_{1,T}, \eta_2 \cdot u_{2,1} \ldots \eta_1 \cdot u_{2,T}, \ldots \ldots \eta_N \cdot u_{N,1} \ldots \eta_1 \cdot u_{N,T})$, after being exponentiated following Equation (4). The computation of Equation 4 uses Multivariate Normal integration. This integration is implemented with the `ptmvnorm` call in the `tmvtnorm` package Wilhelm & Manjunath (2015) in `R`.

3.1.3 Maximum Simulated Likelihood Estimation

Maximum simulated likelihood estimation of the four component model uses a simulated log likelihood to remove the individual effects. Following Filippini & Greene (2016), the simulated log likelihood:

$$\ln L_S(\beta, \lambda, \sigma, \sigma_\mu, \sigma_\eta) = \sum_{i=1}^{N} \ln \left[\frac{1}{R} \sum_{r=1}^{R} \left\{ \prod_{t=1}^{T} \left[\frac{2}{\sigma} \phi\Big((y_{it} - \beta' x_{it} - \sigma_\mu \mu_{ir} - \sigma_\eta |\eta_{ir}|)/\sigma \Big) \cdot \Phi\Big(-(y_{it} - \beta' x_{it} - \sigma_\mu \mu_{ir} - \sigma_\eta |\eta_{ir}|)\lambda/\sigma \Big) \right] \right\} \right], \quad (5)$$

is deployed. The summation over R replaces the integral in the marginal (unconditional) density of ε_{it}, given this integral has no closed form. By the law of large numbers, μ_i and η_i can be removed Train (2009), thus revealing the standard Normal-Half Normal density, where $\phi(z)$ and $\Phi(z)$ represent the Normal PDF and CDF, respectively. In practice, the simulated values for μ_i and η_i are given by Halton draws as to provide the best available approximation for the variables of integration. The usual parameterization $\lambda = \sigma_u/\sigma_v$ and $\sigma = (\sigma_v^2 + \sigma_u^2)^{1/2}$ is followed, where σ_u and σ_v are easily recovered.

Estimation of the two inefficiency components follows Equation 4 as discussed in Section 3.1.2. Filippini & Greene (2016) adopt this methodology from Colombi (2010).

3.2 Meta-Analysis

A challenge with drawing broad conclusions from any Monte Carlo analysis is the dependency upon the choices made by the analysts. In order to quickly synthesize the full set of simulation results and allow for broader comparison across estimators, we adapt Banzhaf & Smith's (2007) approach to "internal" meta-analysis. Meta-analysis has traditionally provided a simple and informative way to summarize results across a large number of distinct empirical studies that investigate a common problem.

However, method-analysis can also be used in the context of a simulation study to summarize how subjective modeling decisions influence statistical outcomes. Our approach builds on the standard multi-level meta-regression model (MRM), which is recommended for meta-analysis of this type of metadata (Nelson & Kennedy 2009). Multi-level models allow for within-study correlation across the different observations of the statistical objective (Bateman & Jones 2003, Johnston et al. 2005). The benchmark multi-level MRM which estimates the impact of methodological characteristics ($\bar{x}_{js}$) on reported measures of say mean squared error (or absolute bias) $\bar{y}_{js}$ for estimate s from simulation j is:

$$\bar{y}_{js} = \bar{x}'_{js}\beta + u_s + \varepsilon_{js}. \tag{6}$$

Here, u_s captures systemic setting level effects and ε_{js} is a standard *iid* observation specific error with constant variance. Clustering by study is standard in the meta regression literature, though alternative clustering strategies may be deployed (clustering by author or region, for example). β is a vector of parameters to be estimated to discern the impact that modelling decisions (e.g. sample size) have on estimated statistical performance. We assume that $E(u_s) = 0$ and $Var(u_s) = \sigma_u^2$. Since we have multiple estimators that we are comparing, we estimate Equation (6) for each of our three estimators, allowing correlation across ε_{js}. Hence, we estimate seemingly unrelated regressions (Zellner 1962).

Finally, another twist that we add to the internal meta-analysis literature is the estimation of a multinomial logit model

$$Pr(y_j = r) = \frac{\exp\left(x_j'\beta_r\right)}{1 + \sum_{r=1}^{3} \exp\left(x_j'\beta_r\right)},\ r = 1,\ 2,\ 3, \tag{7}$$

where y is the rank of the performance of the three distinct estimators. Hence, the dependent variable in Equation (7) is the probability that estimator r performs best in a specific evaluation criterion, such as minimizing the mean squared error or absolute bias. A benefit of this approach is that rather than focus on the impact of a modeling decision on the absolute performance of a given method, we can instead turn attention on the impact that modelling decisions have on the relative performance of the three different estimators.

4 Monte Carlo Simulation

While the generalized panel data stochastic frontier estimator is beginning to see wide spread adoption in practice, there has yet to be a formal comparison across the three different estimation approaches. This is important for a variety of reasons. First, the plug-in likelihood approach can be implemented in a straightforward manner in any language that supports basic matrix manipulation but is inefficient relative to maximum likelihood. How inefficient it is, is unknown. In the cross-sectional setting, both Coelli (1995) and Andor & Parmeter (2017) have found that sequential methods do not perform that poorly relative to maximum likelihood. A notable downside to the plug-in/sequential methods are the fact that there cannot be determinants of inefficiency present. Thus, the full blown maximum likelihood or simulated maximum likelihood approach is more attractive from that angle. One practical issue with simulated maximum likelihood is the number of draws to take to perform the averaging, which maximum likelihood does not suffer from. Again, little evidence in the context of the generalized panel data stochastic frontier model exists on this front.

Thus, we now describe a detailed set of simulations that follow Badunenko & Kumbhakar (2016) but allow for a more robust comparison of the three alternative approaches to estimating the generalized panel data stochastic frontier model.

4.1 The Data Generating Process

For comparability we follow the simulation framework in Badunenko & Kumbhakar (2016). We construct a Cobb-Douglas production frontier with two inputs, X_1 and X_2, $Y = e^{\alpha_0} X_1^{\alpha_1} X_2^{1-\alpha_1}$. In this case our production frontier possesses constant returns to scale. Following Badunenko & Kumbhakar (2016) we set $\alpha_0 = 0.3$ and $\alpha_1 = 0.4$.

The error component, ε_{it} is distributed normally with variance σ_v^2, i.e. $v_{it} \sim N(0, \sigma_v^2)$. Firm level heterogeneity, μ_i is distributed normally as well, with mean 0 and variance σ_μ^2. For technical inefficiency, we assume that the persistent component, η_i is distributed half-normal, with pre-truncation variance σ_η^2 and the time-varying component, u_{it}, is distributed half-normal, with pre-truncation variance σ_u^2. The overall production process is

$$Y_{it} = e^{\alpha_0} X_1^{\alpha_1} X_2^{1-\alpha_1} e^{\mu_i - \eta_i + v_{it} - u_{it}}. \tag{8}$$

We follow the 16 different scenarios in Badunenko & Kumbhakar (2016, Table 1) for the values of $\{\sigma^2_\mu, \sigma^2_\eta, \sigma^2_u, \sigma^2_v\}$. All experiments consist of 1,000 Monte Carlo trials and for each experiment we consider n =50, 100 and 500 with t =3, 6 and 10. X_1 and X_2 are generated from truncated exponential distributions with truncation $\log(2)$ and $\log(10)$, respectively. The data are generated following Equation (8) and then are subsequently logged prior to considering each of our three different estimators of the four component model (plug-in likelihood, full maximum likelihood, and simulated maximum likelihood). We compare the bias and mean square error of the estimates of the overall intercept, the two slope coefficients, and the four variance components across the three methods as well as the run time.[2]

Badunenko & Kumbhakar's (2016) main focus was on how well, and in which relative variance settings, did the simulated maximum likelihood estimator estimate persistent, time-varying and overall technical efficiency. Our focus is slightly different, paying attention to how well the variance parameters are estimated across three seemingly different estimators of the four component model. The plug-in model is conceptually the easiest to implement and deploy, and so if our results here dictate that its performance is similar to the more advanced methods, this may help to usher in its use in empirical work.

4.2 Performance

Our first basic comparison between the three methods is the time that they take to run. Figure 1 presents the run time (in the logarithm of minutes) across the three estimators.[3] A simple, expected result emerges, the sequential estimators (SEQ) is orders of magnitude faster to implement than either full maximum likelihood (FML) or simulated maximum likelihood (SML). Further, SML is an order of magnitude faster than

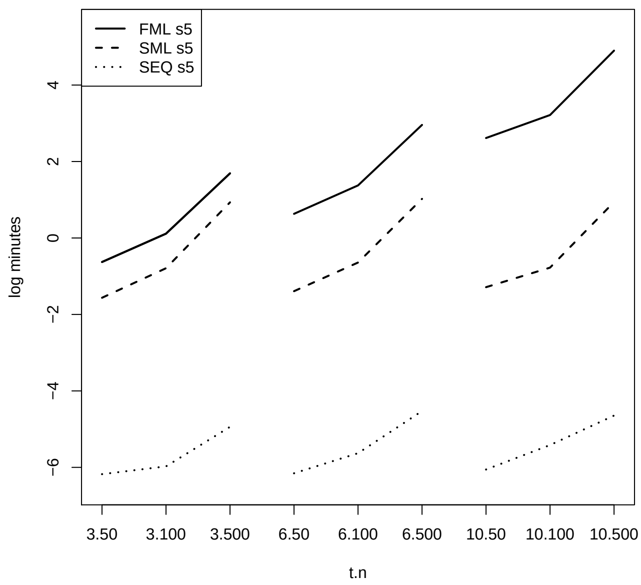
[2] All computations were done in the statistical language `R`. The user written function `psfm()` (available from: `https://github.com/davidhbernstein/sfm`) is used for both simulated and full maximum likelihood while the plug-in estimator used generic matrix oriented programming. All codes are available upon request.

[3] Given the three different estimators across 16 different scenarios it is likely to blur the relative performance, Figure 1 presents the timing runs for scenario 5. The full results can be found in Figure A1.

FML. The relative time differences between FML and SML is due to how the integrals inside the likelihood function are treated as both methods are still performing nonlinear optimization over the same number of parameters.

We immediately see that the plug-in estimator runs orders of magnitude faster than either full maximum likelihood estimation or through simulated maximum likelihood.

Figure 1: Timing for Scenario 5 across all three estimators.



Aside from any issues in the time it takes the estimators to produce results, we now focus on the quality and accuracy of those results relative to the truth. Table 2 presents results on relative bias (R Bias), upward bias (U Bias), correlation (Corr), and mean squared error (MSE) of the three different estimators for scenario 5 (s5), where $\lambda_0 = 1$, $\lambda = 0.2$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$. Table 2 replicates Badunenko & Kumbhakar (2016, Table 4) for SML estimation of s5, and makes the equivalent computations for the FML and SEQ estimation methods. For the full results of all scenarios and the three estimators, see Tables A1 to A9 in the Appendix.

It is important to understand how best to view the results in the table. Take the MSE for example. For persistent technical efficiency, adding more observations in the n dimension should not necessarily lead to an improvement in MSE since we are adding more 'firms' over which to estimate persistent technical efficiency. The requisite com-

Table 2: Finite sample performance of the technical efficiency estimates using FML, SML and SEQ for s5: $\lambda_0 = 1$, $\lambda = 0.2$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$

		Persistent TE				Transient TE				Overall TE			
n	t	R Bias	U Bias	Corr	MSE	R Bias	U Bias	Corr	MSE	R Bias	U Bias	Corr	MSE
s5 (FML)													
50	3	1.17e-02	0.73	0.16	1.80e-03	-3.33e-02	0.48	0.10	7.22e-03	-2.22e-02	0.45	0.16	7.50e-03
100	3	1.34e-02	0.73	0.16	1.54e-03	-2.60e-02	0.50	0.10	5.56e-03	-1.32e-02	0.47	0.16	5.83e-03
500	3	1.37e-02	0.69	0.18	1.10e-03	-1.33e-02	0.51	0.10	3.02e-03	1.06e-04	0.51	0.16	3.48e-03
50	6	7.20e-03	0.66	0.22	1.68e-03	-2.61e-02	0.50	0.11	5.54e-03	-1.93e-02	0.44	0.18	6.12e-03
100	6	6.31e-03	0.63	0.23	1.50e-03	-2.25e-02	0.48	0.11	4.37e-03	-1.65e-02	0.43	0.18	4.91e-03
500	6	7.23e-03	0.62	0.24	1.22e-03	-1.09e-02	0.50	0.11	2.56e-03	-3.92e-03	0.47	0.18	3.19e-03
50	10	1.08e-03	0.58	0.27	1.67e-03	-1.91e-02	0.52	0.11	4.30e-03	-1.81e-02	0.43	0.19	5.36e-03
100	10	2.49e-03	0.58	0.28	1.49e-03	-1.59e-02	0.51	0.11	3.50e-03	-1.35e-02	0.44	0.20	4.47e-03
500	10	1.62e-03	0.53	0.29	1.22e-03	-6.71e-03	0.51	0.11	2.07e-03	-5.17e-03	0.46	0.20	2.95e-03
s5 (SML)													
50	3	4.10e-03	0.64	0.16	2.00e-03	-3.32e-02	0.49	0.10	7.31e-03	-2.93e-02	0.43	0.16	8.33e-03
100	3	8.07e-03	0.67	0.15	1.69e-03	-2.61e-02	0.50	0.10	5.59e-03	-1.81e-02	0.47	0.16	6.60e-03
500	3	5.98e-03	0.60	0.17	1.27e-03	-1.34e-02	0.51	0.10	3.05e-03	-7.35e-03	0.49	0.16	4.13e-03
50	6	4.39e-03	0.62	0.22	1.64e-03	-2.63e-02	0.50	0.11	5.59e-03	-2.21e-02	0.44	0.18	6.45e-03
100	6	3.52e-03	0.58	0.23	1.43e-03	-2.31e-02	0.48	0.11	4.42e-03	-1.97e-02	0.43	0.18	5.33e-03
500	6	4.38e-03	0.58	0.22	1.28e-03	-1.21e-02	0.49	0.11	2.60e-03	-7.76e-03	0.46	0.18	3.53e-03
50	10	1.98e-04	0.55	0.28	1.54e-03	-1.95e-02	0.52	0.11	4.31e-03	-1.93e-02	0.44	0.19	5.36e-03
100	10	1.23e-03	0.55	0.28	1.41e-03	-1.66e-02	0.50	0.11	3.52e-03	-1.54e-02	0.44	0.19	4.50e-03
500	10	2.24e-03	0.55	0.28	1.30e-03	-8.56e-03	0.49	0.11	2.09e-03	-6.35e-03	0.46	0.19	3.13e-03
s5 (SEQ)													
50	3	1.93e-02	0.77	0.17	1.06e-03	-3.29e-02	0.47	0.10	6.83e-03	-1.42e-02	0.50	0.16	6.53e-03
100	3	2.15e-02	0.79	0.17	1.03e-03	-2.64e-02	0.49	0.11	5.42e-03	-5.41e-03	0.54	0.16	5.29e-03
500	3	2.26e-02	0.81	0.18	1.01e-03	-1.51e-02	0.49	0.11	3.10e-03	7.19e-03	0.57	0.16	3.39e-03
50	6	1.94e-02	0.77	0.23	1.04e-03	-2.69e-02	0.49	0.11	5.44e-03	-8.05e-03	0.52	0.17	5.28e-03
100	6	2.02e-02	0.77	0.24	9.95e-04	-2.39e-02	0.47	0.11	4.38e-03	-4.23e-03	0.52	0.18	4.29e-03
500	6	2.06e-02	0.77	0.24	9.41e-04	-1.31e-02	0.48	0.11	2.62e-03	7.27e-03	0.56	0.18	2.91e-03
50	10	1.69e-02	0.73	0.28	1.01e-03	-2.00e-02	0.50	0.11	4.24e-03	-3.44e-03	0.53	0.19	4.44e-03
100	10	1.84e-02	0.74	0.29	9.52e-04	-1.74e-02	0.49	0.11	3.51e-03	6.41e-04	0.54	0.19	3.68e-03
500	10	1.83e-02	0.73	0.29	8.73e-04	-9.44e-03	0.48	0.11	2.12e-03	8.73e-03	0.57	0.19	2.53e-03

parison is for a given n, how does MSE change as t changes. Here we see the expected behavior, MSE decreases as t goes up (for a given n). For transient TE, it is the same comparison, but in this case we can look over n or t. We again see (for all three estimators) that MSE decreases either holding t fixed and increasing n or holding n fixed and increasing t. These two results carry directly over to overall TE as well. Lastly, one might be alarmed that the relative biases reported are all quite small. This is for two reasons. First, all three estimation methods are consistent and are doing a good job estimating the corresponding quantities. Second, because persistent, transient and overall TE are bounded between 0 and 1 and are tightly clustered given the size of the variances used in the simulation design, it is no surprise that the relative errors are small.

For s5, we observe that upward bias is largest for the SEQ method for persistent and overall TE, but similar to FML and SML for transient TE. For all four metrics and all three estimation methodologies, FML and SML appear to be fairly similar. For some

$n - t$ tuples, FML is superior to SML, and vice versa. As $n \cdot t$ increases for s5 persistent TE, upward bias decreases, relative bias decreases for FML and SML, and correlation increases.

As $n \cdot t$ increases for s5 transient TE, relative bias and MSE decrease, upward bias is flat, and correlation is flat. As $n \cdot t$ increases for s5 overall TE, relative bias and MSE decrease, correlation increases, and upward bias increases somewhat (especially for SEQ). An interesting finding for s5 is that although bias tends to suffer from SEQ, correlation performs just as well for SEQ, and sometimes SEQ outperforms FML and SML for correlation.

However, rather than focus on the specific performance of an estimator for one metric or another for a given scenario (of which we have 16), we rather turn to an internal meta-analysis to allow us to draw broader conclusions on the performance of the estimators. Recall that we have run a total of $1,000$ simulations for each of 16 scenarios across nine different (n, t) pairs. With three estimators and nine different outcome metrics, this results in a total of $1,000 \cdot 16 \cdot 9 \cdot 3 \cdot 9 = 3,888,000$ observations to pore over.

With this surfeit of information it is difficult to present visually appealing tables that can concisely display which estimator to select; in essence we lose the forest through the trees. Rather, we turn to, what we view as, a novel approach to large scale Monte Carlo analysis more broadly: an internal meta-analysis. Our hope is that the internal meta-analysis can more easily distill all of these findings into a more easily interpretable set of insights.

4.3 An Internal Meta-Analysis

To get a better understanding of the performance of the three estimators, we conduct two distinct internal meta-analyses. First, we regress our outcomes of interest, namely mean squared error, relative bias, upward bias, and correlation, on the parameters of the simulation process ($N \times t, \lambda,\ \lambda_0,\ \Lambda$), using Equation (6) and data from all iterations. This analysis allows us to identify the contribution of each parameter on the performance of the estimators. Second, we estimate a multinomial logit model as in Equation (7) with the rank of the estimator as the dependent variable to infer their relative overall performance.

Regarding our first internal meta-analysis, we focus on the analysis of MSE and start with overall TE (Table 3). For our purpose, we take the natural logarithm of MSE, such that the coefficients are directly interpretable in semi-elasticities. We find that a larger sample size (N) reduces the MSE of overall TE in each methodology. Using FML on average in the base scenario ($N = 50; t = 3$) the MSE is 0.012 ($exp(-4.402)$). Increasing sample size to N=100 and keeping the panel length t=3 equal reduces MSE by 18%, while N=500 reduces the MSE by almost 50%. The average MSE is comparable for the SML and SEQ estimators. However, the effects of increasing the panel dimensions are somewhat less pronounced. For instance, regarding the SML estimator, the MSE when $N = 100$ and $t = 3$ is reduced by roughly 15%.

The big advantage of an illustration as in Table 3 lies in the straightforward inference. We can easily extract the change in MSE when N or t change instead of comparing numbers as we would need to do in the more traditional way of displaying the results of a Monte Carlo analysis as in Table 2. In addition, Table 2 displays the performance only for one scenario. The concise illustration of the results based on this kind of internal meta-analysis furthermore allows the researcher to easily compare the performance across scenarios. As indicated by the coefficient on λ in Table 3, MSE of overall TE shrinks as the ratio $\frac{\sigma_u}{\sigma_v}$ grows for all estimators. Hence, the more time-variant inefficiency we observe, the lower is the MSE on average. Similarly, a higher $\lambda_0 = \frac{\sigma_\eta}{\sigma_\mu}$ lowers the MSE of overall TE as time-invariant inefficiency increases. The magnitude is very similar across for FML and SML but lower for SEQ. To get the same information from the traditional way of illustrating the results requires drawing information from multiple sources, in this very example it would be based on comparing values in Tables A1–A9.

Table 3: MSE overall technical efficiency

	FML	SML	SEQ
Constant	−4.402 (0.012)	−4.495 (0.012)	−4.516 (0.012)
N=100, t=3	−0.180 (0.011)	−0.137 (0.011)	−0.150 (0.011)
N=500, t=3	−0.495 (0.011)	−0.391 (0.011)	−0.429 (0.011)
N=50, t= 6	−0.158 (0.011)	−0.161 (0.011)	−0.161 (0.011)
N=100, t=6	−0.311 (0.011)	−0.273 (0.011)	−0.297 (0.011)
N=500, t=6	−0.594 (0.011)	−0.502 (0.011)	−0.553 (0.011)
N=50, t=10	−0.221 (0.011)	−0.234 (0.011)	−0.230 (0.011)
N=100, t=10	−0.384 (0.011)	−0.359 (0.011)	−0.385 (0.011)
N=500, t=10	−0.656 (0.011)	−0.558 (0.011)	−0.641 (0.011)
λ_0=1	−0.178 (0.007)	−0.107 (0.007)	−0.170 (0.007)
λ_0=5	−0.785 (0.009)	−0.743 (0.009)	−0.626 (0.009)
λ=1	−0.037 (0.007)	−0.010 (0.007)	0.007 (0.007)
λ=5	−0.414 (0.009)	−0.372 (0.009)	−0.238 (0.009)
Λ=1	−0.106 (0.007)	−0.104 (0.007)	−0.067 (0.007)
Λ=5	0.222 (0.010)	0.258 (0.011)	0.226 (0.010)
R^2	0.147	0.131	0.098
Adj. R^2	0.147	0.131	0.098
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the log of mean squared error of overall technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

In addition to analyzing the determinants of MSE of overall TE, we can repeat the previous analysis with different dependent variables in Equation (6). Tables 4 and 5 display the impact of $N \times t$, λ_0, λ, and Λ on the log of MSE in transient and persistent TE, respectively. Table 4 shows for instance that the reduction in the MSE in transient TE is very similar for FML and SML for increasing the panel dimensions, while the effects are lower for SEQ. For example going from $N = 50$ to $N = 100$, while keeping $t = 6$ reduces the MSE by roughly 15% for FML and SML, while the reduction amounts to 11% in the case of SEQ. Moreover, we detect that across all estimators increasing λ_0 raises the MSE in transient TE substantially compared to the base category of $\lambda = 0.2$. This kind of information is extremely easy to extract from displaying the results as in Table 4. Relying on the more traditional way of displaying results would involve the tedious task of comparing the rows of different tables. Table 5 shows that the MSE in persistent TE is on average much lower across all estimators, amounting to just 0.001 ($exp(-6.314)$). For larger panels, irrespective of increasing N or t, the MSE in persistent TE is further reduced. Merely for larger λ and Λ, the MSE increases.

Last, Table 6 reports how correlation of overall TE levels vary when we vary our parameters of interest. Using FML on average correlation in the base case $(N = 50, t = 3)$ is 0.228. We can increase it by 0.012 when we increase sample size to $N = 100$ and by 0.036 if we increase sample size further to $N = 500$. Similarly, also longer panels increase the correlation in overall TE levels. In addition, an increase in λ_0 and λ also has a postive bearing on the correlation, while increasing Λ result in a lower correlation. The effects are very similar across all three estimators. Tables A10 and A11 in the appendix document the correlation of transient and persistent TE levels, respectively, while Tables A12 to A17 show the results of the internal meta analysis for relative and upward bias.

In the second part of our internal meta-analysis, we aggregate the data of the simulation process by computing means of our evaluation criteria MSE, correlation, relative bias, and upward bias. Subsequently, we identify the best performing estimator of each

Table 4: MSE transient technical efficiency

	FML	SML	SEQ
Constant	−4.202 (0.011)	−4.177 (0.011)	−4.184 (0.011)
N=100, t=3	−0.149 (0.011)	−0.156 (0.011)	−0.114 (0.011)
N=500, t=3	−0.410 (0.011)	−0.405 (0.011)	−0.297 (0.011)
N=50, t= 6	−0.280 (0.011)	−0.298 (0.011)	−0.248 (0.011)
N=100, t=6	−0.404 (0.011)	−0.421 (0.011)	−0.348 (0.011)
N=500, t=6	−0.612 (0.011)	−0.615 (0.011)	−0.502 (0.011)
N=50, t=10	−0.444 (0.011)	−0.473 (0.011)	−0.408 (0.011)
N=100, t=10	−0.545 (0.011)	−0.570 (0.011)	−0.492 (0.011)
N=500, t=10	−0.709 (0.011)	−0.727 (0.011)	−0.630 (0.011)
λ_0=1	0.539 (0.007)	0.534 (0.007)	0.524 (0.007)
λ_0=5	1.170 (0.009)	1.159 (0.009)	1.184 (0.009)
λ=1	−0.683 (0.007)	−0.692 (0.007)	−0.793 (0.007)
λ=5	−1.395 (0.009)	−1.420 (0.009)	−1.255 (0.009)
Λ=1	−1.280 (0.007)	−1.267 (0.007)	−1.302 (0.007)
Λ=5	−2.605 (0.010)	−2.585 (0.010)	−2.601 (0.010)
R^2	0.344	0.342	0.335
Adj. R^2	0.344	0.342	0.335
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the log of mean squared error of transient technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

Table 5: MSE of persistent technical efficiency

	FML	SML	SEQ
Constant	−6.314 (0.013)	−6.433 (0.013)	−6.519 (0.013)
N=100, t=3	−0.131 (0.012)	−0.079 (0.013)	−0.091 (0.013)
N=500, t=3	−0.417 (0.012)	−0.263 (0.013)	−0.283 (0.013)
N=50, t= 6	−0.090 (0.012)	−0.115 (0.013)	−0.080 (0.013)
N=100, t=6	−0.220 (0.012)	−0.200 (0.013)	−0.191 (0.013)
N=500, t=6	−0.499 (0.012)	−0.377 (0.013)	−0.415 (0.013)
N=50, t=10	−0.125 (0.012)	−0.168 (0.013)	−0.124 (0.013)
N=100, t=10	−0.287 (0.012)	−0.282 (0.013)	−0.272 (0.013)
N=500, t=10	−0.559 (0.012)	−0.432 (0.013)	−0.511 (0.013)
λ_0=1	−0.467 (0.008)	−0.363 (0.008)	−0.611 (0.008)
λ_0=5	−1.210 (0.010)	−1.141 (0.010)	−1.056 (0.010)
λ=1	0.314 (0.008)	0.364 (0.008)	0.382 (0.008)
λ=5	0.770 (0.010)	0.864 (0.010)	0.888 (0.010)
Λ=1	1.140 (0.008)	1.107 (0.008)	1.259 (0.008)
Λ=5	2.110 (0.012)	2.135 (0.012)	2.295 (0.012)
R^2	0.214	0.200	0.224
Adj. R^2	0.214	0.199	0.224
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the log of mean squared error of persistent technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

Table 6: Correlation of overall technical efficiency

	FML	SML	SEQ
Constant	0.228 (0.001)	0.228 (0.001)	0.219 (0.001)
N=100, t=3	0.012 (0.001)	0.011 (0.001)	0.011 (0.001)
N=500, t=3	0.036 (0.001)	0.032 (0.001)	0.031 (0.001)
N=50, t= 6	0.019 (0.001)	0.020 (0.001)	0.018 (0.001)
N=100, t=6	0.029 (0.001)	0.028 (0.001)	0.028 (0.001)
N=500, t=6	0.055 (0.001)	0.051 (0.001)	0.053 (0.001)
N=50, t=10	0.028 (0.001)	0.030 (0.001)	0.029 (0.001)
N=100, t=10	0.041 (0.001)	0.041 (0.001)	0.041 (0.001)
N=500, t=10	0.066 (0.001)	0.063 (0.001)	0.066 (0.001)
λ_0=1	0.139 (0.001)	0.134 (0.001)	0.148 (0.001)
λ_0=5	0.454 (0.001)	0.456 (0.001)	0.458 (0.001)
λ=1	0.154 (0.001)	0.156 (0.001)	0.157 (0.001)
λ=5	0.470 (0.001)	0.473 (0.001)	0.459 (0.001)
Λ=1	−0.173 (0.001)	−0.175 (0.001)	−0.171 (0.001)
Λ=5	−0.016 (0.001)	−0.024 (0.001)	−0.021 (0.001)
R^2	0.853	0.844	0.847
Adj. R^2	0.853	0.844	0.846
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the correlation of overall technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

setting (Tables A1–A9) and estimate a multinomial logit model as in Equation (7), including the same covariates as in the previous analysis as regressors and calculate their marginal effects.[4]

Tables A18–A21 in the Appendix show the marginal effects of multinomial logit models for the MSE, correlation, relative bias, and upward bias, respectively. The main advantage compared to the previous analysis is that it allows us to easily learn about the drivers of the relative ranking and to determine the relative performance of the three estimators. For instance, increasing the sample size from $N = 50$ to $N = 100$ increases the probability that FML is the best performing estimator in terms of relative bias in transient TE by 12.5 percentage points (Table A20). Increasing the sample size even further to $N = 500$ raises the probability by almost 30 percentage points. Similarly, the marginal effects displayed in Table A20 indicate that the probability that SEQ performs best also increases with sample size, although they are not statistically different from zero. In turn, a larger N reduces the probability that SML performs best. To be precise, the probability drops by a striking 50 percentage points if $N = 500$. In contrast, its probability of being ranked first increases substantially when $\lambda_0 = 5$, i.e. when $\sigma_{u0} = 0.02$ and $\sigma_{v0} = 0.2$. This illustration is extremely useful to determine the relative rankings of different estimators and is one of the key advantages of displaying the results in this manner compared to the more traditional way of illustrating the results of Monte Carlo simulations.

Finally, we can summarize our findings by calculating the predicted probabilities that a particular estimator performs best, i.e. the mean predicted probabilities of being ranked first (Table 7). For instance, we detect that in about 70% of the settings, SML yields the lowest relative bias in overall TE, while in the remaining settings, FML performs best. Hence, if scholars or regulators are interested in avoiding relative bias in overall TE, we recommend using the SML procedure. The results look very similar when focusing on relative bias in transient TE, while all three methods avoid relative

[4]For the sake of compactness, we do not include the interaction between N and T, as the calculation of marginal effects of interactions in nonlinear models is cumbersome Ai & Norton (2003).

bias in persistent TE equally frequently. In terms of correlation, the SEQ methods performs relatively well compared to FML and SML. Finally, focusing on the MSE, we detect that SEQ performs best when it comes to both transient and persistent TE. In turn, when focusing on overall TE, FML and SEQ have the same probability to perform best. Hence, practitioners who are interested in reducing overall TE are recommended to use SEQ, as it is computationally easier.

Table 7: Mean predicted probabilities of performing best

	FML	SML	SEQ
Transient TE			
MSE	0.263	0.118	0.618
Relative bias	0.201	0.722	0.076
Upward bias	0.181	0.722	0.097
Correlation	0.181	0.403	0.417
Persistent TE			
MSE	0.298	0.160	0.542
Relative bias	0.375	0.313	0.313
Upward bias	0.341	0.299	0.361
Correlation	0.347	0.479	0.174
Overall TE			
MSE	0.431	0.139	0.431
Relative bias	0.299	0.701	0.000
Upward bias	0.292	0.708	0.000
Correlation	0.375	0.271	0.354

Note: The table displays the predicted probabilities that a particular estimator performs best. The values in each row sum up to unity.

4.4 Limitations of this MC simulation

There are a few caveats to our MC study. First, we have not analyzed or discussed the setting where dynamics might enter into the model. Second, we have left a full accounting of exogenous determinants of inefficiency out given that the plug-in methods are known to lack the ability to handle this important case. Third, one useful recommendation of our work is that even if the researcher may wish to use maximum likelihood, particular datasets and the production structure the analyst wishes to estimate may lead to convergence problems. Here the plug-in method at a minimum can shed light on potential good starting values and be of use as a check on the quality of the simulated or full maximum likelihood setup.

5 Conclusions

Monte Carlo simulations are often used to evaluate the performance of methods as a statistical referee. They are usually used for three purposes: (i) to compare statistical methods, (ii) to identify factors influencing the performance of methods, and (iii) to show that newly developed methods produce meaningful and better results than conventional methods. While Monte Carlo simulations are internally valid and the best and most objective methods for such purposes, they are to some extent arbitrary.

In this paper, we have introduced "internal" meta-analyses for Monte Carlo simulations, which enable a much more standardized procedure and comprehensive analysis of a large variety and number of simulations. This in turn allows clear guidelines to be set for Monte Carlo simulations as to which scenarios need to be considered to allow for a fair and transparent comparison.

To exemplify the procedure, we conducted a detailed set of Monte Carlo simulations to compare three competing estimators for the generalized stochastic frontier panel data model. Among others, we found that plug-in likelihood, full maximum likelihood and simulated maximum likelihood all work comparatively well, with simulated maximum likelihood having the majority of cases as the best estimator. Beyond the contributions of the specific Monte Carlo study results to the efficiency analysis literature, the application of the internal meta-analysis in Monte Carlos simulation studies is new to the operational research and efficiency community and could prove useful far beyond the specific setting here.

In our meta-analysis, we have presented different ways of displaying the results of Monte Carlo simulations. The challenge is to find a concise presentation because of the sheer amount of possible results that is owed to the number of parameters and assumptions that can be chosen in a Monte Carlo simulation. The exact way of showing the results depends on the researcher's or applicant's preference. For instance, an aggregate summary of overall performance can be easiest displayed by predicting the probabilities of a multinomial logit model. In contrast, if one is interested in understanding the determinants that drive a particular evaluation criterion, running a meta-regression would be preferable.

The use of internal meta-analyses in Monte Carlo simulations is an important step towards standardizing the so-called statistical referee. This allows a large number of scenarios to be taken into account and the structured presentation of the results is still possible in a meaningful way. In principle, it is additionally only necessary to agree on essential assumptions that should generally be taken into account. These could be taken from existing, well-known studies (for our case, for example, we referred to Badunenko & Kumbhakar 2016) or the main scientific societies provide recommendations for the specific contexts. With these two elements – clear guidelines and internal meta-analyses – future Monte Carlo studies would be standardized, fair and transparent.

Supplemental Material

Additional results can be found in the accompanying supplemental material files.

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Online Appendix

Internal Meta-Analysis for Monte Carlo Simulations

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JEL Classification: C1, C15.

Table A1: Finite sample performance of the technical efficiency estimates using FML: $\sigma_\eta < \sigma_\mu$, where $\lambda_0 = 0.2$ (BK Table 3)

		Persistent TE			Transient TE			Overall TE		
n	t	R Bias	U Bias	Corr	R Bias	U Bias	Corr	R Bias	U Bias	Corr
s13: $\lambda_0 = 0.2$, $\lambda = 0.2$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.2$, $\sigma_v = 0.2$)										
50	3	-4.04e-02	0.53	0.082	-3.21e-02	0.53	0.086	-7.05e-02	0.37	0.099
100	3	-3.61e-02	0.52	0.077	-2.68e-02	0.51	0.092	-6.16e-02	0.36	0.098
500	3	-2.80e-02	0.47	0.085	-1.24e-02	0.53	0.089	-3.97e-02	0.38	0.1
50	6	-4.85e-02	0.5	0.085	-2.38e-02	0.52	0.1	-7.08e-02	0.34	0.096
100	6	-4.16e-02	0.48	0.084	-1.90e-02	0.52	0.1	-5.97e-02	0.34	0.095
500	6	-2.47e-02	0.49	0.088	-1.02e-02	0.52	0.097	-3.44e-02	0.38	0.096
50	10	-4.91e-02	0.48	0.09	-1.56e-02	0.56	0.11	-6.39e-02	0.34	0.097
100	10	-3.83e-02	0.49	0.091	-1.39e-02	0.54	0.1	-5.17e-02	0.35	0.094
500	10	-2.63e-02	0.48	0.094	-5.37e-03	0.54	0.1	-3.14e-02	0.38	0.091
s14: $\lambda_0 = 0.2$, $\lambda = 1$, $\Lambda = 0.2$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.02$, $\sigma_\mu = 0.2$, $\sigma_v = 0.2$)										
50	3	-4.03e-02	0.53	0.073	6.16e-02	0.63	0.42	1.95e-02	0.52	0.36
100	3	-3.74e-02	0.51	0.067	5.18e-02	0.59	0.43	1.28e-02	0.49	0.38
500	3	-2.75e-02	0.44	0.065	3.59e-02	0.53	0.44	7.24e-03	0.46	0.4
50	6	-4.69e-02	0.51	0.069	4.77e-02	0.58	0.47	-1.54e-03	0.47	0.36
100	6	-4.03e-02	0.49	0.064	3.79e-02	0.54	0.47	-4.01e-03	0.45	0.38
500	6	-2.26e-02	0.49	0.061	1.98e-02	0.48	0.47	-3.14e-03	0.43	0.43
50	10	-4.64e-02	0.49	0.077	4.07e-02	0.55	0.48	-7.75e-03	0.45	0.38
100	10	-3.64e-02	0.5	0.062	2.80e-02	0.51	0.48	-9.48e-03	0.43	0.4
500	10	-2.26e-02	0.5	0.062	1.69e-02	0.48	0.48	-6.02e-03	0.42	0.44
s9: $\lambda_0 = 0.2$, $\lambda = 1$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.2$, $\sigma_v = 0.04$)										
50	3	-3.83e-02	0.52	0.087	1.15e-02	0.66	0.36	-2.70e-02	0.5	0.2
100	3	-3.10e-02	0.52	0.08	9.79e-03	0.62	0.36	-2.13e-02	0.5	0.2
500	3	-2.39e-02	0.47	0.074	7.93e-03	0.57	0.36	-1.62e-02	0.46	0.21
50	6	-4.35e-02	0.49	0.074	8.39e-03	0.59	0.41	-3.55e-02	0.46	0.21
100	6	-3.49e-02	0.5	0.07	6.20e-03	0.55	0.41	-2.89e-02	0.46	0.22
500	6	-2.15e-02	0.5	0.065	2.76e-03	0.48	0.41	-1.88e-02	0.45	0.24
50	10	-4.60e-02	0.47	0.08	5.92e-03	0.54	0.44	-4.03e-02	0.43	0.22
100	10	-3.54e-02	0.49	0.064	3.86e-03	0.5	0.43	-3.17e-02	0.45	0.23
500	10	-2.29e-02	0.49	0.064	2.21e-03	0.47	0.43	-2.07e-02	0.44	0.24
s10: $\lambda_0 = 0.2$, $\lambda = 5$, $\Lambda = 0.2$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.2$, $\sigma_\mu = 0.2$, $\sigma_v = 0.04$)										
50	3	-3.90e-02	0.51	0.061	2.75e-02	0.55	0.77	-1.22e-02	0.46	0.69
100	3	-3.12e-02	0.51	0.06	1.71e-02	0.51	0.79	-1.44e-02	0.44	0.73
500	3	-2.46e-02	0.43	0.076	1.25e-02	0.5	0.79	-1.24e-02	0.41	0.76
50	6	-4.09e-02	0.52	0.056	1.50e-02	0.53	0.84	-2.60e-02	0.44	0.75
100	6	-3.39e-02	0.51	0.058	1.46e-02	0.53	0.85	-1.94e-02	0.44	0.77
500	6	-2.11e-02	0.49	0.063	1.48e-02	0.53	0.84	-6.37e-03	0.46	0.8
50	10	-4.33e-02	0.5	0.064	1.53e-02	0.54	0.87	-2.80e-02	0.44	0.76
100	10	-3.35e-02	0.51	0.053	1.55e-02	0.54	0.86	-1.80e-02	0.46	0.79
500	10	-2.14e-02	0.49	0.062	1.64e-02	0.55	0.86	-5.04e-03	0.47	0.82

Table A2: Finite sample performance of the technical efficiency estimates using SML: $\sigma_\eta < \sigma_\mu$, where $\lambda_0 = 0.2$ (BK Table 3)

		Persistent TE			Transient TE			Overall TE		
n	t	R Bias	U Bias	Corr	R Bias	U Bias	Corr	R Bias	U Bias	Corr
s13: $\lambda_0 = 0.2$, $\lambda = 0.2$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.2$, $\sigma_v = 0.2$)										
50	3	-5.44e-02	0.34	0.09	-3.12e-02	0.57	0.078	-8.33e-02	0.3	0.096
100	3	-4.39e-02	0.4	0.085	-2.40e-02	0.57	0.085	-6.65e-02	0.34	0.096
500	3	-2.76e-02	0.53	0.083	-9.65e-03	0.58	0.086	-3.68e-02	0.41	0.096
50	6	-5.55e-02	0.35	0.094	-2.28e-02	0.55	0.1	-7.68e-02	0.3	0.094
100	6	-4.38e-02	0.42	0.088	-1.80e-02	0.54	0.099	-6.09e-02	0.33	0.091
500	6	-1.95e-02	0.58	0.082	-9.14e-03	0.54	0.097	-2.82e-02	0.43	0.091
50	10	-5.50e-02	0.37	0.094	-1.54e-02	0.56	0.11	-6.95e-02	0.31	0.091
100	10	-3.81e-02	0.45	0.093	-1.38e-02	0.54	0.1	-5.13e-02	0.35	0.092
500	10	-1.97e-02	0.57	0.085	-5.48e-03	0.53	0.1	-2.50e-02	0.43	0.087
s14: $\lambda_0 = 0.2$, $\lambda = 1$, $\Lambda = 0.2$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.02$, $\sigma_\mu = 0.2$, $\sigma_v = 0.2$)										
50	3	-6.22e-02	0.3	0.086	6.06e-02	0.63	0.39	-4.37e-03	0.47	0.33
100	3	-5.32e-02	0.36	0.074	5.25e-02	0.59	0.41	-3.06e-03	0.46	0.36
500	3	-3.38e-02	0.48	0.062	3.65e-02	0.54	0.44	1.40e-03	0.45	0.4
50	6	-5.98e-02	0.34	0.087	4.83e-02	0.58	0.46	-1.45e-02	0.43	0.35
100	6	-4.45e-02	0.41	0.077	3.79e-02	0.55	0.46	-8.30e-03	0.43	0.39
500	6	-1.95e-02	0.59	0.048	2.03e-02	0.49	0.47	4.88e-04	0.45	0.43
50	10	-5.66e-02	0.35	0.09	4.00e-02	0.55	0.48	-1.91e-02	0.42	0.37
100	10	-3.88e-02	0.45	0.074	2.81e-02	0.51	0.48	-1.18e-02	0.42	0.41
500	10	-1.88e-02	0.59	0.05	1.73e-02	0.48	0.48	-1.76e-03	0.44	0.44
s9: $\lambda_0 = 0.2$, $\lambda = 1$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.2$, $\sigma_v = 0.04$)										
50	3	-4.52e-02	0.41	0.094	1.11e-02	0.66	0.36	-3.43e-02	0.44	0.19
100	3	-3.44e-02	0.49	0.076	9.90e-03	0.62	0.37	-2.46e-02	0.48	0.21
500	3	-2.45e-02	0.53	0.063	1.24e-02	0.64	0.37	-1.24e-02	0.51	0.21
50	6	-4.85e-02	0.43	0.086	8.55e-03	0.59	0.42	-4.03e-02	0.43	0.21
100	6	-3.64e-02	0.49	0.067	6.53e-03	0.55	0.42	-3.01e-02	0.46	0.23
500	6	-1.95e-02	0.56	0.052	5.33e-03	0.52	0.42	-1.42e-02	0.51	0.25
50	10	-4.75e-02	0.43	0.086	6.10e-03	0.54	0.44	-4.17e-02	0.41	0.23
100	10	-3.63e-02	0.48	0.073	4.20e-03	0.51	0.44	-3.22e-02	0.44	0.24
500	10	-2.23e-02	0.52	0.059	3.53e-03	0.49	0.43	-1.89e-02	0.47	0.26
s10: $\lambda_0 = 0.2$, $\lambda = 5$, $\Lambda = 0.2$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.2$, $\sigma_\mu = 0.2$, $\sigma_v = 0.04$)										
50	3	-5.47e-02	0.33	0.087	2.48e-02	0.54	0.78	-3.10e-02	0.39	0.69
100	3	-4.19e-02	0.42	0.069	1.58e-02	0.51	0.79	-2.64e-02	0.4	0.73
500	3	-2.22e-02	0.55	0.056	1.66e-02	0.52	0.79	-5.89e-03	0.46	0.75
50	6	-5.55e-02	0.33	0.082	1.17e-02	0.51	0.85	-4.40e-02	0.35	0.76
100	6	-3.90e-02	0.46	0.067	1.31e-02	0.52	0.85	-2.60e-02	0.42	0.78
500	6	-1.62e-02	0.6	0.049	1.55e-02	0.53	0.85	-6.62e-04	0.5	0.81
50	10	-5.63e-02	0.32	0.091	1.27e-02	0.53	0.88	-4.39e-02	0.36	0.78
100	10	-3.94e-02	0.44	0.067	1.34e-02	0.53	0.87	-2.61e-02	0.42	0.8
500	10	-1.48e-02	0.6	0.044	1.60e-02	0.55	0.87	1.24e-03	0.52	0.83

Table A3: Finite sample performance of the technical efficiency estimates using SEQ: $\sigma_\eta < \sigma_\mu$, where $\lambda_0 = 0.2$ (BK Table 3)

		Persistent TE			Transient TE			Overall TE		
n	t	R Bias	U Bias	Corr	R Bias	U Bias	Corr	R Bias	U Bias	Corr
s13: $\lambda_0 = 0.2$, $\lambda = 0.2$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.2$, $\sigma_v = 0.2$)										
50	3	-2.94e-02	0.53	0.078	-2.79e-02	0.51	0.087	-5.52e-02	0.39	0.1
100	3	-2.53e-02	0.52	0.076	-2.48e-02	0.49	0.093	-4.85e-02	0.38	0.1
500	3	-1.91e-02	0.48	0.086	-1.25e-02	0.51	0.09	-3.06e-02	0.4	0.1
50	6	-4.03e-02	0.51	0.082	-2.42e-02	0.49	0.1	-6.28e-02	0.35	0.097
100	6	-3.41e-02	0.49	0.081	-2.03e-02	0.48	0.099	-5.31e-02	0.35	0.096
500	6	-1.98e-02	0.49	0.089	-1.02e-02	0.5	0.097	-2.94e-02	0.39	0.097
50	10	-4.37e-02	0.49	0.088	-1.83e-02	0.5	0.11	-6.08e-02	0.34	0.097
100	10	-3.36e-02	0.5	0.089	-1.52e-02	0.5	0.1	-4.80e-02	0.36	0.095
500	10	-2.25e-02	0.49	0.095	-7.34e-03	0.5	0.1	-2.93e-02	0.39	0.092
s14: $\lambda_0 = 0.2$, $\lambda = 1$, $\Lambda = 0.2$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.02$, $\sigma_\mu = 0.2$, $\sigma_v = 0.2$)										
50	3	-2.84e-02	0.53	0.07	7.00e-02	0.65	0.42	4.13e-02	0.56	0.36
100	3	-2.60e-02	0.51	0.067	6.16e-02	0.62	0.43	3.52e-02	0.54	0.38
500	3	-2.21e-02	0.43	0.068	4.83e-02	0.57	0.44	2.56e-02	0.51	0.4
50	6	-3.77e-02	0.52	0.069	5.46e-02	0.6	0.46	1.55e-02	0.5	0.37
100	6	-3.28e-02	0.49	0.067	4.67e-02	0.57	0.46	1.28e-02	0.48	0.38
500	6	-2.05e-02	0.48	0.065	3.31e-02	0.52	0.47	1.22e-02	0.47	0.42
50	10	-4.14e-02	0.49	0.077	4.73e-02	0.57	0.48	4.36e-03	0.47	0.37
100	10	-3.25e-02	0.49	0.069	3.61e-02	0.54	0.48	2.66e-03	0.46	0.4
500	10	-2.15e-02	0.48	0.067	2.64e-02	0.51	0.48	4.48e-03	0.45	0.43
s9: $\lambda_0 = 0.2$, $\lambda = 1$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.2$, $\sigma_v = 0.04$)										
50	3	-3.80e-02	0.52	0.085	1.35e-02	0.67	0.36	-2.47e-02	0.52	0.2
100	3	-3.11e-02	0.52	0.075	1.31e-02	0.66	0.37	-1.81e-02	0.52	0.2
500	3	-2.47e-02	0.47	0.074	1.27e-02	0.64	0.36	-1.22e-02	0.48	0.2
50	6	-4.34e-02	0.5	0.069	9.83e-03	0.6	0.41	-3.39e-02	0.48	0.21
100	6	-3.52e-02	0.5	0.065	8.88e-03	0.58	0.41	-2.65e-02	0.48	0.21
500	6	-2.27e-02	0.49	0.066	7.57e-03	0.56	0.41	-1.52e-02	0.48	0.23
50	10	-4.61e-02	0.47	0.075	7.62e-03	0.56	0.44	-3.87e-02	0.45	0.22
100	10	-3.58e-02	0.49	0.068	6.31e-03	0.54	0.43	-2.97e-02	0.46	0.22
500	10	-2.38e-02	0.48	0.067	5.40e-03	0.52	0.43	-1.85e-02	0.45	0.23
s10: $\lambda_0 = 0.2$, $\lambda = 5$, $\Lambda = 0.2$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.2$, $\sigma_\mu = 0.2$, $\sigma_v = 0.04$)										
50	3	-3.86e-02	0.5	0.065	5.69e-02	0.66	0.76	1.66e-02	0.53	0.64
100	3	-3.28e-02	0.5	0.058	5.06e-02	0.64	0.77	1.64e-02	0.52	0.67
500	3	-3.04e-02	0.4	0.073	4.68e-02	0.63	0.77	1.49e-02	0.51	0.7
50	6	-4.12e-02	0.5	0.059	3.35e-02	0.62	0.84	-8.76e-03	0.49	0.72
100	6	-3.47e-02	0.49	0.062	3.29e-02	0.62	0.84	-2.65e-03	0.49	0.74
500	6	-2.36e-02	0.47	0.068	3.28e-02	0.62	0.83	8.55e-03	0.51	0.78
50	10	-4.42e-02	0.48	0.065	2.69e-02	0.61	0.86	-1.80e-02	0.47	0.75
100	10	-3.43e-02	0.5	0.06	2.66e-02	0.61	0.86	-8.25e-03	0.48	0.77
500	10	-2.33e-02	0.48	0.063	2.72e-02	0.61	0.86	3.53e-03	0.5	0.81

Table A4: Finite sample performance of the technical efficiency estimates using FML: $\sigma_\eta = \sigma_\mu$, where $\lambda_0 = 1$ (BK Table 4)

		Persistent TE			Transient TE			Overall TE		
n	t	R Bias	U Bias	Corr	R Bias	U Bias	Corr	R Bias	U Bias	Corr
s5: $\lambda_0 = 1$, $\lambda = 0.2$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$)										
50	3	1.17e-02	0.73	0.16	-3.33e-02	0.48	0.1	-2.22e-02	0.45	0.16
100	3	1.34e-02	0.73	0.16	-2.60e-02	0.5	0.1	-1.32e-02	0.47	0.16
500	3	1.37e-02	0.69	0.18	-1.33e-02	0.51	0.1	1.06e-04	0.51	0.16
50	6	7.20e-03	0.66	0.22	-2.61e-02	0.5	0.11	-1.93e-02	0.44	0.18
100	6	6.31e-03	0.63	0.23	-2.25e-02	0.48	0.11	-1.65e-02	0.43	0.18
500	6	7.23e-03	0.62	0.24	-1.09e-02	0.5	0.11	-3.92e-03	0.47	0.18
50	10	1.08e-03	0.58	0.27	-1.91e-02	0.52	0.11	-1.81e-02	0.43	0.19
100	10	2.49e-03	0.58	0.28	-1.59e-02	0.51	0.11	-1.35e-02	0.44	0.2
500	10	1.62e-03	0.53	0.29	-6.71e-03	0.51	0.11	-5.17e-03	0.46	0.2
s15: $\lambda_0 = 1$, $\lambda = 0.2$, $\Lambda = 5$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.04$, $\sigma_\mu = 0.2$, $\sigma_v = 0.2$)										
50	3	6.00e-02	0.64	0.39	-3.60e-02	0.49	0.087	2.24e-02	0.52	0.36
100	3	5.16e-02	0.6	0.4	-3.14e-02	0.47	0.092	1.86e-02	0.5	0.37
500	3	3.44e-02	0.53	0.44	-2.06e-02	0.44	0.089	1.32e-02	0.47	0.41
50	6	4.09e-02	0.58	0.42	-2.50e-02	0.51	0.1	1.47e-02	0.5	0.38
100	6	4.69e-02	0.59	0.42	-2.10e-02	0.5	0.1	2.47e-02	0.51	0.38
500	6	3.16e-02	0.52	0.47	-1.30e-02	0.48	0.1	1.83e-02	0.48	0.43
50	10	4.44e-02	0.59	0.44	-1.60e-02	0.55	0.11	2.74e-02	0.53	0.38
100	10	4.61e-02	0.58	0.45	-1.44e-02	0.53	0.1	3.09e-02	0.53	0.39
500	10	2.45e-02	0.5	0.49	-6.36e-03	0.52	0.1	1.80e-02	0.48	0.45
s6: $\lambda_0 = 1$, $\lambda = 1$, $\Lambda = 0.2$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.2$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$)										
50	3	1.12e-02	0.73	0.13	4.38e-02	0.57	0.49	5.49e-02	0.59	0.47
100	3	1.51e-02	0.75	0.13	3.67e-02	0.54	0.5	5.20e-02	0.58	0.49
500	3	1.65e-02	0.71	0.16	1.55e-02	0.47	0.51	3.21e-02	0.53	0.51
50	6	9.18e-03	0.69	0.19	3.66e-02	0.54	0.51	4.57e-02	0.57	0.48
100	6	9.69e-03	0.67	0.2	2.49e-02	0.5	0.51	3.46e-02	0.54	0.49
500	6	1.07e-02	0.65	0.21	1.18e-02	0.46	0.51	2.25e-02	0.5	0.51
50	10	5.01e-03	0.63	0.24	3.09e-02	0.52	0.51	3.59e-02	0.54	0.48
100	10	4.86e-03	0.6	0.24	1.95e-02	0.49	0.51	2.43e-02	0.51	0.5
500	10	4.47e-03	0.57	0.26	1.14e-02	0.46	0.51	1.58e-02	0.48	0.51
s1: $\lambda_0 = 1$, $\lambda = 1$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.04$)										
50	3	7.61e-03	0.61	0.37	7.93e-03	0.6	0.42	1.56e-02	0.63	0.46
100	3	6.62e-03	0.58	0.37	6.33e-03	0.56	0.43	1.29e-02	0.61	0.46
500	3	4.87e-03	0.53	0.43	4.39e-03	0.51	0.44	9.23e-03	0.57	0.48
50	6	4.57e-03	0.57	0.39	6.73e-03	0.56	0.47	1.13e-02	0.59	0.45
100	6	5.84e-03	0.57	0.39	4.75e-03	0.52	0.47	1.06e-02	0.59	0.46
500	6	4.72e-03	0.52	0.46	2.06e-03	0.47	0.48	6.77e-03	0.54	0.49
50	10	5.93e-03	0.58	0.39	5.27e-03	0.53	0.49	1.12e-02	0.6	0.45
100	10	6.49e-03	0.57	0.41	3.28e-03	0.49	0.49	9.78e-03	0.58	0.46
500	10	3.75e-03	0.51	0.49	1.67e-03	0.46	0.49	5.41e-03	0.53	0.49
s16: $\lambda_0 = 1$, $\lambda = 1$, $\Lambda = 1$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.2$, $\sigma_\mu = 0.2$, $\sigma_v = 0.2$)										
50	3	6.13e-02	0.64	0.36	5.95e-02	0.62	0.41	1.24e-01	0.66	0.45
100	3	5.53e-02	0.61	0.35	4.87e-02	0.58	0.42	1.06e-01	0.64	0.45
500	3	4.34e-02	0.56	0.39	3.43e-02	0.53	0.43	7.80e-02	0.6	0.47
50	6	4.25e-02	0.59	0.37	4.93e-02	0.58	0.46	9.31e-02	0.62	0.44
100	6	4.94e-02	0.6	0.35	3.65e-02	0.54	0.46	8.67e-02	0.61	0.45
500	6	4.09e-02	0.55	0.41	1.92e-02	0.48	0.47	6.05e-02	0.57	0.48
50	10	4.80e-02	0.6	0.37	3.98e-02	0.55	0.48	8.90e-02	0.62	0.44
100	10	5.14e-02	0.6	0.37	2.76e-02	0.51	0.48	8.02e-02	0.6	0.45
500	10	3.22e-02	0.53	0.45	1.64e-02	0.48	0.49	4.88e-02	0.55	0.48
s11: $\lambda_0 = 1$, $\lambda = 1$, $\Lambda = 5$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.04$, $\sigma_\mu = 0.2$, $\sigma_v = 0.04$)										
50	3	5.06e-02	0.61	0.42	1.08e-02	0.65	0.37	6.22e-02	0.62	0.41
100	3	4.82e-02	0.59	0.42	8.55e-03	0.6	0.38	5.73e-02	0.6	0.42
500	3	2.73e-02	0.51	0.47	6.40e-03	0.55	0.39	3.39e-02	0.53	0.47
50	6	4.18e-02	0.58	0.41	8.19e-03	0.59	0.42	5.03e-02	0.6	0.4
100	6	5.01e-02	0.6	0.39	5.89e-03	0.54	0.42	5.62e-02	0.6	0.4
500	6	2.98e-02	0.52	0.47	2.45e-03	0.47	0.43	3.23e-02	0.53	0.47
50	10	4.30e-02	0.59	0.38	5.73e-03	0.54	0.44	4.90e-02	0.59	0.39
100	10	4.75e-02	0.59	0.39	3.68e-03	0.5	0.44	5.14e-02	0.59	0.4
500	10	2.52e-02	0.51	0.48	1.94e-03	0.47	0.45	2.71e-02	0.51	0.48
s2: $\lambda_0 = 1$, $\lambda = 5$, $\Lambda = 0.2$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.2$, $\sigma_\mu = 0.04$, $\sigma_v = 0.04$)										
50	3	3.65e-03	0.58	0.26	6.96e-03	0.51	0.9	1.02e-02	0.55	0.9
100	3	4.08e-03	0.57	0.28	4.55e-03	0.5	0.9	8.23e-03	0.54	0.91
500	3	4.96e-03	0.54	0.3	3.45e-03	0.49	0.91	8.07e-03	0.54	0.91
50	6	5.54e-03	0.6	0.32	4.21e-03	0.5	0.91	9.50e-03	0.55	0.91
100	6	4.98e-03	0.58	0.33	4.22e-03	0.5	0.92	8.97e-03	0.55	0.91
500	6	5.40e-03	0.54	0.37	3.70e-03	0.5	0.92	8.88e-03	0.55	0.92
50	10	7.10e-03	0.62	0.34	5.02e-03	0.51	0.93	1.20e-02	0.58	0.92
100	10	6.94e-03	0.6	0.35	4.42e-03	0.51	0.93	1.12e-02	0.57	0.92
500	10	4.64e-03	0.53	0.42	4.17e-03	0.51	0.93	8.66e-03	0.55	0.92
s12: $\lambda_0 = 1$, $\lambda = 5$, $\Lambda = 1$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.2$, $\sigma_\mu = 0.2$, $\sigma_v = 0.04$)										
50	3	5.59e-02	0.63	0.35	2.82e-02	0.55	0.77	8.51e-02	0.64	0.65
100	3	5.62e-02	0.61	0.36	1.62e-02	0.51	0.78	7.25e-02	0.63	0.67
500	3	3.86e-02	0.55	0.45	1.11e-02	0.49	0.8	4.91e-02	0.58	0.69
50	6	4.59e-02	0.6	0.34	1.40e-02	0.52	0.85	6.03e-02	0.6	0.68
100	6	5.24e-02	0.61	0.36	1.34e-02	0.52	0.85	6.62e-02	0.62	0.69
500	6	3.41e-02	0.53	0.46	1.16e-02	0.51	0.86	4.56e-02	0.57	0.71
50	10	4.58e-02	0.6	0.34	1.42e-02	0.54	0.87	6.07e-02	0.6	0.69
100	10	5.02e-02	0.6	0.36	1.38e-02	0.53	0.87	6.46e-02	0.61	0.7
500	10	2.93e-02	0.52	0.48	1.22e-02	0.53	0.88	4.15e-02	0.56	0.72

Table A5: Finite sample performance of the technical efficiency estimates using SML: $\sigma_\eta = \sigma_\mu$, where $\lambda_0 = 1$ (BK Table 4)

		Persistent TE			Transient TE			Overall TE		
n	t	R Bias	U Bias	Corr	R Bias	U Bias	Corr	R Bias	U Bias	Corr
s5: $\lambda_0 = 1, \lambda = 0.2, \Lambda = 1$ ($\sigma_\eta = 0.04, \sigma_u = 0.04, \sigma_\mu = 0.04, \sigma_v = 0.2$)										
50	3	4.10e-03	0.64	0.16	-3.32e-02	0.49	0.099	-2.93e-02	0.43	0.16
100	3	8.07e-03	0.67	0.15	-2.61e-02	0.5	0.1	-1.81e-02	0.47	0.16
500	3	5.98e-03	0.6	0.17	-1.34e-02	0.51	0.1	-7.35e-03	0.49	0.16
50	6	4.39e-03	0.62	0.22	-2.63e-02	0.5	0.11	-2.21e-02	0.44	0.18
100	6	3.52e-03	0.58	0.23	-2.31e-02	0.48	0.11	-1.97e-02	0.43	0.18
500	6	4.38e-03	0.58	0.22	-1.21e-02	0.49	0.11	-7.76e-03	0.46	0.18
50	10	1.98e-04	0.55	0.28	-1.95e-02	0.52	0.11	-1.93e-02	0.44	0.19
100	10	1.23e-03	0.55	0.28	-1.66e-02	0.5	0.11	-1.54e-02	0.44	0.19
500	10	2.24e-03	0.55	0.28	-8.56e-03	0.49	0.11	-6.35e-03	0.46	0.19
s15: $\lambda_0 = 1, \lambda = 0.2, \Lambda = 5$ ($\sigma_\eta = 0.2, \sigma_u = 0.04, \sigma_\mu = 0.2, \sigma_v = 0.2$)										
50	3	5.12e-02	0.61	0.43	-3.31e-02	0.56	0.079	1.69e-02	0.52	0.36
100	3	5.31e-02	0.61	0.41	-2.72e-02	0.54	0.085	2.45e-02	0.52	0.36
500	3	4.17e-02	0.56	0.41	-1.46e-02	0.54	0.084	2.66e-02	0.51	0.37
50	6	3.67e-02	0.57	0.45	-2.37e-02	0.54	0.1	1.21e-02	0.5	0.37
100	6	5.12e-02	0.6	0.43	-1.96e-02	0.53	0.099	3.05e-02	0.53	0.35
500	6	4.11e-02	0.56	0.44	-1.09e-02	0.52	0.099	3.00e-02	0.52	0.38
50	10	4.00e-02	0.58	0.45	-1.60e-02	0.56	0.11	2.33e-02	0.52	0.37
100	10	5.29e-02	0.61	0.45	-1.46e-02	0.53	0.1	3.75e-02	0.55	0.36
500	10	3.33e-02	0.53	0.47	-7.01e-03	0.52	0.1	2.63e-02	0.51	0.41
s6: $\lambda_0 = 1, \lambda = 1, \Lambda = 0.2$ ($\sigma_\eta = 0.04, \sigma_u = 0.2, \sigma_\mu = 0.04, \sigma_v = 0.2$)										
50	3	1.81e-03	0.64	0.13	4.39e-02	0.57	0.48	4.53e-02	0.57	0.47
100	3	4.98e-03	0.65	0.12	3.69e-02	0.54	0.5	4.19e-02	0.56	0.49
500	3	3.38e-03	0.58	0.14	1.51e-02	0.47	0.51	1.84e-02	0.49	0.51
50	6	3.30e-03	0.61	0.19	3.63e-02	0.54	0.5	3.95e-02	0.55	0.48
100	6	4.61e-03	0.61	0.2	2.43e-02	0.5	0.51	2.88e-02	0.52	0.49
500	6	3.28e-03	0.56	0.19	1.19e-02	0.46	0.51	1.51e-02	0.48	0.51
50	10	1.64e-03	0.57	0.25	3.08e-02	0.52	0.51	3.24e-02	0.53	0.48
100	10	1.77e-03	0.55	0.25	1.95e-02	0.49	0.51	2.11e-02	0.5	0.5
500	10	2.22e-03	0.55	0.23	1.15e-02	0.46	0.51	1.36e-02	0.47	0.51
s1: $\lambda_0 = 1, \lambda = 1, \Lambda = 1$ ($\sigma_\eta = 0.04, \sigma_u = 0.04, \sigma_\mu = 0.04, \sigma_v = 0.04$)										
50	3	6.52e-03	0.59	0.38	7.79e-03	0.6	0.42	1.44e-02	0.62	0.45
100	3	6.97e-03	0.59	0.37	6.26e-03	0.56	0.44	1.32e-02	0.61	0.46
500	3	5.65e-03	0.55	0.35	4.56e-03	0.51	0.44	1.02e-02	0.57	0.47
50	6	5.25e-03	0.57	0.4	6.18e-03	0.55	0.47	1.14e-02	0.59	0.45
100	6	6.95e-03	0.59	0.37	4.41e-03	0.52	0.47	1.14e-02	0.6	0.45
500	6	5.49e-03	0.54	0.38	3.52e-03	0.49	0.48	9.02e-03	0.56	0.48
50	10	5.81e-03	0.58	0.41	5.14e-03	0.53	0.49	1.10e-02	0.6	0.45
100	10	8.00e-03	0.6	0.39	3.60e-03	0.5	0.49	1.16e-02	0.6	0.45
500	10	5.35e-03	0.54	0.42	2.84e-03	0.48	0.49	8.20e-03	0.56	0.48
s16: $\lambda_0 = 1, \lambda = 1, \Lambda = 1$ ($\sigma_\eta = 0.2, \sigma_u = 0.2, \sigma_\mu = 0.2, \sigma_v = 0.2$)										
50	3	4.42e-02	0.59	0.41	6.12e-02	0.63	0.39	1.08e-01	0.64	0.44
100	3	4.79e-02	0.59	0.38	5.14e-02	0.59	0.4	1.01e-01	0.63	0.45
500	3	3.94e-02	0.55	0.36	3.69e-02	0.54	0.43	7.66e-02	0.59	0.46
50	6	3.95e-02	0.58	0.42	5.03e-02	0.59	0.46	9.16e-02	0.61	0.44
100	6	4.93e-02	0.6	0.38	3.71e-02	0.54	0.46	8.77e-02	0.61	0.44
500	6	4.54e-02	0.57	0.36	1.97e-02	0.48	0.47	6.55e-02	0.58	0.47
50	10	4.15e-02	0.58	0.41	3.94e-02	0.55	0.48	8.22e-02	0.61	0.44
100	10	5.58e-02	0.62	0.38	2.76e-02	0.51	0.48	8.46e-02	0.61	0.44
500	10	4.01e-02	0.55	0.4	1.67e-02	0.48	0.49	5.72e-02	0.56	0.47
s11: $\lambda_0 = 1, \lambda = 1, \Lambda = 5$ ($\sigma_\eta = 0.2, \sigma_u = 0.04, \sigma_\mu = 0.2, \sigma_v = 0.04$)										
50	3	4.44e-02	0.59	0.44	1.04e-02	0.65	0.37	5.56e-02	0.61	0.41
100	3	4.64e-02	0.58	0.42	9.20e-03	0.61	0.38	5.62e-02	0.6	0.41
500	3	2.51e-02	0.51	0.45	9.29e-03	0.59	0.39	3.47e-02	0.53	0.45
50	6	3.84e-02	0.57	0.43	8.04e-03	0.58	0.43	4.67e-02	0.59	0.4
100	6	5.06e-02	0.6	0.39	6.09e-03	0.54	0.43	5.70e-02	0.61	0.39
500	6	2.48e-02	0.5	0.45	4.68e-03	0.51	0.44	2.96e-02	0.51	0.45
50	10	4.13e-02	0.58	0.41	5.74e-03	0.54	0.45	4.73e-02	0.59	0.39
100	10	4.68e-02	0.58	0.4	4.16e-03	0.51	0.45	5.12e-02	0.59	0.39
500	10	1.83e-02	0.48	0.47	3.21e-03	0.49	0.46	2.16e-02	0.49	0.47
s2: $\lambda_0 = 1, \lambda = 5, \Lambda = 0.2$ ($\sigma_\eta = 0.04, \sigma_u = 0.2, \sigma_\mu = 0.04, \sigma_v = 0.04$)										
50	3	-2.55e-03	0.52	0.24	6.75e-03	0.51	0.89	3.69e-03	0.5	0.9
100	3	-1.90e-03	0.51	0.23	4.70e-03	0.5	0.9	2.34e-03	0.5	0.91
500	3	1.05e-02	0.66	0.18	3.40e-03	0.49	0.91	1.35e-02	0.58	0.91
50	6	2.04e-03	0.55	0.3	3.78e-03	0.5	0.91	5.56e-03	0.53	0.91
100	6	4.66e-03	0.58	0.29	4.26e-03	0.5	0.92	8.67e-03	0.55	0.91
500	6	1.43e-02	0.7	0.22	4.17e-03	0.5	0.92	1.83e-02	0.62	0.92
50	10	4.79e-03	0.58	0.35	4.78e-03	0.51	0.93	9.43e-03	0.56	0.92
100	10	8.27e-03	0.62	0.31	4.36e-03	0.51	0.93	1.25e-02	0.58	0.92
500	10	1.61e-02	0.73	0.22	5.07e-03	0.51	0.93	2.11e-02	0.64	0.92
s12: $\lambda_0 = 1, \lambda = 5, \Lambda = 1$ ($\sigma_\eta = 0.2, \sigma_u = 0.2, \sigma_\mu = 0.2, \sigma_v = 0.04$)										
50	3	4.38e-02	0.59	0.42	2.55e-02	0.54	0.77	6.99e-02	0.62	0.65
100	3	5.03e-02	0.6	0.39	1.46e-02	0.5	0.79	6.48e-02	0.61	0.66
500	3	4.82e-02	0.58	0.39	1.57e-02	0.51	0.79	6.36e-02	0.6	0.68
50	6	3.44e-02	0.56	0.44	1.11e-02	0.51	0.85	4.54e-02	0.58	0.69
100	6	5.32e-02	0.61	0.38	1.25e-02	0.52	0.85	6.60e-02	0.62	0.69
500	6	4.51e-02	0.57	0.39	1.34e-02	0.52	0.86	5.87e-02	0.6	0.7
50	10	3.20e-02	0.56	0.44	1.20e-02	0.53	0.88	4.43e-02	0.58	0.69
100	10	4.63e-02	0.59	0.4	1.25e-02	0.53	0.88	5.92e-02	0.6	0.7
500	10	3.90e-02	0.55	0.4	1.29e-02	0.53	0.88	5.21e-02	0.58	0.7

Table A6: Finite sample performance of the technical efficiency estimates using SEQ: $\sigma_\eta = \sigma_\mu$, where $\lambda_0 = 1$ (BK Table 4)

		Persistent TE			Transient TE			Overall TE		
n	t	R Bias	U Bias	Corr	R Bias	U Bias	Corr	R Bias	U Bias	Corr
s5: $\lambda_0 = 1$, $\lambda = 0.2$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$)										
50	3	1.93e-02	0.77	0.17	-3.29e-02	0.47	0.1	-1.42e-02	0.5	0.16
100	3	2.15e-02	0.79	0.17	-2.64e-02	0.49	0.11	-5.41e-03	0.54	0.16
500	3	2.26e-02	0.81	0.18	-1.51e-02	0.49	0.11	7.19e-03	0.57	0.16
50	6	1.94e-02	0.77	0.23	-2.69e-02	0.49	0.11	-8.05e-03	0.52	0.17
100	6	2.02e-02	0.77	0.24	-2.39e-02	0.47	0.11	-4.23e-03	0.52	0.18
500	6	2.06e-02	0.77	0.24	-1.31e-02	0.48	0.11	7.27e-03	0.56	0.18
50	10	1.69e-02	0.73	0.28	-2.00e-02	0.5	0.11	-3.44e-03	0.53	0.19
100	10	1.84e-02	0.74	0.29	-1.74e-02	0.49	0.11	6.41e-04	0.54	0.19
500	10	1.83e-02	0.73	0.29	-9.44e-03	0.48	0.11	8.73e-03	0.57	0.19
s15: $\lambda_0 = 1$, $\lambda = 0.2$, $\Lambda = 5$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.04$, $\sigma_\mu = 0.2$, $\sigma_v = 0.2$)										
50	3	7.40e-02	0.67	0.39	-3.33e-02	0.48	0.087	3.99e-02	0.56	0.37
100	3	6.78e-02	0.64	0.4	-3.21e-02	0.44	0.093	3.45e-02	0.54	0.38
500	3	5.35e-02	0.59	0.44	-2.49e-02	0.4	0.091	2.79e-02	0.51	0.42
50	6	5.17e-02	0.61	0.41	-2.81e-02	0.46	0.1	2.28e-02	0.52	0.38
100	6	5.71e-02	0.62	0.42	-2.33e-02	0.46	0.1	3.29e-02	0.53	0.38
500	6	4.29e-02	0.56	0.47	-1.66e-02	0.44	0.1	2.61e-02	0.51	0.43
50	10	5.12e-02	0.61	0.43	-1.96e-02	0.49	0.11	3.10e-02	0.54	0.38
100	10	5.27e-02	0.6	0.45	-1.74e-02	0.48	0.1	3.47e-02	0.54	0.39
500	10	3.21e-02	0.53	0.49	-1.13e-02	0.45	0.1	2.07e-02	0.49	0.45
s6: $\lambda_0 = 1$, $\lambda = 1$, $\Lambda = 0.2$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.2$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$)										
50	3	1.78e-02	0.75	0.15	4.61e-02	0.58	0.5	6.46e-02	0.62	0.47
100	3	2.04e-02	0.78	0.15	3.79e-02	0.55	0.51	5.90e-02	0.61	0.48
500	3	2.17e-02	0.79	0.16	1.57e-02	0.47	0.51	3.76e-02	0.55	0.51
50	6	1.91e-02	0.76	0.2	3.73e-02	0.54	0.51	5.70e-02	0.6	0.48
100	6	2.02e-02	0.77	0.21	2.46e-02	0.5	0.51	4.52e-02	0.57	0.5
500	6	2.05e-02	0.77	0.22	1.20e-02	0.46	0.51	3.26e-02	0.54	0.52
50	10	1.81e-02	0.74	0.25	3.15e-02	0.52	0.51	5.01e-02	0.58	0.48
100	10	1.92e-02	0.76	0.26	1.95e-02	0.49	0.51	3.90e-02	0.55	0.51
500	10	1.91e-02	0.75	0.27	1.13e-02	0.46	0.51	3.05e-02	0.53	0.52
s1: $\lambda_0 = 1$, $\lambda = 1$, $\Lambda = 1$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.04$)										
50	3	1.13e-02	0.65	0.39	9.68e-03	0.61	0.42	2.12e-02	0.68	0.46
100	3	1.10e-02	0.63	0.39	7.76e-03	0.57	0.43	1.89e-02	0.66	0.47
500	3	9.02e-03	0.59	0.44	6.09e-03	0.53	0.44	1.51e-02	0.64	0.49
50	6	7.50e-03	0.59	0.4	7.35e-03	0.56	0.47	1.49e-02	0.63	0.46
100	6	8.29e-03	0.59	0.41	6.02e-03	0.54	0.47	1.44e-02	0.62	0.46
500	6	6.57e-03	0.55	0.47	5.05e-03	0.52	0.47	1.16e-02	0.6	0.49
50	10	7.53e-03	0.6	0.41	6.30e-03	0.54	0.49	1.39e-02	0.62	0.45
100	10	7.87e-03	0.59	0.44	4.99e-03	0.52	0.49	1.29e-02	0.61	0.46
500	10	5.14e-03	0.53	0.5	4.05e-03	0.5	0.49	9.20e-03	0.57	0.49
s16: $\lambda_0 = 1$, $\lambda = 1$, $\Lambda = 1$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.2$, $\sigma_\mu = 0.2$, $\sigma_v = 0.2$)										
50	3	7.66e-02	0.68	0.36	6.62e-02	0.64	0.42	1.49e-01	0.7	0.46
100	3	7.17e-02	0.66	0.36	5.59e-02	0.6	0.43	1.32e-01	0.69	0.46
500	3	5.83e-02	0.61	0.4	4.27e-02	0.55	0.43	1.03e-01	0.66	0.48
50	6	5.43e-02	0.62	0.37	5.33e-02	0.59	0.46	1.11e-01	0.65	0.45
100	6	5.97e-02	0.62	0.35	4.40e-02	0.56	0.46	1.06e-01	0.65	0.45
500	6	4.89e-02	0.58	0.42	3.09e-02	0.52	0.47	8.11e-02	0.62	0.48
50	10	5.48e-02	0.62	0.37	4.60e-02	0.57	0.48	1.03e-01	0.64	0.44
100	10	5.71e-02	0.62	0.38	3.49e-02	0.53	0.48	9.42e-02	0.63	0.45
500	10	3.75e-02	0.54	0.46	2.52e-02	0.5	0.48	6.34e-02	0.58	0.49
s11: $\lambda_0 = 1$, $\lambda = 1$, $\Lambda = 5$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.04$, $\sigma_\mu = 0.2$, $\sigma_v = 0.04$)										
50	3	5.09e-02	0.61	0.41	1.19e-02	0.64	0.37	6.40e-02	0.62	0.4
100	3	4.81e-02	0.59	0.42	1.05e-02	0.61	0.38	5.94e-02	0.6	0.41
500	3	2.64e-02	0.51	0.47	9.70e-03	0.59	0.39	3.65e-02	0.54	0.47
50	6	4.20e-02	0.58	0.4	8.93e-03	0.59	0.42	5.14e-02	0.6	0.39
100	6	4.97e-02	0.6	0.39	7.97e-03	0.57	0.42	5.81e-02	0.61	0.39
500	6	2.83e-02	0.52	0.47	6.69e-03	0.54	0.43	3.52e-02	0.54	0.47
50	10	4.28e-02	0.59	0.39	7.17e-03	0.56	0.44	5.04e-02	0.59	0.38
100	10	4.69e-02	0.59	0.39	5.91e-03	0.53	0.44	5.32e-02	0.59	0.4
500	10	2.41e-02	0.5	0.49	4.90e-03	0.51	0.45	2.91e-02	0.52	0.48
s2: $\lambda_0 = 1$, $\lambda = 5$, $\Lambda = 0.2$ ($\sigma_\eta = 0.04$, $\sigma_u = 0.2$, $\sigma_\mu = 0.04$, $\sigma_v = 0.04$)										
50	3	1.11e-02	0.64	0.28	1.50e-02	0.56	0.89	2.59e-02	0.64	0.9
100	3	1.25e-02	0.65	0.29	1.15e-02	0.55	0.9	2.38e-02	0.64	0.91
500	3	1.14e-02	0.62	0.3	9.85e-03	0.54	0.9	2.10e-02	0.63	0.91
50	6	1.16e-02	0.65	0.33	1.07e-02	0.54	0.91	2.21e-02	0.63	0.91
100	6	1.22e-02	0.65	0.36	1.01e-02	0.54	0.91	2.21e-02	0.64	0.91
500	6	1.06e-02	0.61	0.38	9.33e-03	0.54	0.91	1.98e-02	0.63	0.92
50	10	1.11e-02	0.64	0.37	1.04e-02	0.55	0.92	2.15e-02	0.64	0.92
100	10	1.16e-02	0.64	0.39	9.66e-03	0.55	0.92	2.12e-02	0.64	0.92
500	10	9.66e-03	0.6	0.43	9.42e-03	0.54	0.92	1.90e-02	0.63	0.92
s12: $\lambda_0 = 1$, $\lambda = 5$, $\Lambda = 1$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.2$, $\sigma_\mu = 0.2$, $\sigma_v = 0.04$)										
50	3	5.61e-02	0.62	0.34	5.83e-02	0.66	0.75	1.18e-01	0.69	0.62
100	3	5.42e-02	0.61	0.36	5.01e-02	0.64	0.77	1.07e-01	0.68	0.64
500	3	3.40e-02	0.53	0.45	4.68e-02	0.63	0.78	8.14e-02	0.65	0.67
50	6	4.59e-02	0.6	0.35	3.33e-02	0.62	0.84	8.05e-02	0.64	0.66
100	6	5.14e-02	0.6	0.37	3.26e-02	0.62	0.84	8.54e-02	0.66	0.67
500	6	3.30e-02	0.53	0.46	3.13e-02	0.62	0.85	6.49e-02	0.62	0.7
50	10	4.54e-02	0.6	0.34	2.63e-02	0.61	0.87	7.30e-02	0.63	0.67
100	10	4.93e-02	0.59	0.36	2.57e-02	0.61	0.87	7.61e-02	0.64	0.69
500	10	2.75e-02	0.51	0.48	2.46e-02	0.61	0.88	5.24e-02	0.59	0.71

Table A7: Finite sample performance of the technical efficiency estimates using FML: $\sigma_\eta > \sigma_\mu$, where $\lambda_0 = 5$ (BK Table 5)

		Persistent TE			Transient TE			Overall TE		
n	t	R Bias	U Bias	Corr	R Bias	U Bias	Corr	R Bias	U Bias	Corr
s7: $\lambda_0 = 5$, $\lambda = 0.2$, $\Lambda = 5$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$)										
50	3	5.21e-02	0.6	0.65	-4.48e-02	0.41	0.093	3.87e-03	0.46	0.59
100	3	3.90e-02	0.56	0.67	-4.16e-02	0.36	0.099	-5.05e-03	0.43	0.62
500	3	7.68e-03	0.46	0.71	-3.44e-02	0.26	0.1	-2.72e-02	0.34	0.68
50	6	2.54e-02	0.52	0.77	-2.91e-02	0.47	0.11	-5.00e-03	0.43	0.7
100	6	1.05e-02	0.48	0.79	-2.72e-02	0.43	0.11	-1.73e-02	0.39	0.74
500	6	-2.15e-03	0.44	0.8	-1.97e-02	0.39	0.11	-2.20e-02	0.36	0.78
50	10	1.26e-02	0.47	0.82	-1.95e-02	0.52	0.11	-7.64e-03	0.42	0.77
100	10	2.10e-03	0.45	0.84	-1.84e-02	0.48	0.11	-1.66e-02	0.39	0.8
500	10	-7.99e-03	0.41	0.85	-9.76e-03	0.48	0.11	-1.79e-02	0.38	0.83
s8: $\lambda_0 = 5$, $\lambda = 1$, $\Lambda = 1$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.2$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$)										
50	3	6.38e-02	0.64	0.58	3.99e-02	0.56	0.45	1.03e-01	0.66	0.61
100	3	5.52e-02	0.61	0.6	3.15e-02	0.53	0.46	8.52e-02	0.64	0.62
500	3	1.91e-02	0.5	0.66	2.01e-02	0.49	0.48	3.72e-02	0.55	0.66
50	6	3.46e-02	0.55	0.7	4.02e-02	0.56	0.49	7.43e-02	0.61	0.64
100	6	2.03e-02	0.51	0.74	2.80e-02	0.52	0.49	4.72e-02	0.56	0.67
500	6	3.90e-03	0.46	0.77	1.65e-02	0.48	0.5	1.91e-02	0.5	0.7
50	10	2.18e-02	0.51	0.78	3.40e-02	0.53	0.5	5.51e-02	0.58	0.67
100	10	1.10e-02	0.49	0.81	2.24e-02	0.5	0.5	3.25e-02	0.53	0.7
500	10	-6.04e-04	0.45	0.83	1.41e-02	0.47	0.5	1.26e-02	0.48	0.72
s3: $\lambda_0 = 5$, $\lambda = 1$, $\Lambda = 5$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.04$)										
50	3	9.40e-03	0.52	0.92	5.68e-03	0.57	0.41	1.50e-02	0.56	0.91
100	3	5.28e-03	0.51	0.93	4.49e-03	0.53	0.43	9.62e-03	0.54	0.92
500	3	1.24e-03	0.48	0.93	4.01e-03	0.51	0.43	5.10e-03	0.52	0.92
50	6	8.61e-03	0.52	0.93	6.61e-03	0.56	0.48	1.52e-02	0.57	0.91
100	6	3.92e-03	0.5	0.94	4.63e-03	0.52	0.48	8.49e-03	0.54	0.93
500	6	1.83e-03	0.49	0.94	1.75e-03	0.47	0.48	3.51e-03	0.51	0.93
50	10	7.73e-03	0.51	0.93	4.98e-03	0.53	0.5	1.27e-02	0.55	0.92
100	10	3.68e-03	0.5	0.94	2.90e-03	0.49	0.5	6.54e-03	0.52	0.93
500	10	1.85e-03	0.49	0.94	1.34e-03	0.46	0.5	3.15e-03	0.51	0.93
s4: $\lambda_0 = 5$, $\lambda = 5$, $\Lambda = 1$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.2$, $\sigma_\mu = 0.04$, $\sigma_v = 0.04$)										
50	3	2.58e-02	0.57	0.8	1.16e-02	0.51	0.83	3.52e-02	0.61	0.92
100	3	1.18e-02	0.53	0.84	5.36e-03	0.48	0.85	1.47e-02	0.56	0.94
500	3	3.73e-03	0.5	0.85	4.38e-03	0.48	0.85	5.64e-03	0.52	0.94
50	6	1.48e-02	0.54	0.88	3.84e-03	0.49	0.9	1.74e-02	0.56	0.93
100	6	6.29e-03	0.51	0.9	3.40e-03	0.49	0.9	8.52e-03	0.54	0.94
500	6	2.84e-03	0.5	0.9	2.94e-03	0.48	0.9	4.61e-03	0.52	0.95
50	10	1.19e-02	0.52	0.9	3.67e-03	0.5	0.92	1.49e-02	0.55	0.94
100	10	5.44e-03	0.51	0.92	2.73e-03	0.49	0.92	7.49e-03	0.53	0.94
500	10	2.17e-03	0.49	0.92	2.61e-03	0.49	0.92	4.11e-03	0.51	0.95

Table A8: Finite sample performance of the technical efficiency estimates using SML: $\sigma_\eta > \sigma_\mu$, where $\lambda_0 = 5$ (BK Table 5)

		Persistent TE			Transient TE			Overall TE		
n	t	R Bias	U Bias	Corr	R Bias	U Bias	Corr	R Bias	U Bias	Corr
s7: $\lambda_0 = 5$, $\lambda = 0.2$, $\Lambda = 5$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$)										
50	3	4.17e-02	0.57	0.67	-3.97e-02	0.49	0.084	-4.83e-04	0.45	0.6
100	3	3.14e-02	0.53	0.68	-3.42e-02	0.47	0.09	-4.79e-03	0.43	0.62
500	3	-3.82e-04	0.43	0.71	-2.10e-02	0.47	0.094	-2.16e-02	0.37	0.69
50	6	2.19e-02	0.5	0.77	-2.76e-02	0.5	0.11	-6.91e-03	0.42	0.7
100	6	9.07e-03	0.46	0.79	-2.50e-02	0.48	0.11	-1.65e-02	0.39	0.73
500	6	-6.71e-03	0.41	0.8	-1.54e-02	0.46	0.11	-2.23e-02	0.36	0.78
50	10	1.21e-02	0.47	0.83	-1.95e-02	0.52	0.11	-7.98e-03	0.42	0.76
100	10	-1.32e-03	0.42	0.84	-1.84e-02	0.48	0.11	-1.99e-02	0.37	0.8
500	10	-1.43e-02	0.37	0.85	-1.06e-02	0.47	0.11	-2.50e-02	0.34	0.83
s8: $\lambda_0 = 5$, $\lambda = 1$, $\Lambda = 1$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.2$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$)										
50	3	4.83e-02	0.59	0.61	4.39e-02	0.58	0.42	9.15e-02	0.64	0.61
100	3	3.90e-02	0.56	0.61	3.52e-02	0.54	0.45	7.28e-02	0.6	0.63
500	3	6.37e-03	0.46	0.65	1.98e-02	0.49	0.48	2.40e-02	0.51	0.67
50	6	2.98e-02	0.53	0.72	4.02e-02	0.56	0.48	6.97e-02	0.6	0.64
100	6	1.96e-02	0.5	0.73	2.84e-02	0.52	0.49	4.69e-02	0.56	0.66
500	6	-2.09e-04	0.44	0.77	1.46e-02	0.47	0.5	1.29e-02	0.49	0.7
50	10	1.97e-02	0.5	0.79	3.37e-02	0.53	0.5	5.27e-02	0.57	0.67
100	10	7.02e-03	0.46	0.81	2.22e-02	0.5	0.5	2.83e-02	0.52	0.69
500	10	-6.71e-03	0.42	0.83	1.38e-02	0.47	0.51	6.02e-03	0.46	0.72
s3: $\lambda_0 = 5$, $\lambda = 1$, $\Lambda = 5$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.04$)										
50	3	9.98e-03	0.51	0.92	5.33e-03	0.57	0.42	1.52e-02	0.55	0.91
100	3	1.42e-03	0.47	0.92	4.17e-03	0.53	0.43	5.44e-03	0.51	0.92
500	3	-6.91e-03	0.41	0.93	8.41e-04	0.46	0.44	-6.23e-03	0.43	0.92
50	6	6.30e-03	0.49	0.93	5.99e-03	0.55	0.48	1.22e-02	0.54	0.91
100	6	1.22e-03	0.47	0.94	4.00e-03	0.51	0.48	5.15e-03	0.51	0.92
500	6	-7.15e-03	0.41	0.94	1.47e-03	0.46	0.48	-5.77e-03	0.43	0.93
50	10	6.25e-03	0.49	0.94	4.40e-03	0.52	0.5	1.06e-02	0.53	0.91
100	10	-5.40e-04	0.45	0.94	2.65e-03	0.49	0.5	2.06e-03	0.48	0.93
500	10	-6.63e-03	0.41	0.94	1.58e-03	0.47	0.5	-5.12e-03	0.43	0.93
s4: $\lambda_0 = 5$, $\lambda = 5$, $\Lambda = 1$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.2$, $\sigma_\mu = 0.04$, $\sigma_v = 0.04$)										
50	3	2.01e-02	0.54	0.83	1.06e-02	0.5	0.84	2.84e-02	0.58	0.92
100	3	7.55e-03	0.5	0.84	4.81e-03	0.48	0.84	9.82e-03	0.52	0.93
500	3	-3.50e-04	0.47	0.85	4.17e-03	0.47	0.85	1.33e-03	0.49	0.94
50	6	1.35e-02	0.52	0.88	3.13e-03	0.48	0.9	1.55e-02	0.53	0.93
100	6	3.83e-03	0.49	0.9	3.19e-03	0.48	0.9	5.85e-03	0.51	0.94
500	6	-2.87e-03	0.45	0.9	3.11e-03	0.48	0.9	-9.30e-04	0.48	0.95
50	10	1.28e-02	0.52	0.91	3.40e-03	0.5	0.92	1.55e-02	0.54	0.93
100	10	3.54e-03	0.48	0.91	2.74e-03	0.49	0.92	5.59e-03	0.5	0.94
500	10	-5.12e-03	0.43	0.92	2.64e-03	0.49	0.92	-3.17e-03	0.46	0.95

Table A9: Finite sample performance of the technical efficiency estimates using SEQ: $\sigma_\eta > \sigma_\mu$, where $\lambda_0 = 5$ (BK Table 5)

		Persistent TE			Transient TE			Overall TE		
n	t	R Bias	U Bias	Corr	R Bias	U Bias	Corr	R Bias	U Bias	Corr
s7: $\lambda_0 = 5$, $\lambda = 0.2$, $\Lambda = 5$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$)										
50	3	9.24e-02	0.74	0.65	-4.95e-02	0.38	0.094	3.77e-02	0.55	0.58
100	3	8.91e-02	0.74	0.67	-5.11e-02	0.32	0.1	3.28e-02	0.54	0.6
500	3	7.78e-02	0.71	0.7	-5.73e-02	0.17	0.1	1.54e-02	0.48	0.63
50	6	5.96e-02	0.66	0.76	-3.67e-02	0.41	0.11	2.02e-02	0.52	0.68
100	6	5.04e-02	0.64	0.79	-3.71e-02	0.36	0.11	1.12e-02	0.49	0.72
500	6	4.49e-02	0.63	0.79	-3.84e-02	0.25	0.11	4.45e-03	0.46	0.75
50	10	3.62e-02	0.59	0.83	-2.53e-02	0.46	0.11	9.65e-03	0.49	0.76
100	10	3.08e-02	0.58	0.84	-2.74e-02	0.4	0.11	2.32e-03	0.46	0.79
500	10	2.50e-02	0.58	0.85	-2.65e-02	0.31	0.11	-2.40e-03	0.44	0.82
s8: $\lambda_0 = 5$, $\lambda = 1$, $\Lambda = 1$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.2$, $\sigma_\mu = 0.04$, $\sigma_v = 0.2$)										
50	3	1.02e-01	0.77	0.59	3.79e-02	0.55	0.45	1.42e-01	0.74	0.62
100	3	1.02e-01	0.77	0.61	2.51e-02	0.51	0.46	1.27e-01	0.73	0.63
500	3	9.03e-02	0.73	0.65	7.37e-03	0.45	0.47	9.56e-02	0.7	0.65
50	6	7.22e-02	0.7	0.71	3.64e-02	0.54	0.49	1.10e-01	0.69	0.64
100	6	6.51e-02	0.68	0.74	2.33e-02	0.5	0.49	8.84e-02	0.67	0.66
500	6	5.88e-02	0.67	0.76	1.23e-02	0.46	0.49	7.02e-02	0.65	0.69
50	10	4.91e-02	0.64	0.79	3.32e-02	0.53	0.5	8.27e-02	0.65	0.67
100	10	4.41e-02	0.63	0.81	2.17e-02	0.49	0.5	6.58e-02	0.63	0.69
500	10	3.79e-02	0.62	0.82	1.47e-02	0.47	0.5	5.22e-02	0.61	0.71
s3: $\lambda_0 = 5$, $\lambda = 1$, $\Lambda = 5$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.04$, $\sigma_\mu = 0.04$, $\sigma_v = 0.04$)										
50	3	1.42e-02	0.55	0.92	2.34e-03	0.5	0.42	1.65e-02	0.57	0.91
100	3	1.11e-02	0.55	0.93	-8.20e-04	0.44	0.43	1.01e-02	0.54	0.92
500	3	7.96e-03	0.54	0.93	-2.28e-03	0.4	0.44	5.51e-03	0.52	0.92
50	6	1.07e-02	0.53	0.93	4.76e-03	0.52	0.48	1.54e-02	0.57	0.92
100	6	6.58e-03	0.52	0.94	3.16e-03	0.5	0.48	9.70e-03	0.55	0.93
500	6	4.93e-03	0.52	0.94	2.03e-03	0.47	0.48	6.89e-03	0.54	0.93
50	10	8.78e-03	0.52	0.94	4.65e-03	0.52	0.5	1.34e-02	0.55	0.92
100	10	5.12e-03	0.51	0.94	3.23e-03	0.49	0.5	8.31e-03	0.54	0.93
500	10	3.50e-03	0.51	0.94	2.52e-03	0.48	0.5	5.98e-03	0.53	0.93
s4: $\lambda_0 = 5$, $\lambda = 5$, $\Lambda = 1$ ($\sigma_\eta = 0.2$, $\sigma_u = 0.2$, $\sigma_\mu = 0.04$, $\sigma_v = 0.04$)										
50	3	4.78e-02	0.66	0.81	2.88e-02	0.59	0.82	7.52e-02	0.78	0.92
100	3	4.16e-02	0.65	0.84	2.37e-02	0.57	0.84	6.34e-02	0.79	0.93
500	3	3.58e-02	0.64	0.84	2.23e-02	0.57	0.84	5.62e-02	0.79	0.94
50	6	2.98e-02	0.61	0.88	1.94e-02	0.58	0.89	4.84e-02	0.71	0.93
100	6	2.52e-02	0.61	0.89	1.85e-02	0.58	0.9	4.29e-02	0.72	0.94
500	6	2.22e-02	0.61	0.9	1.75e-02	0.58	0.9	3.88e-02	0.72	0.94
50	10	2.17e-02	0.58	0.9	1.51e-02	0.57	0.92	3.63e-02	0.66	0.93
100	10	1.74e-02	0.58	0.91	1.40e-02	0.57	0.92	3.09e-02	0.67	0.94
500	10	1.43e-02	0.58	0.92	1.38e-02	0.57	0.92	2.76e-02	0.67	0.95

Table A10: Correlation of transient technical efficiency

	FML	SML	SEQ
Constant	0.090 (0.001)	0.081 (0.001)	0.091 (0.001)
N=100, t=3	0.009 (0.001)	0.011 (0.001)	0.009 (0.001)
N=500, t=3	0.015 (0.001)	0.020 (0.001)	0.012 (0.001)
N=50, t= 6	0.041 (0.001)	0.047 (0.001)	0.040 (0.001)
N=100, t=6	0.042 (0.001)	0.048 (0.001)	0.040 (0.001)
N=500, t=6	0.045 (0.001)	0.051 (0.001)	0.042 (0.001)
N=50, t=10	0.056 (0.001)	0.063 (0.001)	0.055 (0.001)
N=100, t=10	0.055 (0.001)	0.063 (0.001)	0.054 (0.001)
N=500, t=10	0.056 (0.001)	0.063 (0.001)	0.055 (0.001)
λ_0=1	0.049 (0.000)	0.046 (0.000)	0.050 (0.000)
λ_0=5	0.083 (0.000)	0.076 (0.000)	0.084 (0.000)
λ=1	0.334 (0.000)	0.337 (0.000)	0.334 (0.000)
λ=5	0.722 (0.000)	0.730 (0.000)	0.715 (0.000)
Λ=1	−0.051 (0.000)	−0.047 (0.000)	−0.051 (0.000)
Λ=5	−0.086 (0.000)	−0.079 (0.001)	−0.086 (0.000)
R^2	0.971	0.966	0.971
Adj. R^2	0.971	0.966	0.971
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the correlation of transient technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

Table A11: Correlation of persistent technical efficiency

	FML	SML	SEQ
Constant	−0.107 (0.002)	−0.093 (0.002)	−0.101 (0.002)
N=100, t=3	0.007 (0.002)	−0.011 (0.002)	0.007 (0.002)
N=500, t=3	0.036 (0.002)	−0.009 (0.002)	0.033 (0.002)
N=50, t= 6	0.033 (0.002)	0.031 (0.002)	0.031 (0.002)
N=100, t=6	0.039 (0.002)	0.020 (0.002)	0.039 (0.002)
N=500, t=6	0.067 (0.002)	0.017 (0.002)	0.066 (0.002)
N=50, t=10	0.052 (0.002)	0.052 (0.002)	0.054 (0.002)
N=100, t=10	0.058 (0.002)	0.041 (0.002)	0.062 (0.002)
N=500, t=10	0.093 (0.002)	0.042 (0.002)	0.092 (0.002)
λ_0=1	0.218 (0.001)	0.198 (0.001)	0.228 (0.001)
λ_0=5	0.634 (0.001)	0.621 (0.001)	0.641 (0.001)
λ=1	0.082 (0.001)	0.085 (0.001)	0.082 (0.001)
λ=5	0.156 (0.001)	0.155 (0.001)	0.152 (0.001)
Λ=1	0.112 (0.001)	0.132 (0.001)	0.104 (0.001)
Λ=5	0.237 (0.002)	0.263 (0.002)	0.222 (0.002)
R^2	0.766	0.767	0.779
Adj. R^2	0.766	0.767	0.779
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the correlation of persistent technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

Table A12: Relative bias of overall technical efficiency

	FML	SML	SEQ
Constant	−0.046 (0.001)	−0.058 (0.001)	−0.028 (0.001)
N=100, t=3	−0.005 (0.001)	−0.001 (0.001)	−0.004 (0.001)
N=500, t=3	−0.015 (0.001)	−0.005 (0.001)	−0.014 (0.001)
N=50, t= 6	−0.012 (0.001)	−0.008 (0.001)	−0.016 (0.001)
N=100, t=6	−0.013 (0.001)	−0.005 (0.001)	−0.017 (0.001)
N=500, t=6	−0.018 (0.001)	−0.007 (0.001)	−0.021 (0.001)
N=50, t=10	−0.014 (0.001)	−0.009 (0.001)	−0.022 (0.001)
N=100, t=10	−0.015 (0.001)	−0.007 (0.001)	−0.023 (0.001)
N=500, t=10	−0.020 (0.001)	−0.009 (0.001)	−0.028 (0.001)
λ_0=1	0.054 (0.001)	0.055 (0.001)	0.058 (0.001)
λ_0=5	0.034 (0.001)	0.032 (0.001)	0.057 (0.001)
λ=1	0.043 (0.001)	0.041 (0.001)	0.038 (0.001)
λ=5	0.036 (0.001)	0.036 (0.001)	0.036 (0.001)
Λ=1	0.009 (0.001)	0.012 (0.001)	0.006 (0.001)
Λ=5	0.009 (0.001)	0.015 (0.001)	−0.007 (0.001)
R^2	0.106	0.100	0.121
Adj. R^2	0.106	0.100	0.121
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the relative bias of overall technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

Table A13: Relative bias of transient technical efficiency

	FML	SML	SEQ
Constant	−0.003 (0.001)	−0.002 (0.001)	−0.003 (0.001)
N=100, t=3	−0.003 (0.001)	−0.003 (0.001)	−0.004 (0.000)
N=500, t=3	−0.006 (0.001)	−0.004 (0.001)	−0.007 (0.000)
N=50, t= 6	−0.002 (0.001)	−0.003 (0.001)	−0.005 (0.000)
N=100, t=6	−0.005 (0.001)	−0.005 (0.001)	−0.007 (0.000)
N=500, t=6	−0.007 (0.001)	−0.007 (0.001)	−0.009 (0.000)
N=50, t=10	−0.002 (0.001)	−0.003 (0.001)	−0.006 (0.000)
N=100, t=10	−0.005 (0.001)	−0.006 (0.001)	−0.009 (0.000)
N=500, t=10	−0.006 (0.001)	−0.006 (0.001)	−0.010 (0.000)
λ_0=1	0.001 (0.000)	0.001 (0.000)	−0.003 (0.000)
λ_0=5	0.002 (0.000)	0.002 (0.000)	−0.008 (0.000)
λ=1	0.034 (0.000)	0.034 (0.000)	0.041 (0.000)
λ=5	0.021 (0.000)	0.019 (0.000)	0.042 (0.000)
Λ=1	−0.009 (0.000)	−0.009 (0.000)	−0.005 (0.000)
Λ=5	−0.021 (0.000)	−0.020 (0.000)	−0.018 (0.000)
R^2	0.128	0.115	0.208
Adj. R^2	0.128	0.115	0.208
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the relative bias of transient technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

Table A14: Relative bias of persistent technical efficiency

	FML	SML	SEQ
Constant	−0.043 (0.001)	−0.055 (0.001)	−0.023 (0.001)
N=100, t=3	−0.002 (0.001)	0.002 (0.001)	−0.000 (0.001)
N=500, t=3	−0.009 (0.001)	−0.002 (0.001)	−0.005 (0.001)
N=50, t= 6	−0.010 (0.001)	−0.005 (0.001)	−0.011 (0.001)
N=100, t=6	−0.009 (0.001)	−0.000 (0.001)	−0.009 (0.001)
N=500, t=6	−0.011 (0.001)	−0.000 (0.001)	−0.011 (0.001)
N=50, t=10	−0.012 (0.001)	−0.006 (0.001)	−0.016 (0.001)
N=100, t=10	−0.010 (0.001)	−0.001 (0.001)	−0.013 (0.001)
N=500, t=10	−0.014 (0.001)	−0.003 (0.001)	−0.017 (0.001)
λ_0=1	0.052 (0.000)	0.054 (0.000)	0.061 (0.000)
λ_0=5	0.033 (0.001)	0.030 (0.001)	0.067 (0.001)
λ=1	0.008 (0.000)	0.008 (0.000)	−0.005 (0.000)
λ=5	0.015 (0.001)	0.017 (0.001)	−0.007 (0.001)
Λ=1	0.017 (0.001)	0.020 (0.001)	0.010 (0.000)
Λ=5	0.030 (0.001)	0.034 (0.001)	0.011 (0.001)
R^2	0.130	0.131	0.182
Adj. R^2	0.130	0.131	0.182
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the relative bias of persistent technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

Table A15: Upward bias of overall technical efficiency

	FML	SML	SEQ
Constant	0.360 (0.003)	0.333 (0.003)	0.411 (0.003)
N=100, t=3	−0.018 (0.003)	−0.006 (0.003)	−0.009 (0.003)
N=500, t=3	−0.052 (0.003)	−0.020 (0.003)	−0.033 (0.003)
N=50, t= 6	−0.028 (0.003)	−0.021 (0.003)	−0.036 (0.003)
N=100, t=6	−0.039 (0.003)	−0.016 (0.003)	−0.040 (0.003)
N=500, t=6	−0.060 (0.003)	−0.022 (0.003)	−0.052 (0.003)
N=50, t=10	−0.035 (0.003)	−0.022 (0.003)	−0.051 (0.003)
N=100, t=10	−0.045 (0.003)	−0.022 (0.003)	−0.055 (0.003)
N=500, t=10	−0.068 (0.003)	−0.031 (0.003)	−0.071 (0.003)
λ_0=1	0.112 (0.002)	0.121 (0.002)	0.143 (0.002)
λ_0=5	0.063 (0.003)	0.046 (0.003)	0.164 (0.002)
λ=1	0.119 (0.002)	0.107 (0.002)	0.087 (0.002)
λ=5	0.123 (0.003)	0.114 (0.003)	0.127 (0.002)
Λ=1	0.039 (0.002)	0.046 (0.002)	0.028 (0.002)
Λ=5	0.034 (0.003)	0.041 (0.003)	−0.034 (0.003)
R^2	0.065	0.059	0.093
Adj. R^2	0.065	0.059	0.093
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the upward bias of persistent technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

Table A16: Upward bias of transient technical efficiency

	FML	SML	SEQ
Constant	0.534 (0.003)	0.562 (0.003)	0.487 (0.003)
N=100, t=3	−0.031 (0.003)	−0.027 (0.003)	−0.028 (0.003)
N=500, t=3	−0.065 (0.003)	−0.046 (0.003)	−0.060 (0.003)
N=50, t= 6	−0.018 (0.003)	−0.028 (0.003)	−0.024 (0.003)
N=100, t=6	−0.043 (0.003)	−0.050 (0.003)	−0.043 (0.003)
N=500, t=6	−0.074 (0.003)	−0.072 (0.003)	−0.066 (0.003)
N=50, t=10	−0.023 (0.003)	−0.037 (0.003)	−0.029 (0.003)
N=100, t=10	−0.048 (0.003)	−0.061 (0.003)	−0.050 (0.003)
N=500, t=10	−0.065 (0.003)	−0.075 (0.003)	−0.069 (0.003)
λ_0=1	−0.027 (0.002)	−0.035 (0.002)	−0.045 (0.002)
λ_0=5	−0.066 (0.002)	−0.073 (0.003)	−0.119 (0.002)
λ=1	0.048 (0.002)	0.027 (0.002)	0.108 (0.002)
λ=5	0.035 (0.002)	0.013 (0.003)	0.170 (0.002)
Λ=1	0.026 (0.002)	0.028 (0.002)	0.058 (0.002)
Λ=5	0.025 (0.003)	0.043 (0.003)	0.044 (0.003)
R^2	0.017	0.014	0.068
Adj. R^2	0.017	0.014	0.068
Num. obs.	144000	144000	144000

Note: The dependent variable is the upward bias of transient technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

Table A17: Upward bias of persistent technical efficiency

	FML	SML	SEQ
Constant	0.580 (0.004)	0.455 (0.004)	0.651 (0.004)
N=100, t=3	−0.018 (0.004)	0.011 (0.004)	−0.005 (0.003)
N=500, t=3	−0.075 (0.004)	0.006 (0.004)	−0.046 (0.003)
N=50, t= 6	−0.036 (0.004)	−0.015 (0.004)	−0.030 (0.003)
N=100, t=6	−0.048 (0.004)	0.007 (0.004)	−0.034 (0.003)
N=500, t=6	−0.079 (0.004)	0.017 (0.004)	−0.058 (0.003)
N=50, t=10	−0.053 (0.004)	−0.021 (0.004)	−0.050 (0.003)
N=100, t=10	−0.058 (0.004)	−0.001 (0.004)	−0.048 (0.003)
N=500, t=10	−0.100 (0.004)	0.002 (0.004)	−0.080 (0.003)
λ_0=1	0.116 (0.002)	0.137 (0.002)	0.197 (0.002)
λ_0=5	0.049 (0.003)	0.037 (0.003)	0.227 (0.003)
λ=1	−0.016 (0.002)	−0.007 (0.002)	−0.087 (0.002)
λ=5	−0.042 (0.003)	−0.003 (0.003)	−0.151 (0.003)
Λ=1	−0.030 (0.002)	0.001 (0.002)	−0.081 (0.002)
Λ=5	−0.072 (0.003)	−0.026 (0.004)	−0.195 (0.003)
R^2	0.030	0.033	0.074
Adj. R^2	0.030	0.033	0.074
Num. obs.	144,000	144,000	144,000

Note: The dependent variable is the upward bias of persistent technical efficiency. All explanatory variables are factors. The table displays results from a seemingly unrelated regression.

Table A18: Marginal effects of multinomial logit model for MSE

	FML		SEQ		SML	
	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
Tansient TE						
N=100	0.104	(0.058)	0.167*	(0.068)	-0.271**	(0.062)
N=500	0.188**	(0.061)	0.062	(0.072)	-0.250**	(0.065)
T=6	0.104	(0.058)	0.021	(0.067)	-0.125*	(0.056)
T=10	0.063	(0.058)	0.021	(0.068)	-0.083	(0.060)
λ_0=1	-0.264**	(0.070)	0.167*	(0.081)	0.097*	(0.043)
λ_0=5	0.472**	(0.091)	-0.639**	(0.081)	0.167**	(0.059)
λ=1	0.083	(0.055)	-0.139*	(0.062)	0.056	(0.051)
λ=5	0.111	(0.065)	-0.139	(0.073)	0.028	(0.059)
Persistent TE						
N=100	0.063	(0.066)	-0.083	(0.056)	0.021	(0.052)
N=500	0.396**	(0.074)	-0.354**	(0.063)	-0.042	(0.053)
T=6	0.083	(0.071)	-0.125*	(0.059)	0.042	(0.049)
T=10	0.062	(0.071)	-0.188**	(0.062)	0.125*	(0.054)
λ_0=1	0.167*	(0.066)	-0.069	(0.060)	-0.097	(0.055)
λ_0=5	0.306**	(0.081)	-0.139	(0.072)	-0.167**	(0.059)
λ=1	-0.264**	(0.073)	0.208**	(0.076)	0.056*	(0.026)
λ=5	0.056	(0.095)	-0.583**	(0.064)	0.528**	(0.071)
Overall TE						
N=100	0.208*	(0.084)	0.042	(0.084)	-0.250**	(0.066)
N=500	0.396**	(0.083)	-0.125	(0.082)	-0.271**	(0.063)
T=6	0.021	(0.085)	0.083	(0.084)	-0.104	(0.055)
T=10	-0.042	(0.085)	0.021	(0.082)	0.021	(0.064)
λ_0=1	0.111	(0.083)	-0.278**	(0.085)	0.167**	(0.047)
λ_0=5	0.389**	(0.096)	-0.500**	(0.091)	0.111*	(0.056)
λ=1	0.167*	(0.082)	-0.319**	(0.084)	0.153**	(0.046)
λ=5	0.389**	(0.095)	-0.528**	(0.090)	0.139*	(0.058)
No. of obs.			144			

Note: The dependent variable is the best performing estimator. Each panel presents the results of a separate MNL model. ** and * denote statistical significance at the 1 % and 5 %, level, respectively. In some instances, the marginal effects or standard errors for a certain estimator cannot be computed owed to a low number of first ranks.

Figure A1: log minutes for FML (top left), SML (top right) and SEQ (bottom).

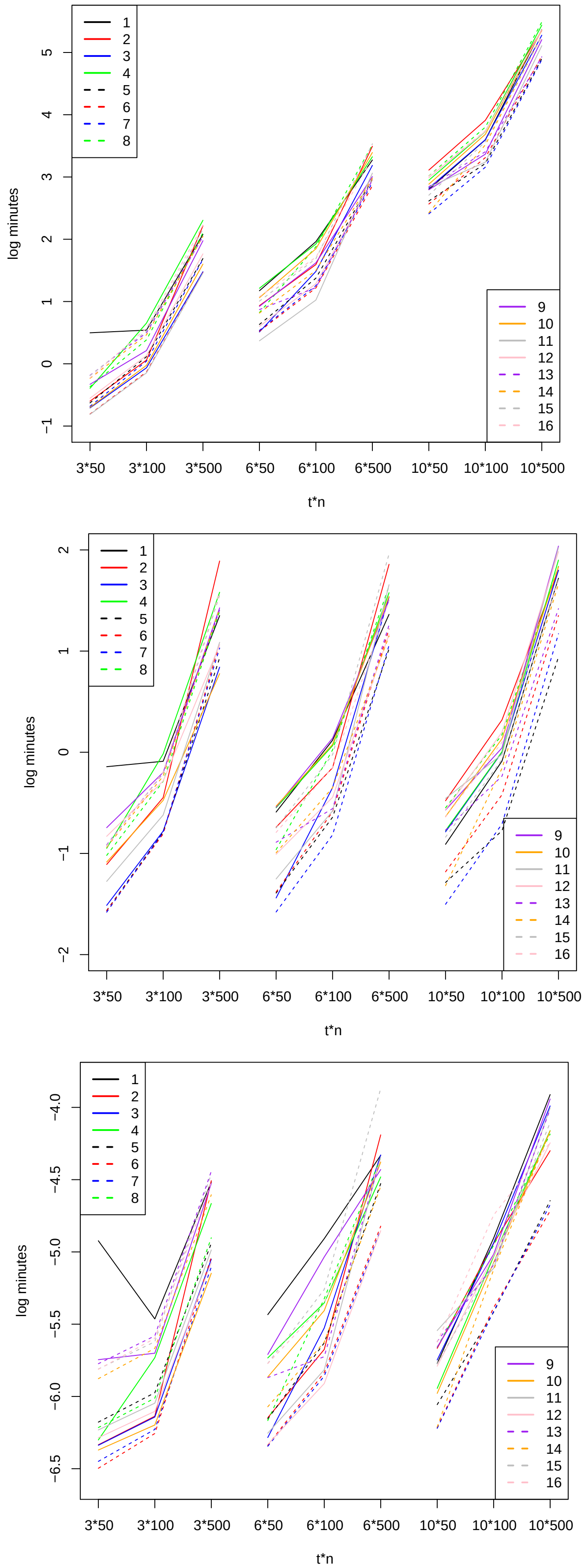

Table A19: Marginal effects of multinomial logit model for correlation

	FML		SEQ		SML	
	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
Tansient TE						
N=100	0.229**	(0.055)	0.083	(0.085)	-0.312**	(0.089)
N=500	0.250**	(0.055)	0.417**	(0.082)	-0.667**	(0.070)
T=6	-0.021	(0.065)	0.042	(0.083)	-0.021	(0.077)
T=10	-0.125*	(0.060)	0.083	(0.083)	0.042	(0.078)
λ_0=1	0.000	(0.054)	0.292**	(0.082)	-0.292**	(0.077)
λ_0=5	0.389**	(0.078)	-0.139	(0.086)	-0.250**	(0.084)
λ=1	0.000	(0.060)	0.056	(0.082)	-0.056	(0.078)
λ=5	0.056	(0.073)	0.000	(0.095)	-0.056	(0.089)
Persistent TE						
N=100	0.000	(0.083)	-0.021	(0.057)	0.021	(0.084)
N=500	0.104	(0.086)	-0.146**	(0.052)	0.042	(0.085)
T=6	0.271**	(0.086)	-0.375**	(0.062)	0.104	(0.086)
T=10	0.083	(0.081)	-0.417**	(0.056)	0.333**	(0.083)
λ_0=1	0.097	(0.082)	0.000	(0.051)	-0.097	(0.085)
λ_0=5	0.194*	(0.097)	0.139*	(0.064)	-0.333**	(0.095)
λ=1	-0.333**	(0.085)	-0.069	(0.062)	0.403**	(0.077)
λ=5	-0.167	(0.104)	-0.278**	(0.052)	0.444**	(0.095)
Overall TE						
N=100	0.083	(0.088)	0.021	(0.084)	-0.104	(0.086)
N=500	0.042	(0.087)	0.042	(0.084)	-0.083	(0.087)
T=6	0.063	(0.086)	-0.042	(0.085)	-0.021	(0.086)
T=10	0.125	(0.087)	-0.083	(0.084)	-0.042	(0.085)
λ_0=1	0.153	(0.083)	0.167*	(0.085)	-0.319**	(0.093)
λ_0=5	0.306**	(0.102)	-0.028	(0.096)	-0.278**	(0.107)
λ=1	-0.167	(0.091)	0.125	(0.097)	0.042	(0.082)
λ=5	0.278**	(0.107)	-0.389**	(0.079)	0.111	(0.098)
No. of obs.			144			

Note: The dependent variable is the best performing estimator. Each panel presents the results of a separate MNL model. ** and * denote statistical significance at the 1 % and 5 %, level, respectively. In some instances, the marginal effects or standard errors for a certain estimator cannot be computed owed to a low number of first ranks.

Table A20: Marginal effects of multinomial logit model for relative bias

	FML		SEQ		SML	
	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
Tansient TE						
N=100	0.125*	(0.063)	0.042	(.)	-0.167*	(0.066)
N=500	0.292**	(0.070)	0.187	(.)	-0.479**	(0.070)
T=6	0.042	(0.068)	0.042	(.)	-0.083	(0.071)
T=10	0.125	(0.072)	-0.000	(.)	-0.125	(0.073)
λ_0=1	0.125	(0.074)	-0.097	(.)	-0.028	(0.078)
λ_0=5	-0.111	(0.067)	-0.167	(.)	0.278**	(0.072)
λ=1	-0.125	(0.076)	0.056	(.)	0.069	(0.077)
λ=5	-0.167*	(0.084)	0.194	(.)	-0.028	(0.086)
Persistent TE						
N=100	0.146	(0.079)	0.042	(0.047)	-0.188*	(0.075)
N=500	0.354**	(0.076)	0.021	(0.049)	-0.375**	(0.068)
T=6	0.000	(0.078)	0.042	(0.048)	-0.042	(0.070)
T=10	-0.062	(0.078)	0.021	(0.049)	0.042	(0.071)
λ_0=1	-0.139	(0.085)	0.069	(0.046)	0.069	(0.072)
λ_0=5	-0.444**	(0.085)	0.444**	(0.065)	-0.000	(0.078)
λ=1	0.375**	(0.071)	-0.667**	(0.060)	0.292**	(0.047)
λ=5	0.194*	(0.085)	-0.861**	(0.053)	0.667**	(0.067)
Overall TE						
N=100	0.208**	(0.071)	–	–	-0.208**	(0.071)
N=500	0.500**	(0.076)	–	–	-0.500**	(0.076)
T=6	0.042	(0.080)	–	–	-0.042	(0.080)
T=10	0.042	(0.080)	–	–	-0.042	(0.080)
λ_0=1	0.028	(0.084)	–	–	-0.028	(0.084)
λ_0=5	-0.194*	(0.087)	–	–	0.194*	(0.087)
λ=1	-0.097	(0.081)	–	–	0.097	(0.081)
λ=5	-0.056	(0.094)	–	–	0.056	(0.094)
No. of obs.			144			

Note: The dependent variable is the best performing estimator. Each panel presents the results of a separate MNL model. ** and * denote statistical significance at the 1 % and 5 %, level, respectively. In some instances, the marginal effects or standard errors for a certain estimator cannot be computed owed to a low number of first ranks.

Table A21: Marginal effects of multinomial logit model for upward bias

	FML		SEQ		SML	
	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
Tansient TE						
N=100	0.187**	(0.056)	0.042	(.)	-0.229**	(0.059)
N=500	0.292**	(0.056)	0.250	(.)	-0.542**	(0.054)
T=6	0.083	(0.061)	0.042	(.)	-0.125*	(0.063)
T=10	0.083	(0.061)	-0.000	(.)	-0.083	(0.063)
λ_0=1	0.236**	(0.062)	-0.181	(.)	-0.056	(0.069)
λ_0=5	-0.083*	(0.041)	-0.250	(.)	0.333**	(0.053)
λ=1	-0.125	(0.065)	0.097	(.)	0.028	(0.065)
λ=5	-0.139	(0.074)	0.194	(.)	-0.056	(0.073)
Persistent TE						
N=100	0.083	(.)	0.000	(.)	-0.083	(0.071)
N=500	0.375	(.)	-0.042	(.)	-0.333**	(0.062)
T=6	0.042	(.)	-0.042	(.)	0.000	(0.064)
T=10	0.042	(.)	-0.063	(.)	0.021	(0.064)
λ_0=1	-0.153	(.)	0.042	(.)	0.111	(0.065)
λ_0=5	-0.444	(.)	0.361	(.)	0.083	(0.070)
λ=1	0.542	(.)	-0.778	(.)	0.236**	(0.045)
λ=5	0.278	(.)	-1.000	(.)	0.722**	(0.055)
Overall TE						
N=100	0.229**	(0.064)	–	–	-0.229**	(0.064)
N=500	0.521**	(0.069)	–	–	-0.521**	(0.069)
T=6	0.125	(0.073)	–	–	-0.125	(0.073)
T=10	0.063	(0.072)	–	–	-0.063	(0.072)
λ_0=1	0.069	(0.077)	–	–	-0.069	(0.077)
λ_0=5	-0.194*	(0.078)	–	–	0.194*	(0.078)
λ=1	-0.236**	(0.075)	–	–	0.236**	(0.075)
λ=5	-0.139	(0.088)	–	–	0.139	(0.088)
No. of obs.			144			

Note: The dependent variable is the best performing estimator. Each panel presents the results of a separate MNL model. ** and * denote statistical significance at the 1 % and 5 %, level, respectively. In some instances, the marginal effects or standard errors for a certain estimator cannot be computed owed to a low number of first ranks.