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Vogelpothsweg 87, 44227 Dortmund, Germany

Universität Duisburg-Essen, Department of Economics
Universitätsstr. 12, 45117 Essen, Germany

Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI)
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Christoph Hanck and Robert Czudaj[1]

Nonstationary-Volatility Robust Panel Unit Root Tests and the Great Moderation

Abstract

This paper argues that typical applications of panel unit root tests should take possible nonstationarity in the volatility process of the innovations of the panel time series into account. Nonstationarity volatility arises for instance when there are structural breaks in the innovation variances. A prominent example is the reduction in GDP growth variances enjoyed by many industrialized countries, known as the "Great Moderation". It also proposes a new testing approach for panel unit roots that is, unlike many previously suggested tests, robust to such volatility processes. The panel test is based on Simes' (1986) classical multiple test, which combines evidence from time series unit root tests of the series in the panel. As time series unit root tests, we employ recently proposed tests of Cavaliere and Taylor (2008b). The panel test is robust to general patterns of cross-sectional dependence and yet is straightforward to implement, only requiring valid p-values of time series unit root tests, and no resampling. Monte Carlo experiments show that other panel unit root tests suffer from sometimes severe size distortions in the presence of nonstationary volatility, and that this defect can be remedied using the test proposed here. We use the methods developed here to test for unit roots in OECD panels of gross domestic products and inflation rates, yielding inference robust to the "Great Moderation". We find little evidence of trend stationarity, and mixed evidence regarding inflation stationarity.

JEL Classification: C12, C23, E31, O40

Keywords: Panel unit root test; nonstationary volatility; cross-sectional dependence; GDP stationarity; inflation stationarity

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1 Introduction

Although the problem of testing for unit roots is not new, it still attracts considerable attention. Especially, the additional cross-sectional dimension provided by panel data is seen as a way to overcome the low power of traditional time series unit root tests. However, a major drawback of so-called 'first generation' panel unit root tests (PURTs) provided by Maddala and Wu [1999], Im, Pesaran and Shin [2003], and Levin, Lin and Chu [2002] is that these rely on the assumption that the individual time series in the panel are cross-sectionally independent. However, macroeconomic panel data sets usually do not meet this assumption, since, for instance, common global shocks, that heterogeneously affect different countries, lead to cross-sectional dependence among the test statistics [see, e.g., O'Connell, 1998].

Therefore, 'second generation' PURTs are designed to eliminate this caveat and to provide reliable inference in the presence of cross-sectional dependence. Phillips and Sul [2003], Moon and Perron [2004], and Bai and Ng [2004] assume the dependence to be driven by (multiple) factors in the error terms. Suitably 'de-factoring' the observations, e.g. by the principal component method, asymptotically removes the common factors, then allowing for the application of standard PURTs. Breitung and Das [2005], in turn, propose a feasible generalized least-squares approach that can be applied when $T > n$, where T denotes the number of time series observations on each of the n series. Pesaran [2007] adds the cross-section averages of lagged levels and of first-differences of the individual series to Augmented Dickey-Fuller [1979] (ADF) regressions. PURTs can then be based on the simple averages of the individual cross-sectionally augmented ADF statistics. In case of a homogenous panel Herwartz and Siedenburg [2008] suggest a test based on a generalized variance estimator, the application of refined residuals in this framework and a wild bootstrap technique. The approach most closely related to the one to be put forward here is by Demetrescu, Hassler and Tarcolea [2006] and Hanck [2013], who draw on the meta-analytic literature to derive their p-value combination tests.

All of the above-cited tests are, in some way or another, suitable panel generalizations of traditional Dickey and Fuller [1979] or other well-known time series unit root tests. As such, they also invoke the traditional assumption in the unit root testing literature that the variance of the innovations driving the time series stays constant over time. Hamori and Tokihisa [1997] and Kim, Leybourne and Newbold [2002] show that traditional unit root tests perform poorly if this assumption is not met, e.g. because there is abrupt change in the innovation variance at some point during the sample period. We show that similarly negative results obtain for popular second generation panel unit root tests, many of which overreject severely while others are overly conservative.[1]

The main objective of this study is to provide a novel panel unit root test that avoids this potential shortcoming. The test is based on Simes' [1986] classical intersection test of the 'global' null hypothesis H_0 that all individual null hypotheses $H_{i,0}$, $i = 1, \ldots, n$, are true, i.e., that all n time series

[1]Therefore, Demetrescu and Hanck [2012] propose an instrumental variable (IV) Cauchy estimator which uses the sign of the first lag as the instrument and thus accounts for time-varying volatility of the innovations as well as cross-sectional dependence.

are unit root processes. Simes' [1986] test is widely applied in, among many other areas, genetical micro-array experiments [e.g., Dudoit, Shaffer and Boldrick, 2003]. Our new panel test is straightforward to implement, only requiring valid p-values of time series unit root tests. As pointed out by e.g. Maddala and Wu [1999] such easy-to-implement and intuitive p-value combination tests are typically competitive in terms of size and power to conceptually and computationally much more complicated procedures. The Simes-type approach of constructing p-value based panel unit root tests was already successfully exploited to construct standard (i.e. Dickey-Fuller based) panel tests in a companion paper [Hanck, 2013]. Suitable p-values for the present situation of nonstationary volatility are obtained from recently proposed time series unit root tests by Cavaliere and Taylor [2008b] that are robust to general patterns of nonstationary volatility. Moreover, the multiple testing approach of Simes [1986] yields a panel test that is robust to cross-sectional dependence.

As an additional advantage, our Simes-type approach allows to identify the units in the panel for which the alternative of stationarity appears to hold. Doing so, it still controls the 'Familywise Error Rate' (FWER), i.e. the probability to falsely reject at least one true individual time series null hypothesis, at some chosen level α. This would not be achieved by the widely applied strategy to reject for all those time series unit root tests statistics that exceed some fixed level-α critical value, as this latter approach ignores the multiple testing nature of the problem.

Our methodology is of great relevance in macroeconomic and financial applications where large panel data sets with changing variances of the individual series appear. To illustrate this, we use the methods developed here to discuss two longstanding questions in empirical macroeconomics. First, we revisit the debate of whether output levels contain a unit root. Second, we study whether OECD inflation rates are nonstationary. Nonstationary-volatility robust tests are particularly important in this context in view of what has come to be known as the 'Great Moderation' [Stock and Watson, 2002], i.e. the reduction in the volatility of economic growth rates and other variables enjoyed by many industrialized countries since the 1980s. This change in volatility implies that traditional (panel) unit root tests of e.g. output level stationarity may be misspecified. The application of our new panel test yields little evidence of trend stationarity in the investigated panel of OECD countries. We find mixed evidence regarding inflation stationarity.

The next section motivates the need for nonstationary-volatility robust PURTs and develops the new test. Section 3 reports results of a Monte Carlo study. Section 4 presents the empirical results and the last section concludes.

2 The Panel Unit Root Test

As usual, we consider the following dynamic panel model:

$$y_{i,t} = \mu_i(1 - \phi_i) + \phi_i y_{i,t-1} + \epsilon_{i,t} \qquad (i \in \mathbb{N}_n,\ t \in \mathbb{N}_T), \tag{1}$$

where $j \in \mathbb{N}_a$ is shorthand for $j = 1, \ldots, a$, $\phi_i \in (-1, 1]$, $i \in \mathbb{N}_n$, and n denotes the number of series in the panel. Equation (1) states that the time series $\{y_{i,0}, \ldots, y_{i,T}\}$ are generated by a simple first-order autoregressive process for each cross-sectional unit i. The panel unit root null hypothesis indicates that all time series are unit-root nonstationary [Breitung and Pesaran, 2008]. Formally,

$$H_0 : \phi_1 = \phi_2 = \ldots = \phi_n = 1.$$

Put differently, H_0 states that *all* single time series hypotheses $H_{i,0} : \phi_i = 1$ are true,

$$H_0 = \bigcap_{i \in \mathbb{N}_n} H_{i,0}, \tag{2}$$

where $\bigcap_{i \in \mathbb{N}_n}$ denotes the intersection over the n individual time series hypotheses.

2.1 The Need for Nonstationary-Volatility Robust PURTs

To complete the model in Eq. (1) one needs to specify the properties of $\epsilon_{i,t}$. 'First generation' PURTs assumed the $\epsilon_{i,t}$ to be independent across i, an assumption which is now widely agreed to be overly restrictive and has therefore been relaxed in recent work [e.g., Breitung and Das, 2005; Demetrescu *et al.*, 2006; Moon and Perron, 2004; Pesaran, 2007]. We shall follow that route here. Second, whether or not $\phi_i = 1$, it is often expedient to allow for serial dependence in $\epsilon_{i,t}$. A standard assumption in the (panel) unit root literature [Pesaran, 2007] is

Assumption 1.
The errors are generated as $\epsilon_{i,t} = \sum_{j=0}^{\infty} c_{i,j} u_{i,t-j} =: C_i(L) u_{i,t}$, where $C_i(z) \neq 0$ for $z \leqslant 1$ and $\sum_{j=0}^{\infty} j |c_{i,j}| < \infty$, and $u_{i,t}$ is i.i.d. with finite, constant variance σ^2.[2]

While i.i.d.-ness could be relaxed to a martingale difference assumption on $u_{i,t}$ [Davidson, 1994, Thm. 27.14], heterogeneity in the innovation variances that takes the form of 'nonstationary volatility', e.g., structural breaks or trending variances is not covered by the assumptions made above [Hamori and Tokihisa, 1997]. Thus, currently most popular PURTs are potentially misspecified in the presence of nonstationary volatility.[3]

To verify whether nonstationary volatility matters for recent PURTs, we conduct a small scale simulation experiment. The simulated panel data sets exhibit intermediate degrees of cross-sectional dependence and a relatively early moderately negative break in the innovation variance (see Section 3 for details on the Data-Generating Process, henceforth DGP). We compare the following cross-sectional correlation, but not nonstationary-volatility-robust PURTs[4]: $CIPS^*$ by Pesaran [2007], t_{rob} from Breitung and Das [2005], the S test of Hanck [2013], $t_{\hat{\rho}^*,\kappa}$ from Demetrescu *et al.* [2006] and t_a^* by Moon and Perron [2004]. Table I reports the results. The right panel of Table I shows that all tests perform quite well under homoscedasticity, at least for sufficiently large T. When there is nonstationary volatility (left panel), all considered tests exhibit moderate to strong size distortions.

[2] Pesaran [2007] allows $u_{i,t} \sim$ i.i.d.$(0, \sigma_i^2)$, that is, heterogeneity in the innovation variance across i, not t.
[3] The deleterious effect on the properties of time series unit root tests has long been recognized in the literature. See, e.g., Hamori and Tokihisa [1997] and Kim *et al.* [2002]. See also Sen [2007].
[4] We waive to include first generation tests such as those by Levin *et al.* [2002], which are not robust to cross-sectional dependence, such that we cannot expect reasonable performance even under homoscedasticity.

Table I—Empirical Size of Second Generation PURTs under Nonstationary Volatility.

n	T	Heteroscedasticity 30	50	100	200	Homoscedasticity 30	50	100	200
	S	.306	.344	.355	.357	.053	.058	.058	.049
	t_{rob}	.073	.088	.083	.101	.047	.052	.051	.047
8	$CIPS^*$	.562	.563	.610	.567	.068	.049	.070	.059
	$t_{\hat{\rho}^*,\kappa}$	.188	.233	.255	.284	.075	.081	.072	.074
	t_a^*	.002	.005	.023	.029	.040	.066	.089	.091
	S	.370	.405	.440	.402	.052	.049	.047	.047
	t_{rob}	.082	.090	.102	.089	.042	.043	.041	.039
12	$CIPS^*$	.507	.598	.598	.584	.035	.033	.045	.036
	$t_{\hat{\rho}^*,\kappa}$	.213	.262	.321	.308	.077	.080	.080	.064
	t_a^*	.001	.003	.004	.021	.016	.045	.065	.090
	S	.477	.525	.573	.516	.048	.057	.044	.050
	t_{rob}	.109	.100	.093	.096	.044	.035	.045	.049
24	$CIPS^*$	.624	.664	.658	.607	.024	.037	.044	.035
	$t_{\hat{\rho}^*,\kappa}$	.242	.287	.348	.387	.094	.081	.068	.088
	t_a^*	.000	.000	.001	.006	.012	.023	.048	.084

Homoscedasticity corresponds to $\delta = 1$, heteroscedasticity to $\delta = 5$. $\psi = 0$, $\boldsymbol{\phi} = \boldsymbol{\imath}_n$, $\tau = 0.1$. Equicorrelated disturbances with $\theta = 0.5$. (See Section 3 for a precise description of the DGP.) 2,500 replications.

In particular, while t_a^* appears to be undersized, $CIPS^*$, S and $t_{\hat{\rho}^*,\kappa}$ are severely oversized. The t_{rob} test performs relatively best, though also noticeably worse than under homoscedasticity. Also, the size distortions, not vanishing with either increasing n or T, show no sign of being a small sample phenomenon. We therefore conclude that currently most popular PURTs should not be relied upon when researchers suspect a break (or otherwise nonstationary behavior) in the innovation variances. This is of course not to suggest any inherent shortcoming of these tests, as none was designed to cope with nonstationary volatility.

2.2 A Nonstationary-Volatility Robust PURT

This subsection develops the new Nonstationary-Volatility Robust PURT. We draw on classical results from the multiple testing literature that are well-suited for deriving tests in the present non-standard situation. Simes [1986] provides a simple test for testing the 'global' or 'intersection' null hypothesis given in Equation (2). Suppose for the moment that valid p-values p_i, $i \in \mathbb{N}_n$, of suitable test statistics for the individual hypotheses $H_{i,0}$ are available. Denote by $p_{(1)}, \dots, p_{(n)}$ the ordered p-values $p_{(1)} \leqslant \dots \leqslant p_{(n)}$. Then, Simes' Heteroscedasticity-Robust intersection test (henceforth S^H) rejects H_0 at level α if and only if

$$p_{(j)} \leqslant j \cdot \alpha/n \quad \text{for some } j \in \mathbb{N}_n. \tag{3}$$

More precisely, the p-values are sorted from most to least significant and compared to gradually less challenging critical points $j\alpha/n$. If there exists at least one p-value sufficiently small so as to be smaller than the corresponding critical point, the S^H test rejects the panel unit root null. Reassuringly, Hanck

[2013] finds Simes' test to work well under constant volatility when employing standard Dickey and Fuller [1979] t-statistics.[5]

To obtain p-values valid under nonstationary volatility we make use of the recently proposed time-transformed unit root tests by Cavaliere and Taylor [2008b]. They generalize Assumption 1 to

Assumption 2.
The errors are generated as $\epsilon_{i,t} = \sum_{j=0}^{\infty} c_{i,j} u_{i,t-j} = C_i(L)u_{i,t}$, where $C_i(z) \neq 0$ for $z \leqslant 1$ and $\sum_{j=0}^{\infty} j|c_{i,j}| < \infty$. Further, $u_{i,t} = \sigma_{i,t}\varsigma_{i,t}$, $\varsigma_{i,t} \sim$ i.i.d.$(0,1)$. $\sigma_{i,t}$ satisfies, for all $s \in [0,1]$, $\sigma_{i,\lfloor sT \rfloor} \in \mathcal{D}$, the set of cadlag functions on $[0,1]$.

This assumption covers the above-mentioned cases of structural breaks and trending variances, with $\sigma_{\lfloor sT \rfloor} = \sigma_0 + \sigma_1 \mathbb{I}(s > \tau)$, $\tau \in (0,1)$, and $\sigma_{\lfloor sT \rfloor} = \sigma_0 + \sigma_1 s$, respectively. What is more, recent work by Cavaliere and Taylor [2009] suggests that Assumption 2 is far from being a necessary one.

Defining the 'variance profile' $\eta_i(s) = \left(\int_0^1 \sigma^2_{i,\lfloor rT \rfloor}\,\mathrm{d}r\right)^{-1} \int_0^s \sigma^2_{i,\lfloor rT \rfloor}\,\mathrm{d}r$, Cavaliere and Taylor [2007] show that standard unit root test statistics converge to functionals of 'time-transformed' Brownian Motions $B(\eta(s))$ [Davidson, 1994, Sec. 29.4] under nonstationary volatility, thus invalidating the standard limiting distributions. (Under homoscedasticity, $\eta_i(s)$ boils down to s.) They further demonstrate that transforming $y_{i,t}$ with $g_i(s) := \eta_i^{-1}(s)$, the (unique) inverse of the variance profile, via $\tilde{y}_{i,t} = y_{i,\lfloor g_i(t/T)T \rfloor}$, $t = 0, \ldots, T$ yields a series that satisfies the invariance principle [Cavaliere and Taylor, 2008b, Eq. 19]

$$T^{-1/2}\tilde{y}_{i,\lfloor sT \rfloor} \Rightarrow \sqrt{\int_0^1 \sigma^2_{i,\lfloor rT \rfloor}\,\mathrm{d}r}\, C_i(1)B(s). \tag{4}$$

Numerically inverting the (uniformly consistent) estimator of $\eta_i(s)$,

$$\hat{\eta}_i(s) = \frac{\sum_{t=1}^{\lfloor sT \rfloor} \hat{u}^2_{i,t} + (sT - \lfloor sT \rfloor)\hat{u}^2_{i,\lfloor sT \rfloor + 1}}{\sum_{t=1}^{T} \hat{u}^2_{i,t}} \tag{5}$$

to obtain $\hat{g}_i(s)$, one can then transform the series via $y_{i,\lfloor \hat{g}_i(t/T)T \rfloor}$ so as to converge to standard Brownian Motions. Here, $\hat{u}_{i,t}$ denotes the residuals of a regression of $y_{i,t}$ on $y_{i,t-1}$. Conveniently, these transformations are 'non-parametric' in the sense that they require no knowledge of either break type, number or date. Unit root statistics applied to the transformed data will then satisfy their well-known homoscedastic limiting null distributions. More specifically, Cavaliere and Taylor [2008b] consider the $\mathcal{M}$ tests by Ng and Perron [2001].[6] Let $s^2_{i,\mathrm{AR}}(k_i) := \frac{\hat{\sigma}^2_i}{1 - \sum_{j=1}^{k_i} \hat{\beta}_{i,j}}$, where $\hat{\beta}_{i,j}$ and $\hat{\sigma}^2_i$ can be estimated with an OLS regression of $\hat{u}_{i,t}$ on k_i lagged values. The lag orders k_i can be chosen by one of the common selection criteria. The tests are then defined by the statistics

[5] Simes [1986, Thm. 1] proves that the S^H test has type I error probability equal to α when the test statistics are independent. As argued in the Introduction, the assumption of independence is unlikely to be met in most, if not all, applications of panel unit root tests. Fortunately, Sarkar [1998] shows that the assumption of independence is not necessary and can, in fact, be weakened substantially. Specifically, his Proposition 3.1 proves that Simes' test is level α if the test statistics are *multivariate totally positive of order 2* (MTP_2). See Hanck [2013] for further discussion of the test's properties.

[6] As given here, the statistics are for the no deterministics case $\mu_i = 0$. See Cavaliere and Taylor [2008b, Sec. 5] for the suitable modifications in the presence of deterministic trends.

$\mathcal{MZ}_{\alpha,i} := T^{-1}y_{i,T}^2 - s_{i,\mathrm{AR}}^2(k)/(2T^{-2}\sum_{t=1}^T y_{i,\lfloor\hat{g}_i(t/T)T\rfloor}^2)$, $\mathcal{MSB}_i := \left(T^{-2}\sum_{t=1}^T y_{i,\lfloor\hat{g}_i(t/T)T\rfloor}^2/s_{i,\mathrm{AR}}^2(k)\right)^{1/2}$ and $\mathcal{MZ}_{t,i} := \mathcal{MZ}_{\alpha,i} \times \mathcal{MSB}_i$, for which Cavaliere and Taylor [2008b] derive the associated limiting distributions

$$\mathcal{MZ}_{\alpha,i} \Rightarrow \frac{B(1)^2 - 1}{2\int_0^1 B(s)^2\,\mathrm{d}s}, \qquad \mathcal{MSB}_i \Rightarrow \left(\int_0^1 B(s)^2\,\mathrm{d}s\right)^{1/2},$$
$$\mathcal{MZ}_{t,i} \Rightarrow \frac{B(1)^2 - 1}{\left(4\int_0^1 B(s)^2\,\mathrm{d}s\right)^{1/2}}. \tag{6}$$

$\mathcal{MZ}_{\alpha,i}$ and $\mathcal{MZ}_{t,i}$ reject for large negative values, whereas $\mathcal{MSB}_i$ rejects for small values.

In addition, nonstationary-volatility robust versions of the well-known and more widely used Dickey and Fuller [1979] tests are available, given by the t-statistic t_{ϕ_i} of the augmented regression $\Delta y_{i,\lfloor\hat{g}_i(t/T)T\rfloor} = (\phi_i - 1)y_{i,\lfloor\hat{g}_i(\frac{t-1}{T})T\rfloor} + \sum_{j=1}^{k_i}\delta_{i,j}\Delta y_{i,\lfloor\hat{g}_i(\frac{t-j}{T})T\rfloor} + u_{i,t}$, and the coefficient statistic $T(\hat{\phi}_i - 1)/(1 - \sum_{j=1}^{k_i}\hat{\delta}_{i,j})$. The asymptotic null distributions of t_{ϕ_i} and $T(\hat{\phi}_i - 1)/(1 - \sum_{j=1}^{k_i}\hat{\delta}_{i,j})$ then are

$$(i)\quad \frac{T(\hat{\phi}_i - 1)}{1 - \sum_{j=1}^{k_i}\hat{\delta}_{i,j}} \Rightarrow \frac{B(1)^2 - 1}{2\int_0^1 B(s)^2\,\mathrm{d}s} \quad \text{and} \quad (ii)\quad t_{\phi_i} \Rightarrow \frac{B(1)^2 - 1}{\left(4\int_0^1 B(s)^2\,\mathrm{d}s\right)^{1/2}}. \tag{7}$$

To see this, let $\omega_i = \sqrt{\int_0^1 \sigma_{i,\lfloor rT\rfloor}^2\,\mathrm{d}r}$, $\breve{y}_{i,t} = y_{i,\lfloor\hat{g}_i(t/T)T\rfloor}$ and $\breve{u}_{i,t} = \Delta\breve{y}_{i,t}$. The result then follows straightforwardly from (4), uniform consistency of the $\hat{g}_i$ and the Continuous Mapping Theorem, analogously to Cavaliere and Taylor [2008b]: as in Hamilton [1994, Sec. 17.7], under the null we jointly have $T^{-1}\sum_{t=1}^T \breve{y}_{i,t-1}\breve{u}_{i,t} \Rightarrow 1/2\omega_i^2 C_i(1)(B(1)^2 - 1)$ and $T^{-2}\sum_{t=1}^T \breve{y}_{i,t-1}^2 \Rightarrow \omega_i^2 C_i(1)^2\int_0^1 B(s)^2\,\mathrm{d}s$. Asymptotically, the estimation error of the $\delta_{i,j}$ vanishes [Hamilton, 1994, Eq. 17.7.18] and

$$\begin{aligned} T(\hat{\phi}_i - 1) &= T^{-1}\sum_{t=1}^T \breve{y}_{i,t-1}\breve{u}_{i,t}\Big/ T^{-2}\sum_{t=1}^T \breve{y}_{i,t-1}^2 + o_p(1) \\ &\Rightarrow \frac{0.5\omega_i^2 C_i(1)(B(1)^2 - 1)}{\omega^2 C_i(1)^2\int_0^1 B(s)^2\,\mathrm{d}s} = \frac{0.5(B(1)^2 - 1)}{C_i(1)\int_0^1 B(s)^2\,\mathrm{d}s}. \end{aligned}$$

Result (i) then follows by Hamilton [1994, Eq. 17.7.34], from which $1/(1 - \sum_{j=1}^{k_i}\hat{\delta}_{i,j}) \to_p C_i(1)$. Result (ii) follows analogously.

The p-values required for the S^H test can thus be obtained by simulating the asymptotic distributions (6) and (7).[7]

Remark 1. Simes' test is likely to be most useful for small to moderate n and large T. This is because (3) becomes more severe with n. Also, as $T \to \infty$, the p-values corresponding to the false $H_{i,0}$ will tend to 0 in probability. Hence (3) will be satisfied for any finite n. Put differently, S^H is consistent for $T \to \infty$ and $n < \infty$ [Hanck, 2013]. We corroborate the above intuition in the Monte Carlo section. What is more, unlike for most other panel unit root tests it is not necessary for consistency of the S^H

[7] We also worked with MacKinnon's [1996] response surface p-values. These did however not perform consistently better than the ones relying on (6) and (7). This may be because the finite sample distribution of Cavaliere and Taylor-type tests need not coincide with those of the Dickey-Fuller tests, even if the asymptotic ones agree. Detailed results are available upon request.

test that the fraction of stationary series is strictly positive. Intuitively, this is because S^H rejects the global null already if one sufficiently small p-value can be found.

Remark 2. Of course, other nonstationary-volatility robust time series unit root tests might also be employed to construct panel tests using (3), cf. e.g. Beare [2008] or the bootstrap based tests of Cavaliere and Taylor [2008a]. As such, our choice of the present Cavaliere and Taylor-type tests could be extended in other directions (see however the remarks in the Monte Carlo section).

Remark 3. Existing panel unit root tests are silent about the size of the fraction or the identity of the stationary units.[8] As further discussed in Hanck [2013], one can easily determine the units for which the alternative of stationarity can be said to hold, using the p-values from the S^H test. Hommel [1988, Sec. 2] proves that the following procedure controls the FWER, i.e. the probability to falsely reject at least one true $H_{i,0}$, at multiple level α whenever the S^H test is level-α for the hypothesis (2).[9]

Hommel's Procedure

(i) Compute

$$j = \max\{i \in \mathbb{N}_n : p_{(n-i+k)} > k\alpha/i \ \text{ for } \ k \in \mathbb{N}_i\}. \tag{8}$$

(ii) If $p_{(n)} \leqslant \alpha$, reject all $H_{i,0}$. Else, reject all $H_{i,0}$ with $p_i \leqslant \alpha/j$.

Remark 4. This approach can easily be extended to the case of nonstationary-volatility robust panel cointegration by using the p-values of a suitable cointegration test such as, for instance, the wild bootstrap-based implementation of the classical Johansen [1988] rank test proposed by Cavaliere, Rahbek and Taylor [2010].

3 Monte Carlo Simulations

This section investigates the size and power of S^H. We use the following simple DGP:

$$y_{i,t} = \phi_i y_{i,t-1} + \epsilon_{i,t} \qquad (i \in \mathbb{N}_n,\ t \in \mathbb{N}_T)$$

To introduce nonstationary volatility, we generate a permanent break in the innovation variance of normal variates $\xi_{i,t}$ at $\lfloor \tau T \rfloor$, where $\text{Var}(\xi_{i,t}) = 1$ for $t = 1, \ldots, \lfloor \tau T \rfloor$ and $\text{Var}(\xi_{i,t}) = 1/\delta^2$ for $t = \lfloor \tau T \rfloor + 1, \ldots, T$. We consider $\tau \in \{0.1, 0.5, 0.9\}$, corresponding to early, middle and late breaks, and $\delta \in \{1/5, 5\}$ to generate positive and negative breaks, respectively. To gauge the effect of serial correlation, we generate $\tilde{\xi}_{i,t} = \xi_{i,t} + \psi \xi_{i,t-1}$, where $\psi \in \{0, 0.5\}$. Finally, we consider two schemes to generate cross-sectional correlation among the $\epsilon_{i,t}$.[10]

[8] For instance, Shin, Park and Oh [2009] and Demetrescu and Hanck [2012] suggest a PURT that indeed allow for heteroscedastic errors as well as cross-sectional dependence of the series, but is not designed to give the portion of units that are responsible for the rejection of the 'global' unit-root null.

[9] Recently, Romano and Wolf [2005], Chortareas and Kapetanios [2009], Smeekes [2011] and Moon and Perron [2012] have proposed alternative sequential testing procedures, which upon rejection of the 'global' null also allow for the separation of stationary from non-stationary units in a panel. Although these could also be combined with a nonstationary-volatility robust time series unit root test, our choice is motivated by the straightforwardness of the implementation implied by our framework.

[10] We create 30 initial observations before using the $y_{i,t}$ to mitigate the effect of initial conditions under H_A.

Table II—Size of the S^H Test Using Different Time Series Tests.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
(i) $\delta = 1/5$																
	$\mathcal{MZ}_t$	.070	.058	.064	.059	.051	.057	.048	.047	.050	.045	.158	.079	.040	.018	.032
	$T(\hat{\phi} - 1)$	.163	.103	.067	.040	.044	.092	.070	.056	.049	.044	.018	.026	.017	.022	.024
8	$\mathcal{MZ}_\alpha$	.045	.042	.053	.055	.042	.051	.036	.039	.033	.036	.145	.071	.034	.014	.024
	t_ϕ	.036	.041	.038	.033	.035	.036	.042	.047	.039	.040	.007	.013	.011	.012	.018
	$\mathcal{MSB}$	.045	.038	.040	.040	.032	.040	.034	.033	.033	.032	.134	.059	.026	.011	.021
	$\mathcal{MZ}_t$	.067	.056	.063	.056	.048	.064	.042	.042	.044	.044	.231	.083	.036	.020	.028
	$T(\hat{\phi} - 1)$	.198	.131	.072	.039	.048	.107	.067	.058	.046	.052	.018	.024	.014	.029	.025
12	$\mathcal{MZ}_\alpha$	.054	.048	.044	.049	.030	.052	.036	.035	.032	.037	.212	.074	.028	.012	.022
	t_ϕ	.031	.036	.040	.027	.038	.034	.029	.038	.034	.043	.012	.010	.011	.017	.013
	$\mathcal{MSB}$	.043	.034	.042	.038	.030	.044	.027	.022	.028	.030	.197	.064	.022	.007	.017
	$\mathcal{MZ}_t$	.056	.046	.060	.046	.048	.074	.040	.042	.034	.035	.330	.115	.033	.009	.022
	$T(\hat{\phi} - 1)$	.263	.193	.091	.049	.052	.123	.091	.070	.059	.052	.015	.021	.017	.018	.018
24	$\mathcal{MZ}_\alpha$	.053	.032	.056	.035	.039	.060	.037	.029	.028	.033	.312	.106	.028	.008	.018
	t_ϕ	.036	.051	.043	.032	.041	.028	.044	.045	.040	.040	.010	.010	.010	.009	.010
	$\mathcal{MSB}$	.039	.030	.041	.030	.030	.053	.028	.027	.021	.020	.288	.091	.022	.006	.012
	$\mathcal{MZ}_t$	.051	.052	.064	.060	.039	.085	.042	.031	.029	.031	.483	.186	.036	.007	.014
	$T(\hat{\phi} - 1)$	.420	.253	.106	.062	.052	.184	.117	.074	.065	.065	.016	.021	.019	.017	.024
48	$\mathcal{MZ}_\alpha$	.051	.049	.048	.032	.031	.072	.028	.033	.025	.021	.460	.169	.031	.006	.010
	t_ϕ	.032	.044	.046	.044	.039	.035	.042	.041	.043	.046	.009	.010	.010	.010	.015
	$\mathcal{MSB}$	.031	.028	.042	.036	.019	.068	.027	.016	.017	.020	.437	.153	.024	.004	.007
(ii) $\delta = 5$																
	$\mathcal{MZ}_t$	.018	.015	.012	.012	.017	.016	.013	.018	.022	.025	.016	.023	.024	.031	.031
	$T(\hat{\phi} - 1)$	.106	.111	.112	.098	.087	.090	.087	.057	.050	.052	.068	.056	.047	.048	.048
8	$\mathcal{MZ}_\alpha$	.010	.011	.010	.010	.013	.009	.006	.018	.016	.018	.014	.014	.023	.024	.021
	t_ϕ	.011	.018	.026	.027	.034	.013	.018	.021	.022	.026	.006	.012	.016	.025	.029
	$\mathcal{MSB}$	.011	.009	.007	.008	.011	.008	.006	.012	.015	.015	.010	.011	.016	.014	.018
	$\mathcal{MZ}_t$	.019	.013	.013	.014	.016	.014	.012	.017	.020	.019	.016	.017	.026	.027	.024
	$T(\hat{\phi} - 1)$	.120	.146	.132	.120	.094	.106	.082	.070	.054	.058	.070	.056	.052	.052	.051
12	$\mathcal{MZ}_\alpha$	.011	.009	.010	.010	.013	.007	.009	.014	.016	.017	.011	.011	.012	.026	.023
	t_ϕ	.008	.022	.029	.034	.033	.007	.010	.024	.021	.027	.006	.010	.022	.022	.025
	$\mathcal{MSB}$	.008	.007	.008	.007	.009	.005	.007	.011	.010	.010	.008	.007	.014	.016	.013
	$\mathcal{MZ}_t$	.020	.010	.012	.011	.011	.008	.012	.015	.016	.018	.011	.018	.023	.023	.025
	$T(\hat{\phi} - 1)$	.157	.185	.152	.137	.123	.136	.108	.069	.064	.063	.086	.078	.054	.056	.058
24	$\mathcal{MZ}_\alpha$	.012	.006	.006	.009	.010	.012	.006	.010	.013	.018	.011	.011	.015	.022	.019
	t_ϕ	.011	.020	.030	.034	.041	.010	.011	.017	.026	.028	.003	.009	.019	.026	.031
	$\mathcal{MSB}$	.010	.005	.003	.006	.008	.004	.006	.009	.012	.013	.006	.010	.014	.015	.018
	$\mathcal{MZ}_t$	.021	.010	.010	.008	.014	.011	.006	.019	.016	.014	.016	.013	.016	.020	.015
	$T(\hat{\phi} - 1)$	.217	.252	.220	.180	.160	.176	.138	.092	.078	.060	.115	.092	.072	.060	.048
48	$\mathcal{MZ}_\alpha$	.011	.006	.007	.006	.012	.015	.008	.007	.014	.010	.012	.008	.017	.020	.017
	t_ϕ	.010	.023	.044	.042	.044	.005	.016	.024	.026	.026	.004	.012	.020	.025	.020
	$\mathcal{MSB}$	.009	.005	.005	.006	.010	.007	.002	.010	.007	.007	.008	.008	.008	.011	.011

Note: $\psi = 0$, $\boldsymbol{\phi} = \boldsymbol{\imath}_n$. Factor dependent Disturbances.

A. *Equicorrelation*: Let $\tilde{\boldsymbol{\xi}}_t = (\tilde{\xi}_{1,t}, \ldots, \tilde{\xi}_{n,t})'$. Then, generate $\boldsymbol{\varepsilon}_t := (\epsilon_{1,t}, \ldots, \epsilon_{n,t})' = \boldsymbol{\Sigma}^{1/2}\tilde{\boldsymbol{\xi}}_t$, where $\boldsymbol{\Sigma} = \theta\boldsymbol{\imath}_n\boldsymbol{\imath}_n' + (1-\theta)\boldsymbol{I}_n$ with $\boldsymbol{\imath}_n = (1, \ldots, 1)'$, $(n \times 1)$, $\theta = 0.5$ and $\boldsymbol{I}_n$ the $(n \times n)$ identity matrix.

B. *Factor Structure*: Let $\epsilon_{i,t} := \lambda_i \cdot \nu_t + \tilde{\xi}_{i,t}$, where ν_t are i.i.d. $\mathcal{N}(0,1)$ and $\lambda_i \sim \mathcal{U}(-1,3)$, with $\mathcal{U}$ denoting the uniform distribution.

Remark 5. Another relevant scenario would be that of $I(1)$ common factors ν_t. Hanck [2013] finds Simes' test to work well when applied to Bai and Ng [2004]-type defactored idiosyncratic components.

TABLE III—POWER OF THE S^H TEST USING DIFFERENT TIME SERIES TESTS.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
(*i*) $\delta = 1/5$																
	$\mathcal{MZ}_t$	.262	.551	.992	1.00	1.00	.218	.313	.752	.978	.999	.580	.514	.602	.610	.798
	$T(\hat{\phi}-1)$	.289	.356	.860	.996	1.00	.160	.200	.516	.857	.981	.063	.150	.225	.306	.432
8	$\mathcal{MZ}_\alpha$	.207	.429	.955	.999	1.00	.162	.232	.573	.888	.988	.546	.478	.562	.565	.765
	t_ϕ	.091	.196	.784	.993	1.00	.081	.138	.447	.823	.977	.053	.120	.200	.294	.420
	$\mathcal{MSB}$	.194	.429	.961	1.00	1.00	.160	.224	.612	.943	.995	.505	.426	.500	.486	.712
	$\mathcal{MZ}_t$	.268	.542	.992	1.00	1.00	.218	.302	.774	.982	1.00	.457	.353	.375	.354	.580
	$T(\hat{\phi}-1)$	.321	.419	.885	1.00	1.00	.159	.213	.522	.892	.996	.045	.084	.132	.188	.265
12	$\mathcal{MZ}_\alpha$	.198	.370	.930	1.00	1.00	.172	.190	.513	.838	.980	.428	.309	.335	.304	.534
	t_ϕ	.080	.208	.808	.999	1.00	.084	.136	.440	.847	.992	.030	.063	.101	.167	.241
	$\mathcal{MSB}$	.179	.398	.970	1.00	1.00	.166	.204	.618	.944	.999	.394	.276	.289	.260	.478
	$\mathcal{MZ}_t$	.275	.579	1.00	1.00	1.00	.180	.194	.502	.873	.990	.697	.501	.535	.470	.786
	$T(\hat{\phi}-1)$	.442	.532	.964	1.00	1.00	.181	.204	.379	.697	.949	.052	.124	.188	.279	.424
24	$\mathcal{MZ}_\alpha$	.228	.530	.998	1.00	1.00	.218	.235	.647	.963	1.00	.671	.461	.492	.428	.750
	t_ϕ	.077	.223	.905	1.00	1.00	.075	.106	.290	.620	.925	.041	.071	.148	.242	.393
	$\mathcal{MSB}$	.186	.417	.995	1.00	1.00	.143	.140	.351	.742	.962	.644	.414	.433	.364	.689
	$\mathcal{MZ}_t$	.315	.699	1.00	1.00	1.00	.306	.287	.775	.999	1.00	.871	.690	.686	.593	.921
	$T(\hat{\phi}-1)$	.600	.687	.996	1.00	1.00	.282	.294	.620	.967	1.00	.068	.139	.249	.360	.542
48	$\mathcal{MZ}_\alpha$	.186	.356	.990	1.00	1.00	.279	.258	.718	.996	1.00	.855	.660	.642	.545	.895
	t_ϕ	.088	.264	.979	1.00	1.00	.081	.151	.476	.927	1.00	.031	.084	.188	.305	.492
	$\mathcal{MSB}$	.224	.506	1.00	1.00	1.00	.232	.196	.600	.986	1.00	.826	.604	.564	.456	.833
(*ii*) $\delta = 5$																
	$\mathcal{MZ}_t$	.218	.456	.904	.977	.990	.120	.283	.887	.996	1.00	.161	.493	.991	1.00	1.00
	$T(\hat{\phi}-1)$	.483	.694	.970	.997	.999	.338	.535	.956	.999	1.00	.295	.622	.994	1.00	1.00
8	$\mathcal{MZ}_\alpha$	.182	.407	.886	.973	.988	.070	.186	.720	.949	.994	.079	.196	.780	.992	1.00
	t_ϕ	.126	.395	.922	.991	.998	.097	.285	.894	.998	1.00	.082	.374	.979	1.00	1.00
	$\mathcal{MSB}$	.150	.338	.861	.968	.987	.072	.186	.807	.990	.999	.103	.358	.979	1.00	1.00
	$\mathcal{MZ}_t$	.092	.175	.560	.815	.928	.078	.176	.737	.964	.994	.067	.157	.692	.976	1.00
	$T(\hat{\phi}-1)$	.369	.486	.780	.934	.978	.263	.423	.874	.991	1.00	.167	.294	.777	.981	.999
12	$\mathcal{MZ}_\alpha$	.063	.143	.522	.794	.914	.038	.078	.388	.747	.913	.034	.066	.291	.669	.915
	t_ϕ	.042	.161	.588	.857	.952	.047	.184	.728	.975	.999	.019	.108	.596	.958	.999
	$\mathcal{MSB}$	.052	.115	.472	.765	.891	.046	.110	.626	.932	.987	.040	.101	.561	.939	.998
	$\mathcal{MZ}_t$	.167	.283	.837	.985	.999	.066	.170	.737	.989	1.00	.142	.446	.999	1.00	1.00
	$T(\hat{\phi}-1)$	.559	.695	.954	.998	1.00	.340	.492	.908	.999	1.00	.339	.708	.999	1.00	1.00
24	$\mathcal{MZ}_\alpha$	.125	.236	.801	.980	.999	.088	.256	.946	1.00	1.00	.087	.275	.975	1.00	1.00
	t_ϕ	.067	.281	.852	.991	1.00	.044	.180	.751	.994	1.00	.047	.349	.994	1.00	1.00
	$\mathcal{MSB}$	.091	.196	.750	.970	.998	.042	.106	.602	.966	.999	.097	.293	.993	1.00	1.00
	$\mathcal{MZ}_t$	.245	.316	.899	.999	1.00	.090	.221	.912	1.00	1.00	.142	.474	1.00	1.00	1.00
	$T(\hat{\phi}-1)$	.702	.820	.988	1.00	1.00	.454	.656	.984	1.00	1.00	.435	.818	1.00	1.00	1.00
48	$\mathcal{MZ}_\alpha$	.190	.271	.862	.997	1.00	.076	.175	.891	.999	1.00	.100	.262	.991	1.00	1.00
	t_ϕ	.070	.336	.921	.998	1.00	.041	.242	.927	1.00	1.00	.050	.361	1.00	1.00	1.00
	$\mathcal{MSB}$	.160	.219	.821	.994	1.00	.054	.136	.822	.998	1.00	.095	.315	.998	1.00	1.00

Note: $\psi = 0$, $\boldsymbol{\phi} = (\boldsymbol{\imath}'_{n/2}, \widetilde{\boldsymbol{\phi}}'_{n/2})'$ with $(\widetilde{\boldsymbol{\phi}}_{n/2})_i \sim \mathcal{U}(.75, 1)$. Factor dependent Disturbances.

To keep the present designs manageable, we waive to analyze this scenario here. Also, one could study the behavior of S^H in the presence of deterministic trends or drifts. However, the distribution of the single-unit $\mathcal{M}$ tests from (6) would then depend on the particular variance profile, see Cavaliere and Taylor [2008b, Thm. 2]. This makes a large-scale simulation study somewhat inconvenient to implement. A similar comment applies to allowing for broken deterministic trend functions. Some unreported pilot simulations indicate that, as expected, the power of S^H is somewhat lower in the presence of deterministic trends. See Section 4 for empirical results in the presence of (broken) trends.

When $\boldsymbol{\phi} := (\phi_1, \ldots, \phi_n)' = \boldsymbol{\imath}_n$, $H_0 = \bigcap_{i \in \mathbb{N}_n} H_{i,0}$ is true, allowing us to study the size of the tests. Choosing $\boldsymbol{\phi}$ such that $\min_i |\boldsymbol{\phi}| < 1$, we analyze power of the tests. More specifically, we let $\boldsymbol{\phi} = (\boldsymbol{\imath}'_{n/2}, \widetilde{\boldsymbol{\phi}}'_{n/2})'$ and $\boldsymbol{\phi} = \widetilde{\boldsymbol{\phi}}_n$ to investigate stationary alternatives. The components of $\widetilde{\boldsymbol{\phi}}$ are distributed as $(\widetilde{\boldsymbol{\phi}})_i \sim \mathcal{U}(3/4, 1)$ in the first power experiment and as $(\widetilde{\boldsymbol{\phi}})_i \sim \mathcal{U}(0.9, 1)$ in the second power experiment. Based on $2,500$ replications, we calculate the rejection rates of the S^H test based on the statistics from (6) and (7). When $\psi \neq 0$, we select k_i using the criterion of Ng and Perron [1995].

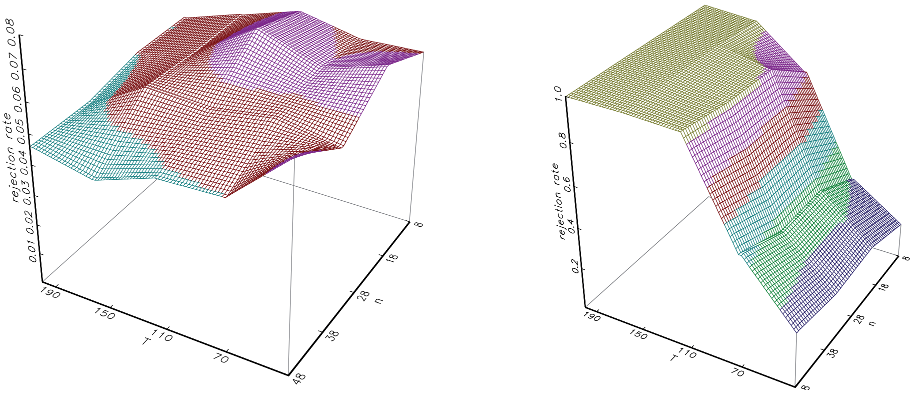
Selected results for scenario B (factor dependent disturbances) are reported in Tables II to III.[11] The entries after, e.g., $\mathcal{MSB}$ denote the rejection rates for the S^H test when the p-values (3) are calculated from n $\mathcal{MSB}_i$ test statistics, defined in (6). The main findings can be summarized as follows.

The S^H test is a level-α test throughout, at least for moderate and large T. There are some exceptions to this finding for $T(\hat{\phi} - 1)$. When δ is large and τ is small, or vice-versa, S^H is somewhat conservative. This is not surprising as this is precisely the case when the time series tests of Cavaliere and Taylor [2008b] underreject, too. Apparently, $T = 30$ is an insufficient time series length to achieve satisfactorily accurate estimation of the variance profile. Indeed, for small T the profile estimation may produce stretches of identical observations, that obviously poorly approximate the actual time series. (Cavaliere and Taylor [2008b] only start their simulations at $T = 100$.) Some unreported simulations suggest that there are indeed size distortions in the time series tests that then inevitably carry over into the panel tests via erroneously small (the oversized case) or large (the undersized case) p-values. We therefore also experimented with the wild bootstrap unit root tests of Cavaliere and Taylor [2008a]. However, these exhibit similar small-T size distortion as the $\mathcal{M}$ tests, which feed analogously into the panel tests.

In other cases, size is well-controlled. No clear pattern emerges as to which underlying time series yields the best performance of the S^H test. Thus, a case can be made to recommend the popular and widely implemented Dickey and Fuller [1979] nonstationary-volatility robust t_ϕ test developed here.

Concerning power (Table III), we again find no clear ranking of the different underlying tests, as power quickly grows with T for all variants of the S^H test. (In view of the size distortions of the second-generation panel unit root tests found in Table I it does not seem useful to include these here.) Similar to the findings for size, the tests perform better in the sense of having higher power when the breaks in the innovation variance are either early positive (both δ and τ small) or late negative (both δ and τ large), consistent with the time series evidence of Cavaliere and Taylor [2008b]. Also note that power is higher in those panels where only half of the series are stationary ($\boldsymbol{\phi} = (\boldsymbol{\imath}'_{n/2}, \widetilde{\boldsymbol{\phi}}'_{n/2})'$)

[11] We do not report the qualitatively similar results for equicorrelation for brevity. The full set of results is available upon request. In particular, we further do not report results for $\psi \neq 0$. As one would expect, these are worse than those under no autocorrelation, with some severe upward size distortions for small T, which however vanish with increasing T. These size distortions are caused by the well-known sensitivity of time series unit root tests to moving-average disturbances, which then carry over into the panel test. It is also worth noting that in case of $\delta = 1$ the Cavaliere and Taylor [2008b, Tables I, IV and VIII] testing procedure does not suffer from noticeable size or power distortions. Unreported simulations indicate that this also holds for the panel test.



Size scales: Magenta > 0.06, $0.05 <$ Red < 0.06, $0.04 <$ Green < 0.05

Power scales: Blue < 0.35, $0.35 <$ Green < 0.6, $0.6 <$ Cyan < 0.75, $0.75 <$ Magenta < 0.9, $0.9 <$ Brown

Figure I—Rejection Rates for the $\mathcal{MZ}_t$ test

than in the entirely stationary panel ($\boldsymbol{\phi} = \widetilde{\boldsymbol{\phi}}_n$). This is because the cutoff criterion for the S^H test, (3), is more likely to be satisfied for some i when there are strongly stationary series in the panel, as the corresponding p-values will then be closer to their probability limit of 0 for finite T than if $\boldsymbol{\phi} \approx \boldsymbol{\imath}$.

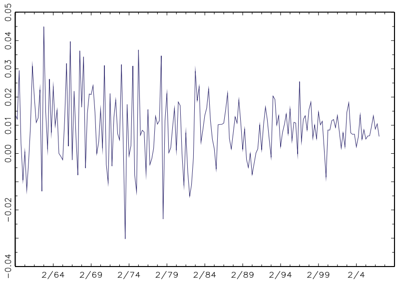
Part of the information contained in the Tables is visualized in Figure I. We provide power results (cf. the right panel) for a setting where size is well-controlled (cf. the left panel). It is seen that power increases quickly in T, but much slower in n, which confirms the intuition offered in Remark 1.

4 Unit Roots in Panel Data and the 'Great Moderation'

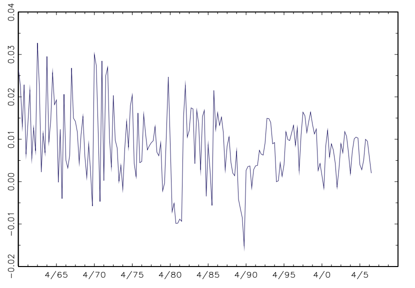
In order to demonstrate the relevance of our methodology for macroeconomic and financial problems, we now apply the tests to two longstanding questions in empirical macroeconomics. Section 4.1 revisits the issue of whether per capita GDP series have a random walk with drift or are better described as stationary around a linear trend. Section 4.2 tests for unit roots in a panel of inflation rates.

4.1 GDP Stationarity

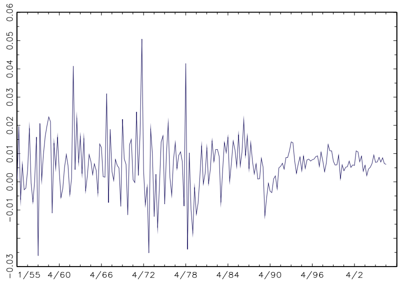
We now apply the S^H test to investigate the null hypothesis that there is a unit root in the (logarithms of) GDPs in a panel of OECD countries. At least since the seminal work of Nelson and Plosser [1982], the possible nonstationarity of GDPs has been a cornerstone of empirical macroeconomics. As emphasized for instance by Campbell and Perron [1991], the distinction between trend stationarity and difference stationarity is potentially important in many contexts, such as forecasting, because the trend- and difference stationary models may imply very different dynamics. If the series contain



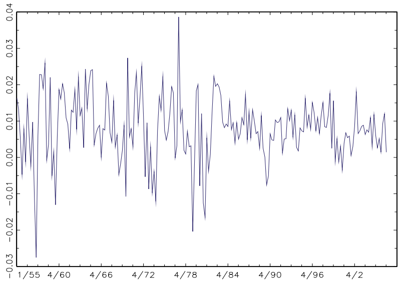
Australia



Canada



United Kingdom



United States

Figure II—Some historical growth rates

a unit root, shocks have persistent effects. As a result, the series do not return to their former path following a random disturbance, and the level of the series shifts permanently. On the other hand, if the series do not contain a unit root, the underlying trends are deterministic. In this case the series return to their steady trend after the shock. Hence, the forecasts implied by the two models are vastly different.

Some prominent papers in this literature are Cochrane [1988], who finds a small random walk component in U.S. GDP, whereas Cogley [1990] and Kormendi and Meguire [1990] identify stronger random-walk behavior in multi-country studies. Kwiatkowski, Phillips, Schmidt and Shin [1992] report weak evidence against the null of trend stationarity for U.S. GNP. Relying on Bayesian techniques, DeJong and Whiteman [1991] forcefully argue against the presence of a unit root in U.S. GDP. The debate appears to be far from settled, with recent contributions both supporting [Vougas, 2007] and rejecting [Murray and Nelson, 2000; Darné, 2009] stationarity.

Recently, panel methods have been used to investigate GDP stationarity in industrialized countries. Using first generation tests, Strauss [2000] finds evidence of stationarity in a panel of U.S. states. On the contrary, Rapach's [2002] study using Levin *et al.*'s [2002] and Im *et al.*'s [2003] tests cannot reject the null of nonstationarity for a panel of international output levels.

However, all of the above results are obtained within the paradigm of homoscedastic (panel) unit root tests. We believe that these results may not be reliable in view of what is known as the 'Great

Table IV—Sorted p-values of Nonstationary-Volatility Robust Unit Root Tests on OECD Output Series.

	$\mathcal{MZ}_t$		t_ϕ		$\mathcal{MZ}_\alpha$		$T(\hat{\phi}-1)$		$\mathcal{MSB}$		Simes' cutoff
$p_{(1)}$	.009	AUT	.000	IRL	.023	ISL	.003	IRL	.018	ISL	.002
$p_{(2)}$	.009	IRL	.026	AUT	.119	DNK	.107	FRA	.112	DNK	.003
$p_{(3)}$	.036	ISL	.173	ISL	.166	IRL	.165	ISL	.192	NOR	.005
$p_{(4)}$	.090	FIN	.201	FRA	.182	GBR	.224	NOR	.197	LUX	.007
$p_{(5)}$	.149	DNK	.262	GER	.193	NOR	.246	POL	.202	GBR	.008
$p_{(6)}$	.199	GBR	.271	POL	.226	LUX	.272	LUX	.431	GER	.010
$p_{(7)}$	.223	NOR	.276	CAN	.251	AUT	.278	GER	.615	POL	.012
$p_{(8)}$	.280	CAN	.296	LUX	.412	GER	.297	AUT	.619	IRL	.013
$p_{(9)}$	.283	LUX	.322	NOR	.472	POL	.411	DNK	.772	PRT	.015
$p_{(10)}$	.343	POL	.404	GBR	.638	PRT	.411	CZE	.785	AUT	.017
$p_{(11)}$	.422	GER	.415	DNK	.720	NZL	.417	GBR	.799	NLD	.018
$p_{(12)}$	.529	PRT	.566	NZL	.746	CAN	.665	NZL	.846	NZL	.020
$p_{(13)}$	.606	NZL	.656	SWE	.752	NLD	.704	SWE	.858	US	.022
$p_{(14)}$	.708	NLD	.682	AUS	.762	FIN	.705	FIN	.860	FRA	.023
$p_{(15)}$	.712	AUS	.708	FIN	.794	FRA	.714	MEX	.868	MEX	.025
$p_{(16)}$	.742	FRA	.728	TUR	.802	AUS	.727	CAN	.907	SVK	.027
$p_{(17)}$	.843	TUR	.755	BEL	.920	BEL	.749	AUS	.910	KOR	.028
$p_{(18)}$	.861	SWE	.759	PRT	.920	MEX	.789	TUR	.943	AUS	.030
$p_{(19)}$	.872	BEL	.781	MEX	.925	SWE	.791	BEL	.963	GRC	.032
$p_{(20)}$	.920	MEX	.809	NLD	.928	US	.802	CHE	.976	JPN	.033
$p_{(21)}$	.928	US	.828	CHE	.965	ESP	.845	JPN	.987	CHE	.035
$p_{(22)}$	.947	ESP	.852	ESP	.967	CHE	.854	GRC	.988	BEL	.037
$p_{(23)}$	.950	CHE	.869	JPN	.976	JPN	.863	NLD	.988	HUN	.038
$p_{(24)}$	.970	JPN	.887	GRC	.981	GRC	.872	US	.991	CAN	.040
$p_{(25)}$	.978	GRC	.888	CZE	.984	TUR	.890	ESP	.996	SWE	.042
$p_{(26)}$	1.000	ITA	.890	US	.999	CZE	.906	PRT	.998	FIN	.043
$p_{(27)}$	1.000	KOR	.968	ITA	.999	ITA	.958	ITA	1.000	ITA	.045
$p_{(28)}$	1.000	HUN	.994	HUN	1.000	KOR	.964	SVK	1.000	TUR	.047
$p_{(29)}$	1.000	CZE	.997	SVK	1.000	HUN	.995	HUN	1.000	CZE	.048
$p_{(30)}$	1.000	SVK	.998	KOR	1.000	SVK	.998	KOR	1.000	ESP	.050

The sorted p-values from the test statistics described in (6) and (7) applied to OECD output data.

Moderation.' It is a well-established stylized fact that many countries enjoy a moderation of the business cycle and, more generally, reduced volatility in the growth rates of GDPs. See, for instance, Blanchard and Simon [2001] for some international evidence. The reasons for this decline are surveyed in Stock and Watson [2002] and include structural changes in output from goods to services, information-technology-led improvements in inventory management and innovations in financial markets.

See Figure II for some selected time series of historical growth rates of OECD countries (see below for a description of the dataset). It is readily apparent that the volatility of GDP growth is smaller since, in most cases, the 1980s. Concretely, the United States and Australia appear to experience reduced GDP-growth variance since the mid-80s, whereas the reduction seems to have set in somewhat later in Canada and the United Kingdom. As we saw in Section 2.1, traditional (panel) unit root tests produce misleading results in the presence of such nonstationary volatility. Furthermore, the above-

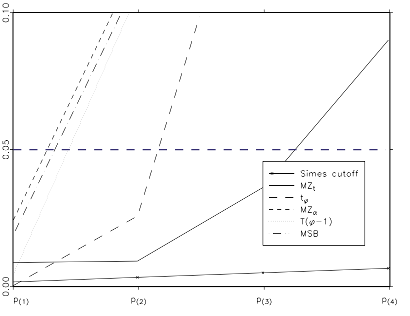


Figure III—First sorted p-values of different tests required for Simes' test

mentioned panel studies disregard the unquestionable presence of cross-sectional dependence among output levels—better known as 'Globalization.'

We therefore compute the S^H test to provide potentially more reliable inference in the presence of nonstationary volatility. An attractive feature of constructing a panel test of GDP stationarity using Cavaliere and Taylor's approach is that we can easily sidestep the debate whether the reduction in volatility is due to a break or a continuing downward trend in innovation variances. (The former view is held by Kim and Nelson [1999] and McConnell and Perez-Quiros [2000] whereas the findings of Blanchard and Simon [2001] support the latter.) As long as the innovation variances satisfy the mild assumption (see ass. 2) that $\sigma_{i,\lfloor sT \rfloor} \in \mathcal{D}$, the time-transformed time series unit root tests employed here will automatically adjust to the variance patterns in the different countries.

Our dataset comprises the seasonally adjusted quarterly GDP levels from 30 OECD members, constructed from data made available on the OECD website (series `LNBQRSA`, at 2000 prices in most cases). It runs until 2007Q4 and therefore does not cover the recent developments clouded by the financial crisis and the euro debt crisis that may imply the end of the Great Moderation.[12] The series start at varying dates, ranging from 1955Q1 in the case of the United Kingdom and the United States to 2000Q1 for Greece, Hungary and Ireland, yielding time series lengths ranging from $T = 32$ to $T = 212$. In view of the secular trend in Gross Domestic Products, we need to accommodate time trends to construct the test statistics in the present application. We thus calculate the time series test statistics from (6) and (7) employing the trend-corrected statistics as outlined in Cavaliere and Taylor [2008b, Sec. 5]. The p-values are then computed from the corresponding limiting distributions that are then functionals of detrended Brownian Motions. In the trend case, separate distributions arise for each country, as these then depend on the variance profile. E.g.,

$$\mathcal{MZ}_{\alpha,i} \Rightarrow \frac{F_{B_i|\breve{Z}}(1)^2 - F_{B_i|\breve{Z}}(0)^2 - 1}{2\int_0^1 F_{B_i|\breve{Z}}(s)^2 \, \mathrm{d}s}$$

where $F_{B_i|\breve{Z}}(s) := B_i(s) - \breve{Z}_i(s)'\big(\int_0^1 \breve{Z}_i(r)\breve{Z}_i(r)' \, \mathrm{d}r\big)^{-1} \int_0^1 \breve{Z}_i(r)B_i(r) \, \mathrm{d}r$ and $\breve{Z}_i(s) = Z(g_i(s))$, where

[12] To check for robustness of our findings, we have also included the recent period until 2012Q4 and found qualitatively the same results regarding the question of GDP stationarity.

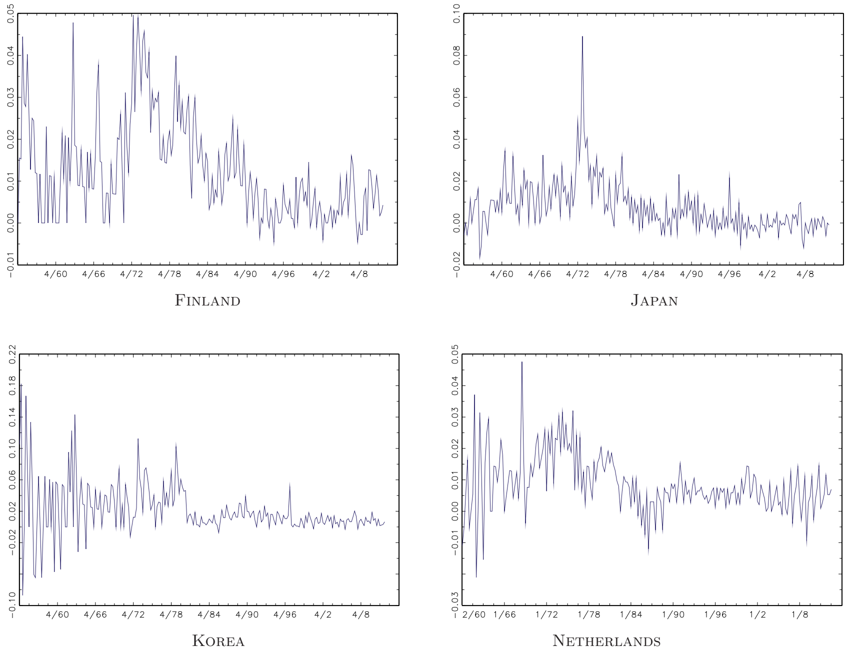


Figure IV—Some quarterly inflation rates

$\breve{Z}_i(s)$ is the suitably scaled limit of the trend function. We obtain these distributions from Cavaliere and Taylor's Theorem 2 from 50,000 draws from the functionals, approximating the Wiener processes with suitably normalized Gaussian random walks of length $T = 1,000$ and estimating the variance profile as in (5). The lag orders k_i required to account for autocorrelation in growth rates are chosen with the automatic criterion of Ng and Perron [1995].

Results are reported in Table IV. It is apparent that there is rather little evidence of stationarity of GDPs in the present OECD panel dataset. We only find a rejection based on the t_ϕ test for Ireland. We are, however, cautious about this finding as the Irish series only has $T = 32$, the shortest series in the panel. This rejection may therefore well be caused by small-sample size distortions. Figure III plots the first few sorted p-values of the different tests along with the cutoff values of Simes' test. It is seen that the sorted p-values all increase rather quickly, so as to move away from Simes' cutoff value. (As such, it is also not interesting to calculate Hommel's procedure.) Only in the case of the t_ϕ-test is the first p-value sufficiently small so as to lead to a rejection of H_0. Our results suggest that previous rejections of the (panel) unit root null may have been driven by the upward size distortions that result when ignoring nonstationary volatility in homoscedastic panel tests (cf. Table I).

Note also that the first one or two p-values of all tests are below the 5%-line. Specifically, these are the p-values for Austria, Ireland and Iceland for $\mathcal{MZ}_t$, Ireland and Austria for t_ϕ, and Iceland for $\mathcal{MZ}_\alpha$ and $\mathcal{MSB}$. That is, if one had conducted separate nonstationary volatility-robust unit root

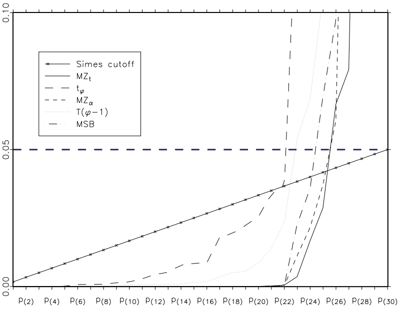


Figure V—Sorted p-values of countries' inflation tests

tests on the series and rejected the single null for each series for which $p_i \leqslant 0.05$, one would have found a certain amount of evidence in favor of GDP trend stationarity.[13] Our multiple testing approach, however, suggests that these rejections are to be seen as spurious (except, perhaps, for t_ϕ), as that approach does not control the FWER—by conducting a sufficient amount of hypotheses tests, one is bound to eventually reject some null hypothesis even if all are correct.

4.2 Unit Roots in Inflation Panels

We now revisit another longstanding question in empirical macroeconomics, viz. that of testing for a unit root in inflation. Whether or not inflation contains a unit root has important implications for the plausibility of many sticky price [Taylor, 1979] and Phillips curve [Calvo, 1983] models [Culver and Papell, 1997]. E.g., the latter assumes stationarity of inflation. Furthermore, upon accepting that the nominal interest rate contains a unit root, stationarity of the real interest rate requires inflation to have a unit root [Rose, 1988]. Indeed, it is often argued in the applied time series literature that price levels are potentially integrated of order two, i.e. $I(2)$ [Juselius and MacDonald, 2004]. This would imply that inflation rates are $I(1)$. On the other hand, finding that inflation rates contain a unit root would call the credibility of the corresponding central banks into question at least in countries with the explicit goal to achieve stable prices. Unsurprisingly, therefore, commensurately many empirical studies have investigated the issue of inflation stationarity, using a variety of techniques. A selective list of contributions includes the early work of Nelson and Schwert [1977], Rose [1988] and Johansen [1992], who use univariate techniques and find mixed results. More recently, Culver and Papell [1997] or Lee and Wu [2001] use panel methods, and are mostly in favor of inflation stationarity. The interest in the issue continues unabated, as evidenced by e.g. Romero-Ávila and Usabiaga [2009]. At the same time, the inflation rate is one of the prominent examples of a time series that enjoyed a 'Great Moderation'. Among many others, Stock and Watson [2002] or Cogley and Sargent [2005] note that there has been a downward trend in the *innovation variances of* inflation in recent decades, with the standard deviation of U.S. inflation from 1981 to 2001 being roughly half as high as that from

[13]Our full sample results show that these rejections even disappear when including the recent crises into the sample period.

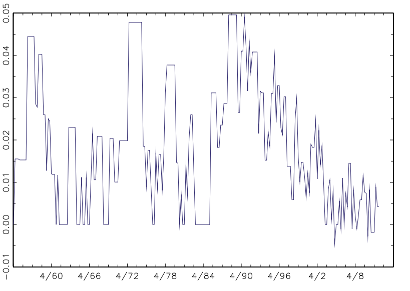


Figure VI—The variance transformed Finnish inflation rate

1960 to 1983. See Figure IV for a few examples of pronounced reductions in the innovation variances of inflation. These appear to have set in the late 1970s, thus several years earlier than the variance reductions for GDP.

However, to the best of our knowledge, just as with testing for GDP stationarity, these two strands of literature are typically not connected. Concretely, the above (panel) unit root studies do not allow for variance nonstationarity. Moreover, most panel tests again neglect possible cross-sectional dependence. Here, we attempt to take these features of the data into account, employing the previously developed techniques to conduct inference about inflation (non-)stationarity robust to variance nonstationarity. We again use quarterly data from the same 30 OECD member countries. The earliest starting date is 1955Q1, the latest is 1991Q1 (for some newly formed countries like the Slovak Republic). The time series end in 2013Q1. Since we also want to allow for trends not only in the variances but also in the mean of inflation, we conduct all time series unit root tests with both constants and trends.

The results are presented in Table V. All unit root tests produce a number of very small p-values. Hence, (3) is easily satisfied and the panel unit root null is strongly rejected. Figure V plots sorted p-values of the tests along with Simes' cutoffs. The three $\mathcal{M}$ tests as well as the two ADF tests appear to correlate quite strongly with each other. We also conduct 'standard' homoscedastic unit root tests (detailed results are available upon request) and find that these often produce rather different results. For example, the t_ϕ statistic for Finland has a p-value of 0.138, whereas the statistic applied to the variance-transformed series has a p-value of 0.002. In view of Figure IV, this can be interpreted intuitively. The rather volatile 50s to early 70s produce residuals from detrending that are almost always below the trend line. This leads a standard unit root test to conclude that the persistence in the series is such that it has a stochastic trend. Conversely, the variance transformed time series will 'spread' highly volatile stretches of the data over the sample period (cf. Figure VI). This will ensure that the series spend less consecutive time above or below trend, leading a unit root test to lean in favor of stationarity (for the above period, there is an almost equal split between positive and negative residuals).

We can then use Hommel's procedure to classify the series into stationary and non-stationary ones.

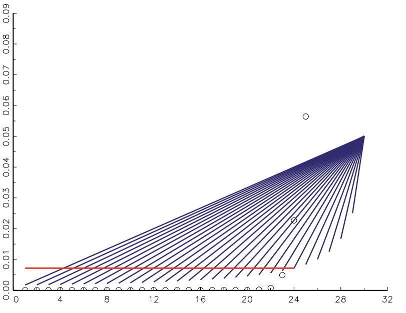


FIGURE VII—INDIVIDUAL REJECTIONS FROM HOMMEL'S j FOR $\mathcal{MZ}_t$

The $\mathcal{MZ}_t$, t_ϕ, $\mathcal{MZ}_\alpha$, $T(\hat{\phi}-1)$ and $\mathcal{MSB}$ tests produce Hommel's j's (8) of 7, 20, 8, 14 and 8 at $\alpha = 0.05$. Hence, p-values smaller than e.g. $\alpha/7$ lead to individual rejections. Figure VII illustrates this for $\mathcal{MZ}_t$. Table V can then be used to read off detailed country results corresponding to these estimates. The $\mathcal{MZ}_t$ test would for instance classify the inflation rates of the countries with the 23 smallest p-values as stationary. Again, some p-values are in the interval $[\alpha/j, \alpha]$ for each underlying time series unit root test. (For instance, those of the Czech Republic and Korea for $\mathcal{MZ}_\alpha$.) The multiple testing approach used here suggests that the corresponding hypotheses would only be spuriously declared false if one rejected whenever a p-value satisfies $p_i \leqslant \alpha$.

To keep the presentation focussed, the previous analysis has allowed for permanent breaks in the innovation variance while assuming the trend function to be constant over time. Clearly, there could simultaneously occur structural breaks in the latter. We therefore now redo the analysis allowing for a broken intercept and trend model; see Perron [1989]. Cavaliere and Taylor [2008b, Sec. 5] show that valid null limiting distributions of the test statistics can be obtained in terms of Hilbert projections of $B_i(s)$ onto the space orthogonal to $\breve{Z}_i(s)$, where $\breve{Z}_i(s) = Z_i(g_i(s))$ is the suitably scaled limit of the trend function. Hence, $\breve{Z}_i(s)$ is the limit of the broken intercept and trend model here. The series of five countries, the Czech Republic, Mexico, Poland, Slovakia and Turkey, are either too short to reliably fit a pre-and post break regime or exhibit little variation in this case and are therefore discarded. This yields $n = 25$ now. We somewhat heuristically identify the break dates through inspection. Figure VIII shows that the results for the broken intercept and trend model are qualitatively similar to those of the trend model. (Detailed country level results are available upon request.) Specifically, all versions of S^H reject the global panel null of inflation nonstationarity, although the $\mathcal{M}$-versions do so more clearly than the ADF-based tests.

5 Conclusion

This paper proposes a new test for a panel unit root against the alternative of a partially stationary panel, making use of Simes' [1986] classical test of the intersection null hypothesis. Unlike most previously proposed panel tests, the one put forward here, extending the idea of Hanck [2013], is

Table V—Sorted p-values of Nonstationary-Volatility Robust Unit Root Tests on Inflation Series.

	$\mathcal{MZ}_t$		t_ϕ		$\mathcal{MZ}_\alpha$		$T(\hat{\phi}-1)$		$\mathcal{MSB}$		Simes' cutoff
$p_{(1)}$	.000	AUS	.000	DNK	.000	AUS	.000	GER	.000	AUS	.002
$p_{(2)}$	.000	AUT	.000	NOR	.000	AUT	.000	NOR	.000	AUT	.003
$p_{(3)}$	.000	BEL	.000	AUT	.000	BEL	.000	AUT	.000	BEL	.005
$p_{(4)}$	.000	CAN	.000	PRT	.000	CAN	.000	PRT	.000	CAN	.007
$p_{(5)}$	.000	GBR	.000	NLD	.000	GBR	.000	DNK	.000	GBR	.008
$p_{(6)}$	.000	DNK	.001	GER	.000	DNK	.000	SWE	.000	DNK	.010
$p_{(7)}$	.000	FIN	.001	SWE	.000	FIN	.000	GRC	.000	FIN	.012
$p_{(8)}$	.000	CHE	.002	KOR	.000	CHE	.000	NLD	.000	CHE	.013
$p_{(9)}$	.000	GER	.002	FIN	.000	GER	.000	AUS	.000	GER	.015
$p_{(10)}$	.000	GRC	.002	AUS	.000	GRC	.001	TUR	.000	GRC	.017
$p_{(11)}$	.000	HUN	.004	IRL	.000	HUN	.001	CAN	.000	HUN	.018
$p_{(12)}$	.000	SWE	.005	GRC	.000	SWE	.001	FIN	.000	SWE	.020
$p_{(13)}$	.000	IRL	.007	MEX	.000	IRL	.001	CHE	.000	IRL	.022
$p_{(14)}$	.000	ITA	.009	TUR	.000	ITA	.002	KOR	.000	ITA	.023
$p_{(15)}$	.000	JPN	.009	CAN	.000	JPN	.002	IRL	.000	JPN	.025
$p_{(16)}$	.000	NOR	.011	CHE	.000	NOR	.003	HUN	.000	NOR	.027
$p_{(17)}$	.000	LUX	.024	BEL	.000	LUX	.004	LUX	.000	LUX	.028
$p_{(18)}$	.000	ESP	.026	LUX	.000	ESP	.007	ESP	.000	ESP	.030
$p_{(19)}$	.000	NLD	.029	ISL	.000	NLD	.010	BEL	.000	NLD	.032
$p_{(20)}$	.000	PRT	.038	HUN	.000	PRT	.012	ITA	.000	PRT	.033
$p_{(21)}$	.000	NZL	.043	ESP	.000	NZL	.031	MEX	.000	NZL	.035
$p_{(22)}$	.001	US	.069	ITA	.000	US	.039	ISL	.000	US	.037
$p_{(23)}$	.005	CZE	.173	US	.015	CZE	.072	SVK	.045	KOR	.038
$p_{(24)}$	.023	SVK	.209	JPN	.045	KOR	.085	US	.048	CZE	.040
$p_{(25)}$	.056	KOR	.226	SVK	.051	SVK	.146	JPN	.113	ISL	.042
$p_{(26)}$	.096	ISL	.272	GBR	.096	ISL	.167	NZL	.117	SVK	.043
$p_{(27)}$	.130	MEX	.318	FRA	.346	FRA	.183	GBR	.320	TUR	.045
$p_{(28)}$	.335	FRA	.326	NZL	.349	TUR	.188	CZE	.388	FRA	.047
$p_{(29)}$	.405	TUR	.393	CZE	.533	MEX	.282	FRA	.907	MEX	.048
$p_{(30)}$	.919	POL	.674	POL	.992	POL	.717	POL	1.000	POL	.050

The sorted p-values from the test statistics described in (6) and (7) applied to OECD inflation data.

robust to the presence of nonstationary volatility. Moreover, the test is intuitive, straightforward to implement and yet robust to general patterns of cross-sectional dependence. Importantly, unlike other tests, Simes' [1986] approach allows to shed light on the important question for how many and also which of the units in the panel the alternative can be said to hold when the null hypothesis is rejected. Hence, the test suggested here allows to decide, for instance, for each unit individually, whether to forecast the respective time series using a deterministic or stochastic trend specification.

Monte Carlo simulations investigate the performance of the new S^H test based on several different underlying nonstationary-volatility robust time series unit root tests, two of which are derived specifically for this paper. The results show that the S^H test controls size and is powerful for different patterns of cross-sectional dependence, nonstationary volatility and serial correlation.

We use the new tests to revisit the question of nonstationarity of output levels and inflation rates. Unlike in previous panel studies, the test results are not contaminated by the 'Great Moderation',

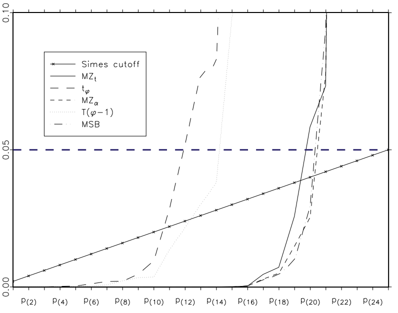


Figure VIII—Sorted p-values of countries' inflation tests, broken intercept and trend model

i.e. the reduction in e.g. the volatility of GDP growth rates experienced by many industrialized countries. We find only very weak evidence of stationarity in the investigated panel of OECD output levels, thus contributing to the view that output levels are well described by a stochastic trend. On the other hand, we find that several OECD countries appear to have a stationary inflation rate.

Obviously, the framework used here is quite flexible and could hence be adopted to other topics in macroeconometrics and finance. Essentially, one only requires valid time series p-values that can then be conveniently combined into a panel test statistic. As such, the present approach could possibly be used to straightforwardly derive, say, panel unit root tests that allow for nonlinearity or panel cointegration tests, the development of which has often proved tedious using other approaches.

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Additional Monte Carlo results

These tables also contain rows with test results for the S^H test using p-values obtained from MacKinnon-type response surface regressions. The respective time series tests underlying the S^H test are then indexed by an M. Dependence scheme 1 corresponds to the benchmark case of cross-sectional independence, scheme 2 to equicorrelation and scheme 3 to factor dependence.

Table A-1—Rejection Rates of the Tests.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.065	.058	.064	.054	.054	.064	.055	.049	.050	.054	.207	.082	.032	.019	.027
	$T(\hat{\phi}-1)$	.154	.108	.067	.036	.043	.094	.071	.054	.052	.047	.013	.024	.019	.023	.022
	$\mathcal{MZ}_\alpha$	.046	.042	.043	.035	.034	.044	.034	.028	.036	.034	.196	.075	.026	.014	.021
	t_ϕ	.028	.036	.035	.026	.034	.034	.041	.039	.041	.039	.010	.012	.014	.013	.014
8	$\mathcal{MSB}$	.046	.042	.043	.035	.034	.044	.034	.028	.036	.034	.175	.066	.020	.008	.015
	$\mathcal{MZ}_{t,M}$	.050	.051	.063	.054	.056	.049	.046	.047	.050	.055	.188	.077	.032	.019	.027
	$t_{\phi,M}$	.132	.101	.066	.036	.044	.077	.067	.053	.052	.048	.010	.021	.019	.023	.023
	$T(\hat{\phi}-1)_M$	.059	.055	.048	.032	.042	.067	.063	.050	.050	.046	.020	.023	.019	.018	.018
	$\mathcal{MZ}_{\alpha,M}$	.117	.088	.076	.058	.056	.103	.078	.055	.052	.055	.250	.101	.038	.020	.027
	$\mathcal{MZ}_t$	.063	.065	.060	.064	.051	.075	.044	.045	.042	.046	.275	.110	.040	.010	.028
	$T(\hat{\phi}-1)$	.168	.131	.079	.050	.045	.108	.079	.052	.050	.048	.013	.016	.017	.018	.027
	$\mathcal{MZ}_\alpha$	.041	.039	.043	.042	.033	.052	.026	.028	.024	.024	.255	.102	.032	.008	.023
	t_ϕ	.030	.040	.037	.038	.035	.034	.036	.036	.037	.034	.009	.008	.010	.010	.015
12	$\mathcal{MSB}$	.041	.039	.043	.042	.033	.052	.026	.028	.024	.024	.241	.090	.026	.006	.015
	$\mathcal{MZ}_{t,M}$	.042	.050	.057	.061	.049	.054	.034	.041	.041	.045	.248	.100	.036	.010	.027
	$t_{\phi,M}$	.144	.119	.074	.049	.044	.085	.069	.049	.047	.046	.006	.013	.016	.017	.024
	$T(\hat{\phi}-1)_M$	.061	.057	.050	.046	.041	.078	.063	.046	.044	.047	.020	.019	.014	.014	.021
	$\mathcal{MZ}_{\alpha,M}$	.123	.090	.068	.067	.051	.126	.072	.050	.044	.046	.332	.130	.044	.011	.027
	$\mathcal{MZ}_t$	.068	.054	.059	.060	.044	.076	.049	.036	.037	.036	.386	.129	.039	.011	.028
	$T(\hat{\phi}-1)$	.251	.166	.094	.057	.042	.146	.098	.068	.050	.053	.014	.022	.021	.018	.024
	$\mathcal{MZ}_\alpha$	.048	.034	.040	.037	.030	.062	.028	.023	.023	.021	.366	.118	.035	.010	.024
	t_ϕ	.026	.037	.040	.037	.030	.040	.045	.043	.038	.044	.010	.015	.010	.014	.019
24	$\mathcal{MSB}$	.048	.034	.040	.037	.030	.062	.028	.023	.023	.021	.351	.109	.028	.008	.016
	$\mathcal{MZ}_{t,M}$	.038	.035	.050	.054	.043	.053	.030	.032	.035	.032	.331	.108	.032	.010	.026
	$t_{\phi,M}$	.208	.142	.086	.052	.040	.111	.085	.059	.046	.050	.007	.014	.016	.018	.022
	$T(\hat{\phi}-1)_M$	.076	.061	.052	.045	.036	.096	.075	.058	.048	.049	.029	.025	.012	.016	.022
	$\mathcal{MZ}_{\alpha,M}$	.150	.097	.072	.063	.045	.137	.078	.043	.040	.038	.468	.163	.047	.013	.028
	$\mathcal{MZ}_t$	.064	.046	.054	.051	.050	.102	.052	.033	.024	.034	.564	.198	.042	.008	.012
	$T(\hat{\phi}-1)$	.381	.250	.118	.052	.059	.195	.113	.080	.058	.067	.015	.020	.020	.016	.024
	$\mathcal{MZ}_\alpha$	.046	.028	.033	.027	.031	.078	.031	.017	.013	.022	.542	.182	.036	.007	.011
	t_ϕ	.027	.040	.041	.033	.042	.041	.038	.048	.034	.048	.011	.009	.010	.008	.012
48	$\mathcal{MSB}$	.046	.028	.033	.027	.031	.078	.031	.017	.013	.022	.519	.160	.030	.004	.009
	$\mathcal{MZ}_{t,M}$	.034	.027	.040	.034	.041	.062	.029	.022	.017	.031	.494	.160	.033	.007	.010
	$t_{\phi,M}$	.296	.202	.104	.045	.054	.126	.079	.062	.052	.061	.008	.008	.014	.013	.020
	$T(\hat{\phi}-1)_M$	.094	.075	.057	.038	.047	.112	.075	.063	.044	.057	.033	.023	.014	.010	.016
	$\mathcal{MZ}_{\alpha,M}$	.168	.087	.064	.053	.050	.184	.085	.042	.027	.034	.657	.242	.050	.009	.012

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \boldsymbol{\imath}_n$, $\delta = 0.2$. Dependence scheme 1.

Table A-2—Rejection Rates of the Tests.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.059	.058	.062	.056	.050	.068	.042	.048	.049	.046	.201	.100	.037	.020	.034
	$T(\hat{\phi}-1)$	.142	.120	.067	.036	.047	.090	.068	.049	.047	.045	.014	.024	.016	.018	.027
	$\mathcal{MZ}_\alpha$	.052	.059	.054	.046	.049	.053	.048	.041	.035	.037	.183	.086	.033	.015	.029
	t_ϕ	.026	.039	.038	.033	.037	.035	.032	.032	.036	.036	.011	.011	.007	.012	.017
8	$\mathcal{MSB}$	.036	.040	.043	.038	.031	.052	.028	.026	.030	.025	.173	.075	.028	.009	.018
	$\mathcal{MZ}_{t,M}$	.045	.050	.059	.056	.050	.055	.038	.046	.050	.048	.182	.090	.036	.020	.034
	$t_{\phi,M}$	.127	.112	.066	.036	.047	.071	.059	.048	.047	.046	.010	.021	.016	.017	.027
	$T(\hat{\phi}-1)_M$	.049	.058	.050	.038	.042	.068	.056	.044	.048	.043	.018	.021	.012	.015	.026
	$\mathcal{MZ}_{\alpha,M}$	.102	.083	.072	.060	.050	.107	.063	.055	.051	.048	.242	.114	.041	.020	.034
	$\mathcal{MZ}_t$	.067	.061	.068	.056	.050	.062	.048	.051	.049	.044	.242	.110	.035	.016	.030
	$T(\hat{\phi}-1)$	.170	.128	.083	.043	.046	.108	.078	.056	.054	.045	.010	.023	.015	.019	.023
	$\mathcal{MZ}_\alpha$	.057	.046	.053	.048	.046	.057	.034	.038	.041	.035	.226	.101	.030	.012	.024
	t_ϕ	.024	.040	.037	.028	.034	.033	.035	.038	.043	.034	.008	.016	.009	.010	.016
12	$\mathcal{MSB}$	.043	.038	.042	.034	.034	.046	.032	.029	.028	.023	.214	.090	.023	.010	.016
	$\mathcal{MZ}_{t,M}$	.046	.046	.063	.052	.049	.049	.041	.046	.047	.042	.216	.102	.034	.015	.028
	$t_{\phi,M}$	.148	.115	.078	.041	.045	.088	.068	.052	.052	.044	.007	.019	.013	.018	.022
	$T(\hat{\phi}-1)_M$	.058	.061	.050	.035	.039	.077	.062	.050	.051	.042	.020	.025	.011	.015	.018
	$\mathcal{MZ}_{\alpha,M}$	.120	.087	.074	.058	.049	.110	.072	.056	.052	.044	.300	.130	.039	.016	.030
	$\mathcal{MZ}_t$	.056	.059	.056	.057	.046	.082	.044	.037	.033	.036	.358	.143	.035	.008	.018
	$T(\hat{\phi}-1)$	.241	.171	.093	.052	.046	.126	.096	.066	.052	.052	.012	.021	.011	.012	.018
	$\mathcal{MZ}_\alpha$	.052	.050	.052	.042	.039	.063	.033	.030	.023	.030	.343	.138	.028	.006	.014
	t_ϕ	.032	.045	.045	.034	.033	.034	.040	.043	.038	.041	.011	.013	.006	.006	.011
24	$\mathcal{MSB}$	.039	.037	.034	.039	.030	.062	.030	.024	.018	.020	.327	.122	.023	.004	.009
	$\mathcal{MZ}_{t,M}$	.036	.040	.046	.051	.042	.056	.031	.032	.029	.033	.318	.127	.032	.007	.017
	$t_{\phi,M}$	.198	.147	.088	.048	.044	.098	.076	.058	.047	.050	.008	.015	.010	.011	.016
	$T(\hat{\phi}-1)_M$	.076	.073	.056	.044	.039	.083	.072	.055	.046	.048	.028	.022	.008	.009	.014
	$\mathcal{MZ}_{\alpha,M}$	.120	.095	.068	.062	.047	.138	.071	.044	.037	.036	.420	.174	.042	.009	.020
	$\mathcal{MZ}_t$	.068	.050	.049	.042	.046	.100	.043	.029	.028	.028	.502	.181	.042	.009	.016
	$T(\hat{\phi}-1)$	.324	.229	.103	.059	.064	.173	.114	.076	.060	.055	.017	.022	.020	.015	.018
	$\mathcal{MZ}_\alpha$	.054	.046	.046	.048	.035	.075	.036	.024	.024	.021	.483	.169	.036	.007	.012
	t_ϕ	.028	.050	.042	.037	.042	.039	.040	.046	.037	.044	.013	.010	.010	.007	.010
48	$\mathcal{MSB}$	.049	.027	.029	.025	.032	.079	.028	.019	.015	.016	.461	.154	.028	.004	.009
	$\mathcal{MZ}_{t,M}$	.030	.027	.036	.035	.041	.066	.027	.022	.020	.023	.444	.150	.033	.007	.014
	$t_{\phi,M}$	.262	.192	.088	.050	.054	.127	.083	.062	.048	.051	.008	.011	.013	.013	.015
	$T(\hat{\phi}-1)_M$	.095	.080	.054	.045	.047	.111	.078	.060	.048	.050	.033	.019	.014	.009	.013
	$\mathcal{MZ}_{\alpha,M}$	.161	.096	.061	.047	.046	.171	.076	.043	.031	.028	.584	.220	.047	.009	.016

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \boldsymbol{\imath}_n$, $\delta = 0.2$. Dependence scheme 2.

Table A-3—Rejection Rates of the Tests.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.070	.058	.064	.059	.051	.057	.048	.047	.050	.045	.158	.079	.040	.018	.032
	$T(\hat{\phi}-1)$	.163	.103	.067	.040	.044	.092	.070	.056	.049	.044	.018	.026	.017	.022	.024
	$\mathcal{MZ}_\alpha$	.045	.042	.053	.055	.042	.051	.036	.039	.033	.036	.145	.071	.034	.014	.024
	t_ϕ	.036	.041	.038	.033	.035	.036	.042	.047	.039	.040	.007	.013	.011	.012	.018
8	$\mathcal{MSB}$	.045	.038	.040	.040	.032	.040	.034	.033	.033	.032	.134	.059	.026	.011	.021
	$\mathcal{MZ}_{t,M}$	.054	.049	.062	.059	.052	.043	.044	.045	.049	.046	.138	.073	.040	.018	.032
	$t_{\phi,M}$	.147	.099	.064	.040	.044	.074	.064	.056	.049	.044	.013	.022	.017	.022	.025
	$T(\hat{\phi}-1)_M$	.074	.061	.051	.040	.042	.066	.060	.056	.045	.045	.018	.022	.016	.018	.022
	$\mathcal{MZ}_{\alpha,M}$	.122	.084	.075	.062	.051	.092	.070	.052	.051	.046	.199	.093	.045	.018	.031
	$\mathcal{MZ}_t$	.067	.056	.063	.056	.048	.064	.042	.042	.044	.044	.231	.083	.036	.020	.028
	$T(\hat{\phi}-1)$	.198	.131	.072	.039	.048	.107	.067	.058	.046	.052	.018	.024	.014	.029	.025
	$\mathcal{MZ}_\alpha$	.054	.048	.044	.049	.030	.052	.036	.035	.032	.037	.212	.074	.028	.012	.022
	t_ϕ	.031	.036	.040	.027	.038	.034	.029	.038	.034	.043	.012	.010	.011	.017	.013
12	$\mathcal{MSB}$	.043	.034	.042	.038	.030	.044	.027	.022	.028	.030	.197	.064	.022	.007	.017
	$\mathcal{MZ}_{t,M}$	.045	.044	.058	.053	.047	.047	.035	.038	.042	.043	.204	.074	.033	.019	.026
	$t_{\phi,M}$	.172	.113	.069	.037	.047	.086	.058	.052	.044	.052	.010	.016	.014	.028	.024
	$T(\hat{\phi}-1)_M$	.084	.060	.055	.036	.043	.072	.050	.050	.045	.050	.022	.020	.014	.022	.020
	$\mathcal{MZ}_{\alpha,M}$	.113	.078	.072	.058	.050	.106	.073	.048	.045	.044	.291	.102	.040	.021	.028
	$\mathcal{MZ}_t$	.056	.046	.060	.046	.048	.074	.040	.042	.034	.035	.330	.115	.033	.009	.022
	$T(\hat{\phi}-1)$	.263	.193	.091	.049	.052	.123	.091	.070	.059	.052	.015	.021	.017	.018	.018
	$\mathcal{MZ}_\alpha$	.053	.032	.056	.035	.039	.060	.037	.029	.028	.033	.312	.106	.028	.008	.018
	t_ϕ	.036	.051	.043	.032	.041	.028	.044	.045	.040	.040	.010	.010	.010	.009	.010
24	$\mathcal{MSB}$	.039	.030	.041	.030	.030	.053	.028	.027	.021	.020	.288	.091	.022	.006	.012
	$\mathcal{MZ}_{t,M}$	.030	.031	.052	.040	.042	.048	.029	.035	.032	.032	.283	.098	.028	.008	.020
	$t_{\phi,M}$	.221	.164	.081	.044	.049	.087	.076	.062	.054	.048	.009	.015	.014	.016	.017
	$T(\hat{\phi}-1)_M$	.089	.084	.057	.040	.046	.077	.075	.062	.051	.047	.028	.021	.013	.011	.012
	$\mathcal{MZ}_{\alpha,M}$	.135	.082	.070	.050	.048	.129	.071	.052	.037	.036	.405	.148	.039	.011	.022
	$\mathcal{MZ}_t$	.051	.052	.064	.060	.039	.085	.042	.031	.029	.031	.483	.186	.036	.007	.014
	$T(\hat{\phi}-1)$	.420	.253	.106	.062	.052	.184	.117	.074	.065	.065	.016	.021	.019	.017	.024
	$\mathcal{MZ}_\alpha$	.051	.049	.048	.032	.031	.072	.028	.033	.025	.021	.460	.169	.031	.006	.010
	t_ϕ	.032	.044	.046	.044	.039	.035	.042	.041	.043	.046	.009	.010	.010	.010	.015
48	$\mathcal{MSB}$	.031	.028	.042	.036	.019	.068	.027	.016	.017	.020	.437	.153	.024	.004	.007
	$\mathcal{MZ}_{t,M}$	.020	.026	.049	.048	.030	.053	.027	.020	.022	.026	.406	.153	.028	.005	.010
	$t_{\phi,M}$	.337	.198	.090	.054	.044	.126	.086	.057	.053	.055	.007	.013	.014	.014	.022
	$T(\hat{\phi}-1)_M$	.108	.084	.060	.049	.043	.109	.076	.059	.053	.054	.027	.022	.012	.013	.016
	$\mathcal{MZ}_{\alpha,M}$	.146	.088	.076	.065	.038	.165	.070	.038	.032	.031	.573	.222	.041	.008	.014

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \boldsymbol{\imath}_n$, $\delta = 0.2$. Dependence scheme 3.

Table A-4—Rejection Rates of the Tests.

n	T	$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
		30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.026	.009	.005	.003	.006	.011	.013	.016	.020	.016	.013	.014	.033	.021	.032
	$T(\hat{\phi}-1)$	.162	.229	.216	.186	.172	.119	.095	.066	.060	.055	.067	.050	.056	.047	.054
	$\mathcal{MZ}_\alpha$	.011	.003	.003	.002	.002	.004	.006	.010	.010	.011	.006	.007	.020	.013	.023
	t_ϕ	.020	.037	.050	.058	.051	.012	.018	.022	.025	.028	.006	.014	.023	.018	.034
8	$\mathcal{MSB}$	.007	.002	.002	.001	.002	.004	.006	.010	.010	.011	.006	.007	.020	.013	.023
	$\mathcal{MZ}_{t,M}$	.020	.008	.004	.003	.006	.008	.011	.016	.020	.017	.010	.013	.032	.021	.032
	$t_{\phi,M}$	.148	.220	.213	.186	.174	.103	.088	.065	.060	.056	.053	.044	.054	.047	.055
	$T(\hat{\phi}-1)_M$	.035	.051	.061	.069	.060	.027	.030	.028	.030	.032	.016	.021	.033	.026	.035
	$\mathcal{MZ}_{\alpha,M}$	.022	.006	.003	.003	.004	.015	.015	.015	.020	.017	.025	.024	.035	.022	.030
	$\mathcal{MZ}_t$	.035	.012	.002	.003	.008	.010	.010	.013	.015	.017	.016	.018	.018	.026	.026
	$T(\hat{\phi}-1)$	.214	.277	.261	.228	.201	.140	.110	.084	.069	.064	.066	.064	.056	.052	.058
	$\mathcal{MZ}_\alpha$	.020	.006	.002	.002	.006	.003	.003	.007	.010	.010	.007	.007	.012	.017	.015
	t_ϕ	.020	.044	.063	.066	.062	.008	.020	.024	.030	.027	.006	.012	.018	.021	.030
12	$\mathcal{MSB}$	.014	.004	.001	.002	.005	.003	.003	.007	.010	.010	.007	.007	.012	.017	.015
	$\mathcal{MZ}_{t,M}$	.029	.007	.002	.002	.008	.006	.008	.011	.014	.016	.008	.014	.016	.024	.026
	$t_{\phi,M}$	.182	.254	.252	.226	.200	.113	.096	.080	.065	.062	.054	.057	.052	.050	.056
	$T(\hat{\phi}-1)_M$	.037	.066	.078	.078	.072	.028	.033	.031	.038	.034	.022	.023	.024	.030	.034
	$\mathcal{MZ}_{\alpha,M}$	.033	.010	.003	.003	.006	.016	.014	.014	.015	.017	.030	.026	.020	.027	.027
	$\mathcal{MZ}_t$	.040	.009	.003	.002	.004	.013	.005	.011	.015	.012	.018	.014	.021	.023	.025
	$T(\hat{\phi}-1)$	.312	.382	.353	.306	.248	.182	.144	.090	.088	.070	.089	.072	.061	.053	.053
	$\mathcal{MZ}_\alpha$	.020	.004	.002	.001	.004	.007	.002	.006	.010	.007	.009	.009	.012	.013	.014
	t_ϕ	.028	.048	.071	.074	.072	.011	.018	.025	.034	.029	.005	.008	.016	.023	.028
24	$\mathcal{MSB}$	.014	.002	.002	.001	.002	.007	.002	.006	.010	.007	.009	.009	.012	.013	.014
	$\mathcal{MZ}_{t,M}$	.031	.008	.003	.002	.004	.010	.004	.009	.013	.011	.010	.012	.017	.020	.024
	$t_{\phi,M}$	.268	.344	.334	.296	.242	.138	.115	.078	.082	.066	.061	.057	.057	.049	.049
	$T(\hat{\phi}-1)_M$	.056	.083	.084	.090	.080	.028	.034	.033	.039	.033	.020	.018	.025	.028	.031
	$\mathcal{MZ}_{\alpha,M}$	.039	.008	.002	.001	.004	.020	.012	.014	.015	.011	.040	.024	.025	.024	.025
	$\mathcal{MZ}_t$	.049	.011	.002	.001	.002	.015	.006	.009	.010	.009	.014	.012	.020	.018	.015
	$T(\hat{\phi}-1)$	.435	.516	.488	.416	.335	.262	.197	.112	.099	.087	.119	.090	.062	.058	.055
	$\mathcal{MZ}_\alpha$	.024	.006	.001	.000	.002	.008	.002	.006	.007	.003	.006	.005	.014	.010	.010
	t_ϕ	.032	.065	.082	.095	.084	.010	.019	.024	.026	.036	.004	.010	.016	.018	.026
48	$\mathcal{MSB}$	.018	.003	.000	.000	.001	.008	.002	.006	.007	.003	.006	.005	.014	.010	.010
	$\mathcal{MZ}_{t,M}$	.032	.007	.002	.001	.001	.007	.002	.006	.008	.006	.005	.006	.016	.014	.013
	$t_{\phi,M}$	.353	.453	.457	.387	.311	.181	.146	.092	.084	.075	.072	.065	.048	.049	.050
	$T(\hat{\phi}-1)_M$	.072	.100	.099	.108	.092	.031	.037	.034	.031	.042	.018	.026	.024	.023	.031
	$\mathcal{MZ}_{\alpha,M}$	.050	.012	.002	.001	.002	.025	.008	.012	.011	.007	.030	.024	.024	.018	.015

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \boldsymbol{\imath}_n$, $\delta = 5$. Dependence scheme 1.

Table A-5—Rejection Rates of the Tests.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.028	.007	.003	.004	.006	.014	.010	.019	.016	.016	.018	.020	.025	.032	.029
	$T(\hat{\phi}-1)$	.164	.198	.194	.188	.160	.118	.092	.071	.059	.058	.068	.054	.051	.054	.048
	$\mathcal{MZ}_\alpha$	.015	.004	.002	.004	.002	.007	.008	.014	.013	.010	.012	.014	.020	.025	.026
	t_ϕ	.019	.036	.056	.057	.050	.010	.016	.021	.022	.025	.007	.010	.021	.027	.027
8	$\mathcal{MSB}$	.012	.002	.002	.003	.002	.005	.004	.008	.008	.011	.010	.009	.015	.018	.019
	$\mathcal{MZ}_{t,M}$	.024	.006	.003	.004	.006	.010	.008	.018	.016	.017	.015	.017	.025	.032	.030
	$t_{\phi,M}$	.151	.189	.190	.188	.162	.103	.086	.069	.059	.058	.054	.048	.049	.054	.050
	$T(\hat{\phi}-1)_M$	.036	.054	.066	.067	.060	.026	.024	.028	.030	.030	.020	.017	.027	.034	.034
	$\mathcal{MZ}_{\alpha,M}$	.026	.006	.003	.004	.005	.018	.013	.021	.018	.016	.030	.025	.028	.032	.028
	$\mathcal{MZ}_t$	.028	.008	.006	.002	.004	.011	.011	.011	.017	.013	.020	.016	.020	.031	.026
	$T(\hat{\phi}-1)$	.205	.256	.227	.185	.180	.126	.107	.070	.070	.071	.074	.058	.053	.060	.055
	$\mathcal{MZ}_\alpha$	.012	.005	.004	.001	.003	.006	.010	.011	.014	.010	.012	.010	.021	.024	.020
	t_ϕ	.020	.042	.063	.052	.055	.008	.014	.025	.024	.033	.005	.010	.020	.032	.028
12	$\mathcal{MSB}$	.010	.004	.002	.001	.002	.004	.004	.004	.009	.006	.007	.007	.014	.018	.017
	$\mathcal{MZ}_{t,M}$	.020	.006	.006	.002	.004	.006	.008	.009	.017	.013	.011	.012	.019	.029	.026
	$t_{\phi,M}$	.177	.234	.218	.183	.179	.106	.094	.065	.069	.068	.052	.050	.050	.058	.053
	$T(\hat{\phi}-1)_M$	.041	.058	.075	.063	.061	.025	.026	.030	.030	.040	.021	.020	.030	.038	.031
	$\mathcal{MZ}_{\alpha,M}$	.025	.008	.005	.002	.003	.017	.015	.012	.017	.012	.034	.020	.023	.030	.027
	$\mathcal{MZ}_t$	.040	.011	.003	.002	.003	.011	.006	.014	.011	.013	.016	.017	.024	.017	.023
	$T(\hat{\phi}-1)$	.295	.336	.304	.254	.222	.164	.126	.080	.072	.071	.081	.076	.057	.044	.052
	$\mathcal{MZ}_\alpha$	.020	.008	.002	.001	.002	.009	.003	.010	.011	.012	.015	.010	.017	.021	.016
	t_ϕ	.028	.058	.072	.065	.068	.012	.014	.019	.024	.032	.004	.010	.019	.019	.024
24	$\mathcal{MSB}$	.018	.004	.001	.001	.001	.004	.003	.006	.007	.006	.010	.008	.016	.012	.012
	$\mathcal{MZ}_{t,M}$	.028	.008	.003	.002	.003	.007	.006	.011	.010	.012	.007	.012	.021	.015	.023
	$t_{\phi,M}$	.257	.308	.290	.246	.216	.127	.106	.071	.068	.068	.056	.057	.050	.041	.049
	$T(\hat{\phi}-1)_M$	.057	.087	.089	.076	.080	.031	.030	.026	.031	.038	.018	.022	.024	.024	.030
	$\mathcal{MZ}_{\alpha,M}$	.037	.011	.002	.001	.002	.019	.012	.014	.013	.014	.033	.028	.026	.020	.024
	$\mathcal{MZ}_t$	.055	.008	.003	.002	.004	.010	.010	.008	.012	.011	.016	.016	.015	.018	.020
	$T(\hat{\phi}-1)$	.372	.441	.401	.352	.308	.216	.159	.097	.084	.079	.087	.087	.063	.058	.053
	$\mathcal{MZ}_\alpha$	.029	.005	.002	.001	.003	.008	.004	.008	.008	.010	.009	.008	.017	.016	.018
	t_ϕ	.029	.060	.084	.093	.099	.010	.016	.022	.022	.030	.003	.010	.017	.016	.027
48	$\mathcal{MSB}$	.018	.005	.001	.001	.001	.006	.006	.005	.006	.007	.006	.008	.006	.008	.010
	$\mathcal{MZ}_{t,M}$	.033	.005	.002	.002	.002	.006	.006	.006	.009	.009	.006	.007	.008	.012	.015
	$t_{\phi,M}$	.308	.394	.368	.328	.290	.151	.118	.079	.070	.071	.051	.068	.050	.051	.046
	$T(\hat{\phi}-1)_M$	.063	.092	.104	.106	.110	.030	.031	.029	.028	.037	.014	.022	.025	.023	.032
	$\mathcal{MZ}_{\alpha,M}$	.050	.008	.002	.002	.003	.019	.014	.008	.012	.011	.035	.024	.019	.020	.018

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \boldsymbol{\imath}_n$, $\delta = 5$. Dependence scheme 2.

Table A-6—Rejection Rates of the Tests.

n	T	$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
		30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.018	.015	.012	.012	.017	.016	.013	.018	.022	.025	.016	.023	.024	.031	.031
	$T(\hat{\phi}-1)$	.106	.111	.112	.098	.087	.090	.087	.057	.050	.052	.068	.056	.047	.048	.048
	$\mathcal{MZ}_\alpha$	.010	.011	.010	.010	.013	.009	.006	.018	.016	.018	.014	.014	.023	.024	.021
	t_ϕ	.011	.018	.026	.027	.034	.013	.018	.021	.022	.026	.006	.012	.016	.025	.029
8	$\mathcal{MSB}$	.011	.009	.007	.008	.011	.008	.006	.012	.015	.015	.010	.011	.016	.014	.018
	$\mathcal{MZ}_{t,M}$	.014	.013	.012	.012	.018	.012	.010	.018	.022	.025	.011	.018	.023	.031	.031
	$t_{\phi,M}$	.091	.108	.110	.098	.089	.076	.080	.056	.050	.053	.056	.052	.046	.048	.049
	$T(\hat{\phi}-1)_M$	.025	.031	.033	.032	.041	.022	.029	.029	.028	.032	.017	.022	.022	.032	.033
	$\mathcal{MZ}_{\alpha,M}$	.021	.018	.013	.012	.017	.024	.018	.022	.022	.024	.028	.031	.025	.032	.032
	$\mathcal{MZ}_t$	.019	.013	.013	.014	.016	.014	.012	.017	.020	.019	.016	.017	.026	.027	.024
	$T(\hat{\phi}-1)$	.120	.146	.132	.120	.094	.106	.082	.070	.054	.058	.070	.056	.052	.052	.051
	$\mathcal{MZ}_\alpha$	.011	.009	.010	.010	.013	.007	.009	.014	.016	.017	.011	.011	.012	.026	.023
	t_ϕ	.008	.022	.029	.034	.033	.007	.010	.024	.021	.027	.006	.010	.022	.022	.025
12	$\mathcal{MSB}$	.008	.007	.008	.007	.009	.005	.007	.011	.010	.010	.008	.007	.014	.016	.013
	$\mathcal{MZ}_{t,M}$	.014	.011	.010	.014	.016	.009	.009	.016	.020	.019	.012	.013	.022	.025	.024
	$t_{\phi,M}$	.098	.129	.124	.118	.092	.080	.071	.067	.053	.056	.052	.050	.048	.050	.050
	$T(\hat{\phi}-1)_M$	.022	.037	.038	.038	.038	.019	.021	.030	.029	.033	.016	.020	.027	.030	.030
	$\mathcal{MZ}_{\alpha,M}$	.025	.019	.015	.015	.016	.023	.016	.020	.020	.018	.030	.024	.030	.026	.024
	$\mathcal{MZ}_t$	.020	.010	.012	.011	.011	.008	.012	.015	.016	.018	.011	.018	.023	.023	.025
	$T(\hat{\phi}-1)$	.157	.185	.152	.137	.123	.136	.108	.069	.064	.063	.086	.078	.054	.056	.058
	$\mathcal{MZ}_\alpha$	.012	.006	.006	.009	.010	.012	.006	.010	.013	.018	.011	.011	.015	.022	.019
	t_ϕ	.011	.020	.030	.034	.041	.010	.011	.017	.026	.028	.003	.009	.019	.026	.031
24	$\mathcal{MSB}$	.010	.005	.003	.006	.008	.004	.006	.009	.012	.013	.006	.010	.014	.015	.018
	$\mathcal{MZ}_{t,M}$	.013	.006	.008	.010	.011	.004	.007	.012	.015	.018	.006	.013	.021	.021	.022
	$t_{\phi,M}$	.124	.158	.140	.131	.118	.099	.084	.063	.061	.060	.060	.060	.048	.052	.055
	$T(\hat{\phi}-1)_M$	.027	.036	.038	.039	.045	.022	.024	.023	.029	.031	.012	.021	.027	.033	.036
	$\mathcal{MZ}_{\alpha,M}$	.028	.015	.011	.012	.012	.023	.021	.018	.016	.019	.032	.030	.030	.026	.026
	$\mathcal{MZ}_t$	.021	.010	.010	.008	.014	.011	.006	.019	.016	.014	.016	.013	.016	.020	.015
	$T(\hat{\phi}-1)$	.217	.252	.220	.180	.160	.176	.138	.092	.078	.060	.115	.092	.072	.060	.048
	$\mathcal{MZ}_\alpha$	.011	.006	.007	.006	.012	.015	.008	.007	.014	.010	.012	.008	.017	.020	.017
	t_ϕ	.010	.023	.044	.042	.044	.005	.016	.024	.026	.026	.004	.012	.020	.025	.020
48	$\mathcal{MSB}$	.009	.005	.005	.006	.010	.007	.002	.010	.007	.007	.008	.008	.008	.011	.011
	$\mathcal{MZ}_{t,M}$	.013	.005	.008	.006	.012	.006	.002	.015	.011	.010	.007	.007	.011	.016	.014
	$t_{\phi,M}$	.155	.208	.194	.162	.146	.115	.102	.075	.067	.053	.069	.065	.056	.053	.038
	$T(\hat{\phi}-1)_M$	.026	.043	.050	.051	.048	.019	.034	.033	.032	.030	.020	.026	.028	.031	.022
	$\mathcal{MZ}_{\alpha,M}$	.031	.014	.010	.008	.016	.020	.011	.021	.018	.014	.038	.021	.020	.022	.015

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \boldsymbol{\imath}_n$, $\delta = 5$. Dependence scheme 3.

Table A-7—Rejection Rates of the Tests.

n	T	$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
		30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.171	.338	.856	.992	1.00	.188	.255	.603	.896	.990	.534	.410	.399	.354	.561
	$T(\hat{\phi}-1)$	.211	.244	.599	.915	.996	.142	.148	.366	.692	.922	.035	.092	.118	.162	.230
	$\mathcal{MZ}_\alpha$	.121	.253	.778	.979	1.00	.138	.177	.466	.802	.961	.502	.370	.359	.306	.518
	t_ϕ	.042	.112	.501	.887	.992	.075	.104	.305	.638	.907	.040	.073	.098	.150	.212
8	$\mathcal{MSB}$	.121	.253	.778	.979	1.00	.138	.177	.466	.802	.961	.471	.331	.315	.261	.457
	$\mathcal{MZ}_{t,M}$	.134	.309	.850	.992	1.00	.152	.229	.592	.895	.990	.496	.386	.389	.352	.562
	$t_{\phi,M}$	.188	.225	.592	.915	.996	.125	.136	.360	.692	.924	.026	.082	.114	.162	.230
	$T(\hat{\phi}-1)_M$	.100	.169	.568	.914	.997	.131	.152	.374	.694	.926	.078	.119	.132	.171	.243
	$\mathcal{MZ}_{\alpha,M}$	.286	.416	.874	.992	1.00	.282	.323	.626	.899	.987	.617	.462	.419	.361	.561
	$\mathcal{MZ}_t$	.300	.617	.998	1.00	1.00	.188	.227	.579	.909	.994	.620	.455	.445	.356	.629
	$T(\hat{\phi}-1)$	.316	.447	.923	1.00	1.00	.158	.172	.377	.735	.950	.035	.094	.115	.178	.262
	$\mathcal{MZ}_\alpha$	.211	.464	.991	1.00	1.00	.143	.151	.436	.813	.980	.588	.421	.398	.311	.583
	t_ϕ	.072	.199	.851	.999	1.00	.073	.114	.294	.674	.934	.040	.072	.107	.150	.231
12	$\mathcal{MSB}$	.211	.464	.991	1.00	1.00	.143	.151	.436	.813	.980	.562	.376	.345	.263	.516
	$\mathcal{MZ}_{t,M}$	.225	.566	.998	1.00	1.00	.148	.192	.558	.904	.994	.574	.420	.425	.346	.625
	$t_{\phi,M}$	.271	.410	.914	1.00	1.00	.126	.150	.353	.726	.949	.025	.079	.106	.170	.258
	$T(\hat{\phi}-1)_M$	.156	.314	.902	1.00	1.00	.134	.165	.366	.730	.950	.088	.113	.132	.177	.270
	$\mathcal{MZ}_{\alpha,M}$	.480	.726	.998	1.00	1.00	.293	.298	.620	.912	.994	.708	.515	.468	.368	.626
	$\mathcal{MZ}_t$	.197	.414	.985	1.00	1.00	.198	.192	.484	.862	.992	.824	.629	.578	.448	.798
	$T(\hat{\phi}-1)$	.358	.393	.866	.997	1.00	.183	.165	.342	.690	.939	.048	.114	.159	.226	.346
	$\mathcal{MZ}_\alpha$	.140	.296	.949	1.00	1.00	.157	.139	.345	.724	.962	.798	.596	.533	.393	.750
	t_ϕ	.060	.134	.732	.993	1.00	.066	.094	.260	.604	.919	.047	.089	.129	.194	.309
24	$\mathcal{MSB}$	.140	.296	.949	1.00	1.00	.157	.139	.345	.724	.962	.768	.554	.468	.333	.691
	$\mathcal{MZ}_{t,M}$	.122	.330	.982	1.00	1.00	.144	.150	.441	.849	.991	.770	.576	.546	.428	.792
	$t_{\phi,M}$	.297	.351	.844	.997	1.00	.138	.138	.312	.674	.938	.031	.082	.142	.213	.335
	$T(\hat{\phi}-1)_M$	.142	.236	.813	.996	1.00	.156	.153	.329	.681	.940	.117	.162	.170	.232	.348
	$\mathcal{MZ}_{\alpha,M}$	.408	.574	.990	1.00	1.00	.331	.290	.541	.876	.992	.896	.711	.620	.464	.799
	$\mathcal{MZ}_t$	.256	.492	.999	1.00	1.00	.270	.246	.656	.985	1.00	.921	.730	.607	.396	.786
	$T(\hat{\phi}-1)$	.526	.574	.953	1.00	1.00	.257	.241	.505	.906	.998	.045	.109	.148	.234	.347
	$\mathcal{MZ}_\alpha$	.187	.350	.991	1.00	1.00	.209	.172	.465	.929	1.00	.908	.698	.548	.352	.742
	t_ϕ	.062	.179	.846	1.00	1.00	.078	.114	.378	.834	.996	.051	.081	.122	.191	.303
48	$\mathcal{MSB}$	.187	.350	.991	1.00	1.00	.209	.172	.465	.929	1.00	.894	.656	.479	.281	.666
	$\mathcal{MZ}_{t,M}$	.127	.349	.999	1.00	1.00	.176	.163	.578	.982	1.00	.874	.655	.543	.354	.767
	$t_{\phi,M}$	.420	.493	.940	1.00	1.00	.180	.177	.450	.885	.998	.026	.069	.124	.210	.314
	$T(\hat{\phi}-1)_M$	.193	.308	.912	1.00	1.00	.200	.204	.472	.893	.999	.122	.153	.166	.230	.340
	$\mathcal{MZ}_{\alpha,M}$	.537	.701	1.00	1.00	1.00	.449	.366	.722	.990	1.00	.966	.803	.650	.415	.791

Note: Case $\psi = 0$, $\boldsymbol{\phi} = (\boldsymbol{\imath}'_{n/2}, \widetilde{\boldsymbol{\phi}}'_{n/2})'$ with $(\widetilde{\boldsymbol{\phi}}_{n/2})_i \sim \mathcal{U}(.75, 1)$, $\delta = 0.2$. Dependence scheme 1.

Table A-8—Rejection Rates of the Tests.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.170	.332	.798	.982	1.00	.166	.206	.451	.740	.927	.446	.312	.322	.256	.437
	$T(\hat{\phi}-1)$	.225	.265	.562	.865	.995	.120	.134	.263	.534	.799	.035	.066	.086	.120	.184
	$\mathcal{MZ}_\alpha$	.251	.478	.960	1.00	1.00	.150	.142	.295	.540	.785	.423	.284	.288	.218	.399
	t_ϕ	.057	.122	.450	.827	.991	.062	.089	.221	.491	.770	.032	.055	.076	.106	.174
8	$\mathcal{MSB}$	.115	.236	.696	.952	.999	.120	.143	.327	.622	.865	.394	.253	.245	.182	.344
	$\mathcal{MZ}_{t,M}$	.137	.302	.791	.982	1.00	.135	.182	.445	.739	.928	.415	.294	.315	.255	.442
	$t_{\phi,M}$	.203	.248	.556	.864	.995	.104	.122	.257	.533	.802	.028	.059	.083	.120	.184
	$T(\hat{\phi}-1)_M$	.118	.176	.514	.864	.994	.114	.136	.261	.540	.798	.058	.090	.094	.127	.194
	$\mathcal{MZ}_{\alpha,M}$	.280	.406	.818	.984	1.00	.242	.259	.475	.744	.925	.528	.362	.338	.259	.438
	$\mathcal{MZ}_t$	.239	.442	.947	.999	1.00	.192	.227	.565	.860	.985	.644	.501	.489	.410	.675
	$T(\hat{\phi}-1)$	.296	.335	.768	.976	.999	.151	.154	.362	.675	.918	.050	.113	.151	.194	.312
	$\mathcal{MZ}_\alpha$	.217	.462	.962	1.00	1.00	.190	.275	.694	.952	.997	.619	.467	.453	.362	.635
	t_ϕ	.072	.157	.663	.964	.999	.063	.094	.306	.624	.894	.054	.090	.137	.175	.290
12	$\mathcal{MSB}$	.166	.320	.892	.998	1.00	.147	.155	.426	.774	.958	.589	.434	.405	.305	.578
	$\mathcal{MZ}_{t,M}$	.180	.389	.944	.999	1.00	.157	.201	.544	.854	.986	.599	.467	.474	.402	.673
	$t_{\phi,M}$	.258	.309	.760	.974	.999	.122	.134	.346	.671	.918	.039	.094	.143	.190	.310
	$T(\hat{\phi}-1)_M$	.150	.223	.734	.977	.999	.132	.148	.361	.677	.916	.114	.150	.165	.209	.319
	$\mathcal{MZ}_{\alpha,M}$	.394	.543	.952	.999	1.00	.293	.298	.594	.864	.986	.729	.564	.512	.418	.669
	$\mathcal{MZ}_t$	.223	.443	.957	1.00	1.00	.230	.226	.548	.861	.986	.728	.529	.463	.329	.626
	$T(\hat{\phi}-1)$	.360	.414	.813	.986	1.00	.186	.189	.386	.715	.934	.040	.095	.132	.168	.276
	$\mathcal{MZ}_\alpha$	.174	.358	.902	.999	1.00	.220	.230	.546	.881	.983	.712	.508	.429	.293	.582
	t_ϕ	.054	.157	.704	.978	.999	.068	.106	.316	.658	.918	.038	.074	.110	.142	.248
24	$\mathcal{MSB}$	.158	.320	.912	.999	1.00	.177	.157	.426	.774	.960	.688	.469	.380	.243	.520
	$\mathcal{MZ}_{t,M}$	.138	.360	.948	1.00	1.00	.163	.178	.522	.856	.986	.682	.488	.439	.317	.619
	$t_{\phi,M}$	.296	.367	.793	.985	1.00	.144	.156	.363	.702	.932	.024	.075	.122	.157	.267
	$T(\hat{\phi}-1)_M$	.141	.253	.770	.985	1.00	.163	.172	.384	.707	.933	.093	.126	.143	.174	.280
	$\mathcal{MZ}_{\alpha,M}$	.413	.567	.966	1.00	1.00	.362	.314	.595	.871	.987	.807	.604	.502	.342	.628
	$\mathcal{MZ}_t$	.265	.535	.990	1.00	1.00	.263	.227	.554	.898	.987	.832	.603	.491	.310	.615
	$T(\hat{\phi}-1)$	.478	.538	.908	.996	1.00	.239	.226	.439	.774	.962	.039	.096	.118	.154	.272
	$\mathcal{MZ}_\alpha$	.212	.412	.956	1.00	1.00	.215	.182	.427	.821	.974	.816	.575	.455	.275	.577
	t_ϕ	.063	.196	.827	.995	1.00	.068	.113	.334	.712	.947	.045	.067	.092	.126	.236
48	$\mathcal{MSB}$	.185	.404	.971	1.00	1.00	.214	.158	.398	.794	.963	.791	.535	.400	.220	.507
	$\mathcal{MZ}_{t,M}$	.142	.421	.987	1.00	1.00	.179	.152	.494	.881	.986	.766	.530	.439	.276	.591
	$t_{\phi,M}$	.385	.463	.890	.996	1.00	.158	.182	.395	.756	.958	.018	.065	.096	.132	.250
	$T(\hat{\phi}-1)_M$	.172	.325	.876	.997	1.00	.186	.196	.416	.762	.958	.108	.129	.120	.152	.262
	$\mathcal{MZ}_{\alpha,M}$	.507	.692	.994	1.00	1.00	.416	.332	.620	.910	.988	.897	.676	.524	.322	.620

Note: Case $\psi = 0$, $\boldsymbol{\phi} = (\boldsymbol{\imath}'_{n/2}, \widetilde{\boldsymbol{\phi}}'_{n/2})'$ with $(\widetilde{\boldsymbol{\phi}}_{n/2})_i \sim \mathcal{U}(.75, 1)$, $\delta = 0.2$. Dependence scheme 2.

Table A-9—Rejection Rates of the Tests.

n	T	$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
		30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.262	.551	.992	1.00	1.00	.218	.313	.752	.978	.999	.580	.514	.602	.610	.798
	$T(\hat{\phi}-1)$	.289	.356	.860	.996	1.00	.160	.200	.516	.857	.981	.063	.150	.225	.306	.432
	$\mathcal{MZ}_\alpha$	.207	.429	.955	.999	1.00	.162	.232	.573	.888	.988	.546	.478	.562	.565	.765
	t_ϕ	.091	.196	.784	.993	1.00	.081	.138	.447	.823	.977	.053	.120	.200	.294	.420
8	$\mathcal{MSB}$	.194	.429	.961	1.00	1.00	.160	.224	.612	.943	.995	.505	.426	.500	.486	.712
	$\mathcal{MZ}_{t,M}$	.219	.515	.989	1.00	1.00	.182	.286	.742	.977	.999	.540	.486	.594	.608	.800
	$t_{\phi,M}$	.260	.334	.851	.996	1.00	.136	.187	.504	.856	.982	.049	.139	.219	.306	.435
	$T(\hat{\phi}-1)_M$	.170	.274	.836	.996	1.00	.152	.198	.524	.869	.981	.103	.177	.243	.329	.458
	$\mathcal{MZ}_{\alpha,M}$	.405	.644	.994	1.00	1.00	.326	.377	.772	.977	.999	.658	.575	.619	.614	.793
	$\mathcal{MZ}_t$	.268	.542	.992	1.00	1.00	.218	.302	.774	.982	1.00	.457	.353	.375	.354	.580
	$T(\hat{\phi}-1)$	.321	.419	.885	1.00	1.00	.159	.213	.522	.892	.996	.045	.084	.132	.188	.265
	$\mathcal{MZ}_\alpha$	.198	.370	.930	1.00	1.00	.172	.190	.513	.838	.980	.428	.309	.335	.304	.534
	t_ϕ	.080	.208	.808	.999	1.00	.084	.136	.440	.847	.992	.030	.063	.101	.167	.241
12	$\mathcal{MSB}$	.179	.398	.970	1.00	1.00	.166	.204	.618	.944	.999	.394	.276	.289	.260	.478
	$\mathcal{MZ}_{t,M}$	.196	.482	.990	1.00	1.00	.170	.258	.758	.981	1.00	.406	.314	.360	.340	.575
	$t_{\phi,M}$	.276	.381	.878	1.00	1.00	.133	.191	.510	.888	.996	.032	.072	.120	.183	.263
	$T(\hat{\phi}-1)_M$	.176	.312	.864	1.00	1.00	.157	.209	.525	.888	.994	.068	.099	.134	.190	.269
	$\mathcal{MZ}_{\alpha,M}$	.450	.644	.993	1.00	1.00	.342	.401	.803	.985	1.00	.540	.416	.403	.360	.580
	$\mathcal{MZ}_t$	.275	.579	1.00	1.00	1.00	.180	.194	.502	.873	.990	.697	.501	.535	.470	.786
	$T(\hat{\phi}-1)$	.442	.532	.964	1.00	1.00	.181	.204	.379	.697	.949	.052	.124	.188	.279	.424
	$\mathcal{MZ}_\alpha$	.228	.530	.998	1.00	1.00	.218	.235	.647	.963	1.00	.671	.461	.492	.428	.750
	t_ϕ	.077	.223	.905	1.00	1.00	.075	.106	.290	.620	.925	.041	.071	.148	.242	.393
24	$\mathcal{MSB}$	.186	.417	.995	1.00	1.00	.143	.140	.351	.742	.962	.644	.414	.433	.364	.689
	$\mathcal{MZ}_{t,M}$	.172	.481	1.00	1.00	1.00	.128	.151	.460	.864	.990	.634	.444	.506	.448	.781
	$t_{\phi,M}$	.366	.468	.952	1.00	1.00	.142	.166	.353	.681	.947	.029	.091	.168	.265	.413
	$T(\hat{\phi}-1)_M$	.207	.352	.948	1.00	1.00	.161	.183	.363	.684	.947	.098	.134	.195	.276	.426
	$\mathcal{MZ}_{\alpha,M}$	.520	.738	1.00	1.00	1.00	.319	.290	.554	.882	.992	.794	.594	.570	.489	.790
	$\mathcal{MZ}_t$	.315	.699	1.00	1.00	1.00	.306	.287	.775	.999	1.00	.871	.690	.686	.593	.921
	$T(\hat{\phi}-1)$	.600	.687	.996	1.00	1.00	.282	.294	.620	.967	1.00	.068	.139	.249	.360	.542
	$\mathcal{MZ}_\alpha$	.186	.356	.990	1.00	1.00	.279	.258	.718	.996	1.00	.855	.660	.642	.545	.895
	t_ϕ	.088	.264	.979	1.00	1.00	.081	.151	.476	.927	1.00	.031	.084	.188	.305	.492
48	$\mathcal{MSB}$	.224	.506	1.00	1.00	1.00	.232	.196	.600	.986	1.00	.826	.604	.564	.456	.833
	$\mathcal{MZ}_{t,M}$	.169	.542	1.00	1.00	1.00	.192	.194	.722	.998	1.00	.804	.612	.631	.552	.908
	$t_{\phi,M}$	.477	.588	.994	1.00	1.00	.192	.224	.561	.958	1.00	.036	.092	.206	.326	.512
	$T(\hat{\phi}-1)_M$	.244	.452	.993	1.00	1.00	.220	.250	.585	.960	1.00	.113	.158	.241	.357	.541
	$\mathcal{MZ}_{\alpha,M}$	.633	.868	1.00	1.00	1.00	.494	.440	.843	.999	1.00	.940	.775	.735	.617	.923

Note: Case $\psi = 0$, $\boldsymbol{\phi} = (\boldsymbol{\imath}'_{n/2}, \widetilde{\boldsymbol{\phi}}'_{n/2})'$ with $(\widetilde{\boldsymbol{\phi}}_{n/2})_i \sim \mathcal{U}(.75, 1)$, $\delta = 0.2$. Dependence scheme 3.

Table A-10—Rejection Rates of the Tests.

n	T	$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
		30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.136	.111	.142	.230	.352	.042	.079	.406	.776	.954	.158	.422	.984	1.00	1.00
	$T(\hat{\phi}-1)$	.573	.616	.721	.790	.847	.272	.393	.735	.951	.996	.292	.583	.992	1.00	1.00
	$\mathcal{MZ}_\alpha$	.102	.084	.124	.192	.315	.028	.048	.301	.644	.890	.105	.302	.955	1.00	1.00
	t_ϕ	.081	.204	.435	.604	.693	.054	.167	.562	.882	.985	.072	.325	.966	1.00	1.00
8	$\mathcal{MSB}$	.081	.074	.105	.159	.271	.028	.048	.301	.644	.890	.105	.302	.955	1.00	1.00
	$\mathcal{MZ}_{t,M}$	.120	.104	.140	.230	.355	.031	.071	.396	.774	.955	.122	.384	.983	1.00	1.00
	$t_{\phi,M}$	.540	.599	.717	.790	.848	.240	.370	.728	.950	.996	.246	.554	.991	1.00	1.00
	$T(\hat{\phi}-1)_M$	.141	.265	.492	.639	.725	.118	.241	.634	.917	.990	.166	.460	.983	1.00	1.00
	$\mathcal{MZ}_{\alpha,M}$	.156	.123	.153	.235	.347	.074	.112	.434	.778	.952	.262	.514	.986	1.00	1.00
	$\mathcal{MZ}_t$	.134	.092	.113	.161	.290	.056	.110	.610	.958	.999	.151	.428	.992	1.00	1.00
	$T(\hat{\phi}-1)$	.660	.685	.750	.787	.848	.357	.563	.906	.998	1.00	.323	.639	.996	1.00	1.00
	$\mathcal{MZ}_\alpha$	.104	.068	.086	.137	.253	.032	.057	.448	.871	.992	.096	.294	.972	1.00	1.00
	t_ϕ	.072	.190	.420	.565	.672	.074	.234	.770	.989	1.00	.074	.330	.979	1.00	1.00
12	$\mathcal{MSB}$	.090	.055	.067	.110	.208	.032	.057	.448	.871	.992	.096	.294	.972	1.00	1.00
	$\mathcal{MZ}_{t,M}$	.110	.079	.103	.156	.289	.038	.084	.588	.954	.998	.108	.375	.990	1.00	1.00
	$t_{\phi,M}$	.608	.660	.738	.784	.846	.302	.512	.898	.998	1.00	.267	.596	.994	1.00	1.00
	$T(\hat{\phi}-1)_M$	.136	.258	.478	.617	.708	.172	.357	.825	.992	1.00	.192	.498	.989	1.00	1.00
	$\mathcal{MZ}_{\alpha,M}$	.161	.100	.121	.165	.293	.099	.164	.642	.958	.999	.272	.540	.995	1.00	1.00
	$\mathcal{MZ}_t$	.238	.149	.186	.258	.411	.054	.073	.534	.939	.998	.128	.395	.997	1.00	1.00
	$T(\hat{\phi}-1)$	.849	.873	.928	.956	.974	.437	.568	.924	.998	1.00	.339	.659	1.00	1.00	1.00
	$\mathcal{MZ}_\alpha$	.191	.119	.163	.229	.374	.040	.049	.387	.851	.988	.084	.266	.987	1.00	1.00
	t_ϕ	.107	.313	.637	.802	.896	.060	.207	.749	.987	1.00	.058	.292	.994	1.00	1.00
24	$\mathcal{MSB}$	.161	.098	.128	.184	.321	.040	.049	.387	.851	.988	.084	.266	.987	1.00	1.00
	$\mathcal{MZ}_{t,M}$	.184	.123	.169	.244	.399	.037	.055	.494	.934	.998	.071	.309	.996	1.00	1.00
	$t_{\phi,M}$	.794	.849	.922	.954	.974	.340	.502	.911	.998	1.00	.259	.591	1.00	1.00	1.00
	$T(\hat{\phi}-1)_M$	.212	.420	.702	.841	.916	.161	.324	.827	.993	1.00	.172	.484	1.00	1.00	1.00
	$\mathcal{MZ}_{\alpha,M}$	.281	.178	.205	.269	.420	.095	.122	.590	.947	.998	.261	.540	.998	1.00	1.00
	$\mathcal{MZ}_t$	.245	.138	.122	.159	.311	.070	.072	.590	.980	1.00	.108	.304	.997	1.00	1.00
	$T(\hat{\phi}-1)$	.909	.936	.962	.975	.986	.564	.697	.984	1.00	1.00	.372	.687	1.00	1.00	1.00
	$\mathcal{MZ}_\alpha$	.184	.113	.104	.135	.276	.049	.036	.416	.919	1.00	.068	.187	.987	1.00	1.00
	t_ϕ	.110	.292	.636	.808	.898	.056	.238	.866	.998	1.00	.025	.263	.995	1.00	1.00
48	$\mathcal{MSB}$	.153	.090	.078	.101	.220	.049	.036	.416	.919	1.00	.068	.187	.987	1.00	1.00
	$\mathcal{MZ}_{t,M}$	.180	.101	.097	.136	.278	.040	.037	.512	.973	1.00	.049	.194	.997	1.00	1.00
	$t_{\phi,M}$	.843	.906	.948	.971	.982	.400	.601	.977	1.00	1.00	.237	.574	1.00	1.00	1.00
	$T(\hat{\phi}-1)_M$	.227	.424	.708	.838	.916	.194	.411	.931	.999	1.00	.135	.452	1.00	1.00	1.00
	$\mathcal{MZ}_{\alpha,M}$	.282	.162	.130	.166	.307	.124	.134	.660	.986	1.00	.279	.490	.999	1.00	1.00

Note: Case $\psi = 0$, $\boldsymbol{\phi} = (\boldsymbol{\imath}'_{n/2}, \widetilde{\boldsymbol{\phi}}'_{n/2})'$ with $(\widetilde{\boldsymbol{\phi}}_{n/2})_i \sim \mathcal{U}(.75, 1)$, $\delta = 5$. Dependence scheme 1.

Table A-11—Rejection Rates of the Tests.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.170	.131	.194	.260	.428	.042	.071	.381	.776	.933	.144	.367	.941	.999	1.00
	$T(\hat{\phi}-1)$	.593	.648	.754	.809	.864	.264	.339	.672	.910	.988	.250	.513	.962	1.00	1.00
	$\mathcal{MZ}_\alpha$	.131	.111	.168	.220	.386	.020	.035	.174	.346	.547	.114	.298	.912	.999	1.00
	t_ϕ	.094	.250	.505	.643	.755	.053	.144	.528	.845	.976	.069	.280	.912	.998	1.00
8	$\mathcal{MSB}$	.110	.101	.140	.190	.344	.022	.041	.288	.682	.881	.087	.266	.881	.998	1.00
	$\mathcal{MZ}_{t,M}$	.153	.122	.188	.260	.431	.032	.058	.376	.775	.936	.113	.331	.934	.999	1.00
	$t_{\phi,M}$	.560	.633	.752	.808	.864	.230	.318	.666	.910	.988	.209	.487	.959	1.00	1.00
	$T(\hat{\phi}-1)_M$	.160	.317	.557	.674	.779	.110	.212	.595	.876	.983	.153	.400	.938	1.00	1.00
	$\mathcal{MZ}_{\alpha,M}$	.191	.151	.214	.263	.426	.068	.095	.413	.788	.935	.228	.451	.948	.999	1.00
	$\mathcal{MZ}_t$	.188	.144	.189	.266	.402	.062	.124	.593	.927	.991	.115	.287	.898	.998	1.00
	$T(\hat{\phi}-1)$	.674	.710	.797	.844	.878	.370	.536	.878	.990	1.00	.235	.476	.936	.999	1.00
	$\mathcal{MZ}_\alpha$	.152	.108	.164	.234	.356	.041	.066	.421	.773	.948	.055	.143	.700	.975	1.00
	t_ϕ	.104	.256	.543	.679	.765	.082	.252	.752	.972	1.00	.042	.218	.856	.996	1.00
12	$\mathcal{MSB}$	.132	.088	.143	.190	.307	.042	.074	.460	.854	.977	.071	.196	.826	.994	1.00
	$\mathcal{MZ}_{t,M}$	.165	.125	.180	.259	.399	.046	.097	.571	.926	.991	.080	.244	.890	.998	1.00
	$t_{\phi,M}$	.631	.686	.786	.842	.877	.310	.500	.868	.990	1.00	.188	.437	.931	.999	1.00
	$T(\hat{\phi}-1)_M$	.198	.348	.596	.710	.787	.178	.358	.814	.980	1.00	.119	.345	.898	.998	1.00
	$\mathcal{MZ}_{\alpha,M}$	.222	.161	.208	.271	.399	.103	.168	.628	.930	.990	.200	.377	.911	.999	1.00
	$\mathcal{MZ}_t$	.191	.113	.154	.184	.320	.045	.080	.420	.794	.948	.160	.470	.989	1.00	1.00
	$T(\hat{\phi}-1)$	.720	.760	.800	.848	.880	.384	.482	.822	.957	.995	.390	.712	.998	1.00	1.00
	$\mathcal{MZ}_\alpha$	.146	.090	.132	.160	.285	.039	.074	.471	.856	.979	.088	.248	.920	1.00	1.00
	t_ϕ	.092	.239	.495	.656	.753	.058	.167	.628	.903	.988	.074	.381	.989	1.00	1.00
24	$\mathcal{MSB}$	.126	.079	.098	.126	.243	.029	.053	.312	.687	.886	.095	.349	.974	1.00	1.00
	$\mathcal{MZ}_{t,M}$	.148	.097	.141	.175	.309	.030	.058	.390	.785	.947	.088	.398	.988	1.00	1.00
	$t_{\phi,M}$	.660	.728	.788	.842	.876	.310	.424	.804	.954	.995	.297	.649	.997	1.00	1.00
	$T(\hat{\phi}-1)_M$	.176	.320	.554	.696	.778	.156	.278	.702	.924	.992	.210	.560	.994	1.00	1.00
	$\mathcal{MZ}_{\alpha,M}$	.227	.126	.163	.198	.323	.090	.121	.475	.810	.950	.320	.603	.992	1.00	1.00
	$\mathcal{MZ}_t$	.293	.188	.191	.256	.424	.049	.052	.386	.785	.950	.142	.376	.984	1.00	1.00
	$T(\hat{\phi}-1)$	.856	.882	.915	.934	.953	.437	.523	.829	.980	.998	.413	.681	.994	1.00	1.00
	$\mathcal{MZ}_\alpha$	.230	.155	.164	.224	.386	.054	.081	.520	.898	.983	.088	.264	.959	1.00	1.00
	t_ϕ	.119	.334	.659	.804	.876	.044	.165	.618	.931	.994	.046	.305	.966	1.00	1.00
48	$\mathcal{MSB}$	.200	.131	.134	.184	.325	.036	.038	.257	.670	.880	.085	.245	.959	1.00	1.00
	$\mathcal{MZ}_{t,M}$	.223	.142	.154	.228	.403	.028	.037	.317	.760	.944	.064	.261	.978	1.00	1.00
	$t_{\phi,M}$	.780	.836	.899	.929	.950	.327	.438	.798	.972	.998	.271	.592	.992	1.00	1.00
	$T(\hat{\phi}-1)_M$	.243	.459	.713	.826	.897	.146	.278	.697	.949	.996	.183	.489	.983	1.00	1.00
	$\mathcal{MZ}_{\alpha,M}$	.341	.216	.211	.267	.429	.098	.095	.443	.801	.952	.300	.542	.987	1.00	1.00

Note: Case $\psi = 0$, $\boldsymbol{\phi} = (\boldsymbol{\imath}'_{n/2}, \widetilde{\boldsymbol{\phi}}'_{n/2})'$ with $(\widetilde{\boldsymbol{\phi}}_{n/2})_i \sim \mathcal{U}(.75, 1)$, $\delta = 5$. Dependence scheme 2.

Table A-12—Rejection Rates of the Tests.

n	T	$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
		30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.218	.456	.904	.977	.990	.120	.283	.887	.996	1.00	.161	.493	.991	1.00	1.00
	$T(\hat{\phi}-1)$	.483	.694	.970	.997	.999	.338	.535	.956	.999	1.00	.295	.622	.994	1.00	1.00
	$\mathcal{MZ}_\alpha$	.182	.407	.886	.973	.988	.070	.186	.720	.949	.994	.079	.196	.780	.992	1.00
	t_ϕ	.126	.395	.922	.991	.998	.097	.285	.894	.998	1.00	.082	.374	.979	1.00	1.00
8	$\mathcal{MSB}$	.150	.338	.861	.968	.987	.072	.186	.807	.990	.999	.103	.358	.979	1.00	1.00
	$\mathcal{MZ}_{t,M}$	.188	.428	.902	.977	.990	.089	.247	.881	.997	1.00	.128	.454	.990	1.00	1.00
	$t_{\phi,M}$	.442	.674	.970	.997	.999	.292	.508	.952	.999	1.00	.253	.591	.994	1.00	1.00
	$T(\hat{\phi}-1)_M$	.226	.502	.944	.992	.998	.190	.402	.930	.999	1.00	.182	.516	.989	1.00	1.00
	$\mathcal{MZ}_{\alpha,M}$	.289	.518	.910	.978	.990	.193	.358	.898	.997	1.00	.270	.586	.992	1.00	1.00
	$\mathcal{MZ}_t$	.092	.175	.560	.815	.928	.078	.176	.737	.964	.994	.067	.157	.692	.976	1.00
	$T(\hat{\phi}-1)$	.369	.486	.780	.934	.978	.263	.423	.874	.991	1.00	.167	.294	.777	.981	.999
	$\mathcal{MZ}_\alpha$	.063	.143	.522	.794	.914	.038	.078	.388	.747	.913	.034	.066	.291	.669	.915
	t_ϕ	.042	.161	.588	.857	.952	.047	.184	.728	.975	.999	.019	.108	.596	.958	.999
12	$\mathcal{MSB}$	.052	.115	.472	.765	.891	.046	.110	.626	.932	.987	.040	.101	.561	.939	.998
	$\mathcal{MZ}_{t,M}$	.069	.147	.545	.810	.927	.051	.146	.715	.962	.994	.044	.130	.674	.975	1.00
	$t_{\phi,M}$	.316	.455	.772	.931	.978	.213	.392	.862	.991	1.00	.133	.260	.764	.979	.999
	$T(\hat{\phi}-1)_M$	.092	.248	.652	.882	.962	.126	.273	.801	.984	1.00	.068	.185	.680	.969	.999
	$\mathcal{MZ}_{\alpha,M}$	.130	.214	.582	.822	.928	.127	.241	.765	.966	.994	.120	.207	.720	.978	1.00
	$\mathcal{MZ}_t$	.167	.283	.837	.985	.999	.066	.170	.737	.989	1.00	.142	.446	.999	1.00	1.00
	$T(\hat{\phi}-1)$	.559	.695	.954	.998	1.00	.340	.492	.908	.999	1.00	.339	.708	.999	1.00	1.00
	$\mathcal{MZ}_\alpha$	.125	.236	.801	.980	.999	.088	.256	.946	1.00	1.00	.087	.275	.975	1.00	1.00
	t_ϕ	.067	.281	.852	.991	1.00	.044	.180	.751	.994	1.00	.047	.349	.994	1.00	1.00
24	$\mathcal{MSB}$	.091	.196	.750	.970	.998	.042	.106	.602	.966	.999	.097	.293	.993	1.00	1.00
	$\mathcal{MZ}_{t,M}$	.114	.233	.820	.985	.999	.042	.122	.705	.989	1.00	.084	.340	.998	1.00	1.00
	$t_{\phi,M}$	.493	.639	.948	.998	1.00	.255	.422	.897	.998	1.00	.246	.640	.999	1.00	1.00
	$T(\hat{\phi}-1)_M$	.169	.406	.894	.994	1.00	.132	.301	.835	.996	1.00	.176	.545	.998	1.00	1.00
	$\mathcal{MZ}_{\alpha,M}$	.241	.354	.861	.985	.999	.130	.257	.781	.990	1.00	.284	.602	1.00	1.00	1.00
	$\mathcal{MZ}_t$	.245	.316	.899	.999	1.00	.090	.221	.912	1.00	1.00	.142	.474	1.00	1.00	1.00
	$T(\hat{\phi}-1)$	.702	.820	.988	1.00	1.00	.454	.656	.984	1.00	1.00	.435	.818	1.00	1.00	1.00
	$\mathcal{MZ}_\alpha$	.190	.271	.862	.997	1.00	.076	.175	.891	.999	1.00	.100	.262	.991	1.00	1.00
	t_ϕ	.070	.336	.921	.998	1.00	.041	.242	.927	1.00	1.00	.050	.361	1.00	1.00	1.00
48	$\mathcal{MSB}$	.160	.219	.821	.994	1.00	.054	.136	.822	.998	1.00	.095	.315	.998	1.00	1.00
	$\mathcal{MZ}_{t,M}$	.157	.241	.874	.998	1.00	.041	.134	.880	1.00	1.00	.071	.331	1.00	1.00	1.00
	$t_{\phi,M}$	.591	.763	.984	1.00	1.00	.320	.562	.976	1.00	1.00	.286	.724	1.00	1.00	1.00
	$T(\hat{\phi}-1)_M$	.210	.488	.954	.999	1.00	.164	.423	.958	1.00	1.00	.201	.609	1.00	1.00	1.00
	$\mathcal{MZ}_{\alpha,M}$	.340	.397	.917	.999	1.00	.182	.338	.938	1.00	1.00	.334	.677	1.00	1.00	1.00

Note: Case $\psi = 0$, $\boldsymbol{\phi} = (\boldsymbol{\imath}'_{n/2}, \widetilde{\boldsymbol{\phi}}'_{n/2})'$ with $(\widetilde{\boldsymbol{\phi}}_{n/2})_i \sim \mathcal{U}(.75, 1)$, $\delta = 5$. Dependence scheme 3.

Table A-13—Rejection Rates of the Tests.

n	T	$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
		30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.134	.165	.392	.685	.911	.169	.186	.333	.551	.778	.370	.222	.150	.137	.278
	$T(\hat{\phi}-1)$	.201	.189	.290	.454	.774	.107	.130	.206	.350	.588	.022	.050	.049	.083	.125
	$\mathcal{MZ}_\alpha$	.089	.117	.292	.551	.824	.126	.127	.226	.390	.611	.346	.196	.126	.107	.236
	t_ϕ	.046	.075	.191	.388	.725	.056	.087	.170	.310	.546	.022	.033	.039	.064	.107
8	$\mathcal{MSB}$	.089	.117	.292	.551	.824	.126	.127	.226	.390	.611	.325	.175	.106	.077	.188
	$\mathcal{MZ}_{t,M}$	.101	.149	.384	.684	.911	.138	.167	.324	.550	.780	.338	.201	.146	.137	.281
	$t_{\phi,M}$	.177	.179	.284	.454	.775	.091	.116	.204	.349	.593	.017	.046	.047	.083	.126
	$T(\hat{\phi}-1)_M$	.082	.114	.237	.447	.767	.104	.130	.218	.360	.594	.044	.051	.051	.082	.130
	$\mathcal{MZ}_{\alpha,M}$	.220	.214	.420	.689	.904	.264	.248	.356	.548	.762	.440	.258	.160	.140	.272
	$\mathcal{MZ}_t$	.154	.213	.547	.871	.991	.156	.158	.280	.436	.670	.578	.366	.313	.246	.460
	$T(\hat{\phi}-1)$	.254	.276	.413	.630	.923	.123	.120	.194	.304	.511	.026	.060	.096	.145	.209
	$\mathcal{MZ}_\alpha$	.099	.142	.385	.726	.948	.113	.104	.180	.298	.503	.547	.330	.272	.197	.410
	t_ϕ	.052	.092	.258	.533	.886	.060	.077	.147	.258	.458	.029	.052	.073	.110	.183
12	$\mathcal{MSB}$	.099	.142	.385	.726	.948	.113	.104	.180	.298	.503	.512	.293	.219	.147	.346
	$\mathcal{MZ}_{t,M}$	.102	.179	.526	.866	.990	.117	.127	.260	.425	.669	.534	.336	.297	.239	.457
	$t_{\phi,M}$	.224	.251	.397	.620	.922	.102	.104	.186	.296	.507	.018	.052	.087	.138	.207
	$T(\hat{\phi}-1)_M$	.116	.147	.324	.600	.917	.119	.117	.193	.299	.510	.066	.079	.098	.148	.216
	$\mathcal{MZ}_{\alpha,M}$	.268	.288	.580	.870	.987	.255	.217	.304	.445	.662	.668	.412	.334	.249	.458
	$\mathcal{MZ}_t$	.178	.270	.754	.993	1.00	.181	.143	.242	.379	.589	.701	.426	.254	.161	.411
	$T(\hat{\phi}-1)$	.395	.396	.622	.879	.998	.166	.140	.202	.304	.502	.030	.054	.077	.126	.201
	$\mathcal{MZ}_\alpha$	.126	.182	.578	.958	1.00	.141	.104	.156	.249	.412	.675	.399	.225	.134	.362
	t_ϕ	.058	.114	.401	.791	.996	.055	.076	.149	.246	.438	.031	.035	.058	.093	.176
24	$\mathcal{MSB}$	.126	.182	.578	.958	1.00	.141	.104	.156	.249	.412	.646	.371	.180	.094	.292
	$\mathcal{MZ}_{t,M}$	.108	.207	.729	.992	1.00	.129	.107	.210	.362	.583	.642	.377	.229	.150	.396
	$t_{\phi,M}$	.335	.347	.591	.870	.998	.122	.119	.185	.286	.494	.018	.039	.069	.114	.191
	$T(\hat{\phi}-1)_M$	.150	.193	.492	.860	.998	.141	.132	.192	.294	.497	.068	.072	.080	.116	.205
	$\mathcal{MZ}_{\alpha,M}$	.364	.410	.795	.994	1.00	.294	.218	.283	.402	.590	.795	.504	.286	.177	.412
	$\mathcal{MZ}_t$	.145	.202	.536	.945	.998	.219	.152	.232	.376	.648	.887	.544	.298	.143	.443
	$T(\hat{\phi}-1)$	.510	.444	.548	.727	.988	.213	.178	.244	.360	.602	.029	.061	.084	.130	.226
	$\mathcal{MZ}_\alpha$	.098	.126	.370	.800	.989	.178	.104	.139	.236	.440	.869	.515	.265	.112	.384
	t_ϕ	.044	.104	.277	.576	.960	.064	.087	.170	.278	.529	.034	.041	.057	.092	.192
48	$\mathcal{MSB}$	.098	.126	.370	.800	.989	.178	.104	.139	.236	.440	.838	.470	.212	.080	.300
	$\mathcal{MZ}_{t,M}$	.068	.118	.461	.931	.998	.141	.093	.177	.329	.614	.824	.473	.252	.109	.410
	$t_{\phi,M}$	.398	.360	.495	.689	.985	.142	.130	.208	.323	.576	.013	.035	.064	.106	.204
	$T(\hat{\phi}-1)_M$	.142	.174	.371	.664	.982	.171	.150	.222	.335	.582	.084	.081	.083	.112	.212
	$\mathcal{MZ}_{\alpha,M}$	.343	.329	.608	.951	.998	.393	.238	.281	.412	.650	.939	.625	.338	.153	.447

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \widetilde{\boldsymbol{\phi}}_n$ with $(\widetilde{\boldsymbol{\phi}}_n)_i \sim \mathcal{U}(.9, 1)$, $\delta = 0.2$. Dependence scheme 1.

Table A-14—Rejection Rates of the Tests.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.180	.248	.587	.860	.973	.141	.140	.225	.361	.486	.425	.311	.271	.245	.398
	$T(\hat{\phi}-1)$	.222	.236	.420	.641	.896	.097	.106	.145	.248	.360	.026	.061	.086	.130	.178
	$\mathcal{MZ}_\alpha$	.130	.169	.363	.641	.834	.099	.098	.123	.175	.233	.397	.281	.237	.206	.352
	t_ϕ	.050	.103	.312	.581	.868	.044	.070	.117	.215	.328	.024	.042	.071	.110	.162
8	$\mathcal{MSB}$	.124	.172	.453	.752	.931	.102	.090	.150	.259	.355	.370	.249	.197	.156	.298
	$\mathcal{MZ}_{t,M}$	.146	.224	.578	.860	.974	.115	.121	.215	.360	.488	.392	.292	.266	.244	.401
	$t_{\phi,M}$	.198	.222	.416	.640	.897	.076	.096	.141	.247	.362	.021	.051	.084	.130	.180
	$T(\hat{\phi}-1)_M$	.108	.140	.363	.631	.892	.091	.106	.148	.246	.362	.055	.082	.098	.134	.187
	$\mathcal{MZ}_{\alpha,M}$	.297	.312	.604	.856	.968	.214	.187	.240	.363	.478	.504	.355	.288	.245	.391
	$\mathcal{MZ}_t$	.127	.187	.416	.670	.880	.166	.170	.308	.461	.634	.518	.341	.279	.227	.404
	$T(\hat{\phi}-1)$	.244	.235	.316	.453	.748	.118	.123	.206	.313	.502	.031	.055	.092	.122	.199
	$\mathcal{MZ}_\alpha$	.169	.236	.568	.855	.968	.146	.153	.249	.442	.633	.490	.316	.237	.186	.360
	t_ϕ	.051	.079	.196	.376	.693	.056	.084	.160	.270	.455	.025	.043	.066	.097	.171
12	$\mathcal{MSB}$	.084	.122	.307	.535	.777	.120	.113	.213	.333	.485	.461	.280	.196	.148	.296
	$\mathcal{MZ}_{t,M}$	.094	.157	.400	.666	.878	.129	.142	.297	.455	.633	.468	.314	.267	.220	.402
	$t_{\phi,M}$	.213	.216	.303	.449	.747	.095	.108	.199	.307	.501	.022	.044	.085	.119	.197
	$T(\hat{\phi}-1)_M$	.098	.127	.249	.437	.732	.112	.126	.206	.309	.500	.055	.078	.091	.126	.200
	$\mathcal{MZ}_{\alpha,M}$	.229	.253	.447	.674	.876	.276	.225	.333	.463	.620	.599	.388	.298	.231	.398
	$\mathcal{MZ}_t$	.163	.211	.530	.819	.961	.171	.149	.254	.434	.592	.668	.431	.319	.220	.444
	$T(\hat{\phi}-1)$	.346	.316	.433	.642	.900	.163	.139	.206	.334	.505	.030	.068	.094	.142	.214
	$\mathcal{MZ}_\alpha$	.128	.162	.434	.721	.905	.160	.139	.219	.378	.530	.645	.400	.279	.184	.399
	t_ϕ	.051	.098	.279	.558	.862	.057	.080	.146	.276	.453	.032	.046	.072	.114	.189
24	$\mathcal{MSB}$	.114	.144	.398	.720	.912	.132	.103	.178	.308	.448	.626	.369	.230	.134	.341
	$\mathcal{MZ}_{t,M}$	.102	.161	.500	.810	.961	.126	.116	.230	.420	.586	.614	.386	.288	.204	.436
	$t_{\phi,M}$	.289	.281	.412	.633	.897	.122	.114	.182	.320	.501	.017	.050	.085	.134	.209
	$T(\hat{\phi}-1)_M$	.138	.161	.347	.610	.890	.140	.135	.191	.324	.502	.070	.092	.100	.139	.218
	$\mathcal{MZ}_{\alpha,M}$	.310	.304	.564	.821	.960	.296	.223	.294	.451	.588	.757	.507	.358	.233	.449
	$\mathcal{MZ}_t$	.164	.188	.475	.776	.932	.214	.150	.205	.356	.508	.768	.465	.253	.140	.379
	$T(\hat{\phi}-1)$	.430	.406	.474	.603	.874	.196	.183	.222	.320	.473	.024	.056	.076	.120	.196
	$\mathcal{MZ}_\alpha$	.150	.211	.546	.814	.956	.176	.114	.146	.248	.346	.755	.442	.228	.112	.338
	t_ϕ	.053	.105	.278	.501	.834	.055	.087	.161	.257	.424	.028	.039	.054	.076	.150
48	$\mathcal{MSB}$	.115	.129	.343	.650	.860	.170	.105	.132	.229	.362	.732	.403	.192	.076	.272
	$\mathcal{MZ}_{t,M}$	.086	.127	.420	.753	.929	.138	.100	.165	.320	.491	.706	.404	.218	.119	.356
	$t_{\phi,M}$	.350	.347	.436	.574	.866	.135	.134	.192	.290	.452	.012	.036	.058	.101	.172
	$T(\hat{\phi}-1)_M$	.150	.184	.356	.561	.857	.160	.149	.204	.300	.465	.074	.077	.079	.101	.170
	$\mathcal{MZ}_{\alpha,M}$	.326	.294	.520	.790	.931	.352	.229	.247	.376	.513	.847	.546	.282	.150	.381

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \widetilde{\boldsymbol{\phi}}_n$ with $(\widetilde{\boldsymbol{\phi}}_n)_i \sim \mathcal{U}(.9, 1)$, $\delta = 0.2$. Dependence scheme 2.

Table A-15—Rejection Rates of the Tests.

n	T	$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
		30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.140	.191	.474	.805	.965	.148	.166	.323	.524	.756	.419	.316	.364	.368	.557
	$T(\hat{\phi}-1)$	.229	.221	.336	.547	.865	.120	.138	.217	.357	.588	.038	.085	.138	.204	.298
	$\mathcal{MZ}_\alpha$	.131	.237	.600	.919	.997	.139	.148	.264	.450	.664	.402	.286	.320	.320	.510
	t_ϕ	.050	.084	.240	.478	.826	.061	.099	.172	.316	.553	.028	.060	.109	.180	.278
8	$\mathcal{MSB}$	.094	.127	.353	.656	.895	.104	.110	.222	.380	.609	.382	.252	.266	.258	.442
	$\mathcal{MZ}_{t,M}$	.106	.171	.467	.805	.966	.122	.140	.312	.523	.758	.396	.293	.353	.367	.562
	$t_{\phi,M}$	.208	.204	.326	.546	.868	.100	.130	.211	.357	.590	.029	.077	.134	.203	.302
	$T(\hat{\phi}-1)_M$	.105	.136	.292	.538	.860	.121	.143	.217	.360	.594	.060	.099	.140	.213	.306
	$\mathcal{MZ}_{\alpha,M}$	.239	.248	.503	.802	.962	.229	.220	.349	.530	.742	.498	.371	.388	.377	.554
	$\mathcal{MZ}_t$	.108	.126	.306	.510	.756	.128	.121	.181	.280	.393	.535	.358	.368	.357	.620
	$T(\hat{\phi}-1)$	.262	.204	.244	.326	.605	.130	.117	.146	.206	.310	.041	.095	.140	.224	.344
	$\mathcal{MZ}_\alpha$	.121	.169	.411	.718	.940	.125	.124	.210	.367	.543	.504	.322	.324	.297	.560
	t_ϕ	.053	.076	.150	.266	.540	.050	.067	.108	.166	.268	.029	.050	.104	.173	.304
12	$\mathcal{MSB}$	.066	.078	.209	.376	.594	.095	.079	.108	.177	.272	.469	.280	.269	.230	.478
	$\mathcal{MZ}_{t,M}$	.071	.102	.290	.501	.753	.099	.096	.162	.268	.392	.484	.327	.358	.349	.618
	$t_{\phi,M}$	.224	.187	.233	.320	.600	.102	.101	.135	.203	.311	.031	.079	.130	.218	.339
	$T(\hat{\phi}-1)_M$	.112	.117	.184	.316	.594	.113	.106	.144	.204	.308	.068	.100	.140	.215	.352
	$\mathcal{MZ}_{\alpha,M}$	.201	.194	.340	.519	.746	.206	.176	.203	.290	.390	.631	.414	.398	.366	.615
	$\mathcal{MZ}_t$	.143	.218	.612	.950	.999	.205	.184	.352	.610	.878	.648	.411	.325	.308	.580
	$T(\hat{\phi}-1)$	.408	.367	.491	.752	.986	.189	.191	.299	.468	.742	.039	.084	.120	.210	.336
	$\mathcal{MZ}_\alpha$	.119	.174	.525	.882	.995	.144	.121	.207	.331	.539	.616	.383	.282	.252	.526
	t_ϕ	.063	.127	.300	.651	.971	.075	.114	.230	.404	.684	.024	.043	.083	.156	.280
24	$\mathcal{MSB}$	.098	.149	.441	.846	.990	.150	.125	.238	.435	.697	.590	.349	.227	.194	.438
	$\mathcal{MZ}_{t,M}$	.086	.164	.580	.946	.999	.136	.137	.314	.592	.878	.588	.361	.293	.286	.571
	$t_{\phi,M}$	.338	.314	.458	.737	.986	.138	.160	.274	.452	.736	.022	.063	.103	.193	.322
	$T(\hat{\phi}-1)_M$	.153	.208	.396	.732	.983	.167	.175	.295	.471	.738	.067	.080	.120	.197	.325
	$\mathcal{MZ}_{\alpha,M}$	.313	.324	.675	.956	.999	.344	.274	.418	.633	.869	.754	.481	.371	.323	.588
	$\mathcal{MZ}_t$	.143	.200	.544	.934	.998	.213	.166	.264	.496	.762	.824	.528	.388	.281	.694
	$T(\hat{\phi}-1)$	.513	.459	.544	.744	.984	.240	.229	.301	.452	.695	.045	.085	.143	.223	.395
	$\mathcal{MZ}_\alpha$	.141	.166	.536	.930	1.00	.184	.133	.194	.328	.564	.803	.487	.342	.226	.619
	t_ϕ	.062	.113	.330	.615	.967	.073	.112	.209	.358	.604	.020	.041	.088	.163	.329
48	$\mathcal{MSB}$	.095	.120	.374	.789	.984	.161	.114	.168	.314	.536	.777	.440	.278	.166	.516
	$\mathcal{MZ}_{t,M}$	.061	.113	.466	.922	.998	.130	.106	.206	.443	.742	.757	.442	.327	.234	.662
	$t_{\phi,M}$	.414	.380	.489	.706	.983	.159	.169	.250	.412	.670	.021	.059	.112	.190	.362
	$T(\hat{\phi}-1)_M$	.169	.200	.417	.701	.983	.188	.187	.266	.419	.666	.067	.089	.125	.193	.374
	$\mathcal{MZ}_{\alpha,M}$	.329	.328	.620	.944	.998	.382	.262	.316	.524	.769	.896	.621	.437	.308	.698

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \widetilde{\boldsymbol{\phi}}_n$ with $(\widetilde{\boldsymbol{\phi}}_n)_i \sim \mathcal{U}(.9, 1)$, $\delta = 0.2$. Dependence scheme 3.

Table A-16—Rejection Rates of the Tests.

n	T	$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
		30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.122	.094	.121	.189	.308	.024	.032	.106	.185	.281	.082	.167	.640	.974	1.00
	$T(\hat{\phi}-1)$	.731	.740	.783	.830	.870	.210	.251	.321	.460	.628	.203	.350	.783	.990	1.00
	$\mathcal{MZ}_\alpha$	.094	.071	.098	.159	.264	.012	.016	.068	.118	.181	.050	.100	.469	.905	.996
	t_ϕ	.086	.202	.412	.576	.664	.023	.057	.138	.262	.431	.039	.126	.567	.953	1.00
8	$\mathcal{MSB}$	.085	.055	.080	.122	.222	.012	.016	.068	.118	.181	.050	.100	.469	.905	.996
	$\mathcal{MZ}_{t,M}$	.108	.085	.119	.189	.311	.018	.029	.101	.185	.283	.064	.143	.627	.974	1.00
	$t_{\phi,M}$	.682	.718	.779	.830	.872	.181	.230	.314	.458	.631	.171	.322	.775	.990	1.00
	$T(\hat{\phi}-1)_M$	.151	.264	.468	.613	.698	.049	.093	.171	.299	.476	.105	.198	.645	.969	1.00
	$\mathcal{MZ}_{\alpha,M}$	.146	.105	.125	.193	.299	.040	.044	.116	.195	.277	.142	.229	.659	.971	.999
	$\mathcal{MZ}_t$	.093	.057	.066	.087	.169	.036	.037	.148	.300	.471	.060	.092	.306	.720	.954
	$T(\hat{\phi}-1)$	.690	.723	.732	.746	.780	.289	.288	.463	.657	.844	.164	.216	.507	.849	.987
	$\mathcal{MZ}_\alpha$	.062	.041	.050	.070	.133	.022	.019	.092	.199	.324	.032	.059	.200	.543	.860
	t_ϕ	.066	.146	.305	.416	.523	.031	.070	.213	.427	.665	.019	.058	.266	.684	.947
12	$\mathcal{MSB}$	.048	.034	.038	.055	.104	.022	.019	.092	.199	.324	.032	.059	.200	.543	.860
	$\mathcal{MZ}_{t,M}$	.074	.049	.059	.082	.166	.026	.029	.136	.292	.468	.035	.078	.281	.710	.954
	$t_{\phi,M}$	.637	.694	.723	.740	.778	.235	.260	.446	.649	.844	.124	.187	.492	.844	.987
	$T(\hat{\phi}-1)_M$	.120	.210	.361	.454	.554	.082	.118	.276	.483	.719	.058	.108	.351	.750	.963
	$\mathcal{MZ}_{\alpha,M}$	.102	.058	.066	.088	.169	.055	.058	.165	.308	.464	.104	.130	.340	.719	.948
	$\mathcal{MZ}_t$	.118	.054	.046	.072	.127	.032	.029	.147	.280	.474	.046	.083	.381	.866	.997
	$T(\hat{\phi}-1)$	.802	.833	.823	.842	.873	.357	.372	.552	.760	.934	.210	.286	.642	.964	1.00
	$\mathcal{MZ}_\alpha$	.072	.038	.035	.056	.108	.021	.016	.096	.191	.318	.027	.047	.252	.695	.963
	t_ϕ	.063	.173	.353	.466	.594	.030	.071	.268	.491	.787	.018	.064	.357	.848	.996
24	$\mathcal{MSB}$	.058	.028	.027	.043	.084	.021	.016	.096	.191	.318	.027	.047	.252	.695	.963
	$\mathcal{MZ}_{t,M}$	.086	.042	.040	.064	.121	.022	.019	.126	.263	.457	.024	.056	.338	.858	.997
	$t_{\phi,M}$	.720	.800	.808	.829	.867	.285	.315	.524	.748	.930	.146	.230	.608	.960	1.00
	$T(\hat{\phi}-1)_M$	.125	.238	.416	.523	.632	.088	.134	.342	.556	.837	.066	.122	.460	.902	.998
	$\mathcal{MZ}_{\alpha,M}$	.132	.061	.049	.072	.122	.056	.051	.171	.300	.476	.108	.140	.440	.881	.997
	$\mathcal{MZ}_t$	.209	.076	.059	.083	.169	.036	.025	.123	.244	.387	.064	.084	.529	.982	1.00
	$T(\hat{\phi}-1)$	.946	.962	.958	.970	.980	.454	.444	.586	.796	.952	.273	.366	.844	1.00	1.00
	$\mathcal{MZ}_\alpha$	.149	.057	.045	.063	.136	.022	.016	.074	.150	.235	.035	.043	.351	.893	1.00
	t_ϕ	.081	.241	.530	.687	.801	.027	.073	.226	.476	.783	.016	.065	.489	.975	1.00
48	$\mathcal{MSB}$	.125	.044	.035	.043	.097	.022	.016	.074	.150	.235	.035	.043	.351	.893	1.00
	$\mathcal{MZ}_{t,M}$	.143	.051	.045	.067	.141	.018	.016	.091	.203	.354	.026	.040	.452	.977	1.00
	$t_{\phi,M}$	.892	.942	.947	.964	.977	.325	.350	.526	.768	.944	.167	.273	.802	1.00	1.00
	$T(\hat{\phi}-1)_M$	.198	.354	.600	.739	.836	.086	.145	.306	.550	.834	.066	.152	.623	.988	1.00
	$\mathcal{MZ}_{\alpha,M}$	.248	.084	.060	.083	.166	.071	.041	.142	.264	.395	.149	.157	.602	.985	1.00

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \widetilde{\boldsymbol{\phi}}_n$ with $(\widetilde{\boldsymbol{\phi}}_n)_i \sim \mathcal{U}(.9, 1)$, $\delta = 5$. Dependence scheme 1.

Table A-17—Rejection Rates of the Tests.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.094	.053	.066	.106	.165	.029	.043	.174	.326	.492	.052	.073	.228	.490	.716
	$T(\hat{\phi}-1)$	.560	.585	.592	.611	.666	.258	.278	.434	.583	.776	.128	.173	.337	.627	.815
	$\mathcal{MZ}_\alpha$	.059	.038	.051	.088	.141	.025	.036	.156	.300	.458	.052	.084	.325	.672	.900
	t_ϕ	.060	.137	.262	.352	.438	.040	.089	.230	.418	.635	.016	.055	.200	.481	.719
8	$\mathcal{MSB}$	.050	.030	.040	.068	.112	.015	.026	.106	.226	.353	.031	.044	.158	.375	.585
	$\mathcal{MZ}_{t,M}$	.080	.045	.062	.106	.166	.023	.038	.166	.325	.493	.041	.062	.221	.488	.718
	$t_{\phi,M}$	.516	.570	.587	.610	.666	.230	.263	.427	.584	.778	.109	.161	.330	.623	.816
	$T(\hat{\phi}-1)_M$	.104	.187	.293	.387	.469	.085	.128	.290	.462	.672	.050	.092	.248	.535	.754
	$\mathcal{MZ}_{\alpha,M}$	.101	.060	.066	.108	.163	.054	.070	.184	.330	.479	.089	.100	.240	.497	.705
	$\mathcal{MZ}_t$	.096	.047	.055	.078	.142	.030	.039	.132	.277	.407	.066	.121	.425	.786	.944
	$T(\hat{\phi}-1)$	.569	.604	.622	.646	.676	.237	.266	.407	.559	.733	.174	.266	.581	.866	.972
	$\mathcal{MZ}_\alpha$	.064	.031	.040	.060	.118	.026	.038	.166	.340	.503	.041	.076	.246	.544	.756
	t_ϕ	.055	.132	.252	.365	.437	.024	.060	.197	.384	.592	.025	.082	.382	.763	.940
12	$\mathcal{MSB}$	.052	.028	.028	.046	.093	.016	.020	.087	.186	.291	.036	.078	.299	.672	.889
	$\mathcal{MZ}_{t,M}$	.080	.040	.050	.074	.140	.022	.029	.123	.269	.403	.045	.100	.406	.781	.944
	$t_{\phi,M}$	.523	.582	.614	.640	.674	.194	.233	.395	.552	.732	.144	.240	.568	.861	.973
	$T(\hat{\phi}-1)_M$	.110	.184	.296	.397	.470	.071	.103	.255	.436	.633	.070	.149	.458	.799	.950
	$\mathcal{MZ}_{\alpha,M}$	.103	.051	.057	.076	.141	.052	.059	.145	.282	.401	.116	.163	.450	.784	.941
	$\mathcal{MZ}_t$	.144	.064	.065	.090	.178	.034	.036	.150	.342	.516	.055	.098	.376	.738	.925
	$T(\hat{\phi}-1)$	.721	.761	.748	.778	.810	.334	.367	.514	.711	.860	.192	.278	.586	.856	.966
	$\mathcal{MZ}_\alpha$	.100	.045	.048	.072	.148	.022	.027	.117	.209	.326	.044	.064	.296	.664	.886
	t_ϕ	.071	.194	.378	.507	.584	.039	.087	.260	.510	.734	.016	.063	.369	.721	.932
24	$\mathcal{MSB}$	.076	.035	.035	.058	.114	.019	.022	.095	.238	.362	.029	.058	.271	.616	.858
	$\mathcal{MZ}_{t,M}$	.114	.054	.054	.084	.168	.020	.022	.128	.328	.515	.032	.068	.354	.728	.924
	$t_{\phi,M}$	.650	.729	.733	.773	.803	.266	.317	.485	.701	.857	.138	.231	.564	.850	.966
	$T(\hat{\phi}-1)_M$	.137	.263	.429	.548	.614	.099	.159	.334	.571	.778	.061	.128	.450	.778	.946
	$\mathcal{MZ}_{\alpha,M}$	.164	.077	.072	.092	.177	.064	.061	.173	.362	.513	.114	.156	.415	.749	.922
	$\mathcal{MZ}_t$	.178	.070	.058	.068	.144	.033	.031	.099	.220	.324	.058	.085	.405	.774	.955
	$T(\hat{\phi}-1)$	.796	.834	.828	.834	.858	.389	.358	.453	.622	.767	.244	.307	.639	.900	.984
	$\mathcal{MZ}_\alpha$	.123	.052	.044	.051	.118	.032	.026	.108	.257	.390	.040	.061	.347	.706	.917
	t_ϕ	.084	.216	.426	.569	.653	.025	.068	.188	.379	.598	.018	.062	.380	.777	.960
48	$\mathcal{MSB}$	.103	.040	.028	.039	.086	.020	.018	.058	.148	.210	.032	.044	.281	.658	.894
	$\mathcal{MZ}_{t,M}$	.120	.048	.042	.051	.123	.018	.018	.072	.197	.300	.022	.048	.359	.758	.952
	$t_{\phi,M}$	.716	.795	.804	.820	.850	.287	.281	.404	.590	.750	.158	.231	.594	.888	.983
	$T(\hat{\phi}-1)_M$	.176	.302	.483	.606	.682	.088	.132	.247	.434	.644	.069	.138	.476	.828	.968
	$\mathcal{MZ}_{\alpha,M}$	.203	.085	.060	.067	.141	.068	.052	.117	.234	.327	.126	.142	.455	.786	.954

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \widetilde{\boldsymbol{\phi}}_n$ with $(\widetilde{\boldsymbol{\phi}}_n)_i \sim \mathcal{U}(.9, 1)$, $\delta = 5$. Dependence scheme 2.

Table A-18—Rejection Rates of the Tests.

		$\tau = 0.1$					$\tau = 0.5$					$\tau = 0.9$				
n	T	30	50	100	150	200	30	50	100	150	200	30	50	100	150	200
	$\mathcal{MZ}_t$	.080	.106	.312	.580	.842	.047	.079	.317	.644	.855	.074	.130	.473	.860	.989
	$T(\hat{\phi}-1)$	.364	.438	.630	.809	.946	.217	.267	.516	.788	.943	.154	.240	.599	.904	.993
	$\mathcal{MZ}_\alpha$	.059	.081	.272	.528	.804	.030	.072	.245	.553	.777	.054	.125	.518	.915	.997
	t_ϕ	.042	.118	.379	.640	.868	.033	.076	.308	.638	.884	.028	.090	.398	.808	.981
8	$\mathcal{MSB}$	.042	.065	.222	.460	.748	.027	.049	.221	.498	.755	.040	.083	.342	.742	.964
	$\mathcal{MZ}_{t,M}$	.065	.093	.303	.578	.844	.035	.068	.311	.644	.856	.059	.115	.458	.860	.989
	$t_{\phi,M}$	.333	.422	.622	.809	.946	.184	.243	.505	.788	.944	.132	.222	.591	.904	.994
	$T(\hat{\phi}-1)_M$	.082	.173	.438	.682	.891	.077	.133	.375	.682	.909	.068	.142	.481	.850	.984
	$\mathcal{MZ}_{\alpha,M}$	.100	.132	.329	.583	.834	.083	.107	.340	.646	.848	.126	.177	.496	.863	.988
	$\mathcal{MZ}_t$	.058	.071	.191	.416	.656	.040	.069	.295	.592	.843	.066	.124	.486	.893	.994
	$T(\hat{\phi}-1)$	.364	.405	.550	.730	.854	.233	.288	.539	.791	.951	.187	.256	.642	.936	.997
	$\mathcal{MZ}_\alpha$	.034	.052	.156	.353	.603	.047	.075	.347	.749	.939	.043	.094	.359	.795	.976
	t_ϕ	.026	.080	.264	.481	.723	.027	.078	.296	.609	.874	.026	.076	.390	.831	.988
12	$\mathcal{MSB}$	.025	.043	.122	.297	.537	.022	.039	.194	.436	.711	.040	.076	.339	.769	.969
	$\mathcal{MZ}_{t,M}$	.038	.060	.180	.409	.653	.025	.056	.282	.586	.840	.044	.098	.465	.888	.994
	$t_{\phi,M}$	.308	.375	.536	.726	.854	.196	.258	.521	.787	.952	.148	.225	.629	.935	.996
	$T(\hat{\phi}-1)_M$	.066	.126	.322	.529	.752	.076	.128	.371	.672	.903	.072	.142	.492	.886	.991
	$\mathcal{MZ}_{\alpha,M}$	.086	.087	.206	.412	.649	.081	.102	.320	.597	.832	.128	.167	.516	.889	.994
	$\mathcal{MZ}_t$	.069	.059	.152	.357	.619	.050	.074	.356	.746	.943	.056	.107	.470	.894	.997
	$T(\hat{\phi}-1)$	.448	.482	.605	.757	.878	.312	.378	.670	.918	.990	.192	.291	.662	.954	1.00
	$\mathcal{MZ}_\alpha$	.049	.041	.125	.312	.563	.039	.064	.350	.728	.947	.043	.078	.344	.776	.978
	t_ϕ	.030	.078	.266	.503	.721	.028	.092	.391	.778	.963	.021	.070	.394	.857	.995
24	$\mathcal{MSB}$	.038	.032	.102	.254	.499	.033	.048	.250	.606	.855	.036	.062	.322	.765	.978
	$\mathcal{MZ}_{t,M}$	.049	.044	.138	.346	.615	.028	.052	.328	.736	.942	.033	.072	.426	.888	.997
	$t_{\phi,M}$	.372	.436	.584	.749	.876	.237	.325	.648	.915	.989	.131	.244	.632	.952	1.00
	$T(\hat{\phi}-1)_M$	.075	.129	.321	.563	.756	.098	.163	.492	.836	.977	.058	.144	.502	.901	.998
	$\mathcal{MZ}_{\alpha,M}$	.095	.080	.177	.370	.622	.102	.119	.407	.759	.943	.119	.176	.526	.900	.997
	$\mathcal{MZ}_t$	.074	.060	.153	.378	.636	.046	.070	.340	.754	.958	.048	.078	.334	.814	.986
	$T(\hat{\phi}-1)$	.562	.641	.721	.850	.928	.379	.457	.708	.941	.996	.241	.294	.623	.934	.998
	$\mathcal{MZ}_\alpha$	.050	.044	.125	.336	.580	.033	.037	.205	.520	.807	.043	.093	.482	.934	1.00
	t_ϕ	.024	.096	.300	.564	.785	.025	.095	.383	.792	.986	.011	.047	.303	.777	.988
48	$\mathcal{MSB}$	.037	.032	.097	.278	.509	.028	.038	.230	.593	.880	.026	.042	.211	.630	.946
	$\mathcal{MZ}_{t,M}$	.044	.037	.127	.353	.612	.022	.040	.294	.726	.955	.023	.043	.272	.783	.985
	$t_{\phi,M}$	.451	.576	.680	.830	.918	.267	.364	.664	.932	.996	.153	.214	.558	.924	.998
	$T(\hat{\phi}-1)_M$	.087	.158	.359	.618	.818	.098	.185	.483	.854	.989	.058	.113	.406	.846	.992
	$\mathcal{MZ}_{\alpha,M}$	.111	.082	.170	.390	.632	.098	.115	.396	.774	.959	.115	.141	.396	.834	.985

Note: Case $\psi = 0$, $\boldsymbol{\phi} = \widetilde{\boldsymbol{\phi}}_n$ with $(\widetilde{\boldsymbol{\phi}}_n)_i \sim \mathcal{U}(.9, 1)$, $\delta = 5$. Dependence scheme 3.